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## Abstract

The fiscal theory of the price level (FTPL) posits that the price-level adjusts to ensure the Government's budget equation is met in equilibrium, but is silent on the exact adjustment mechanism. By modelling the Government as a large, satiable player in a game, we demonstrate that the FTPL's outcome can be understood as a 'dividend equilibrium', achieved via price level driven revaluation of initial debt. It coincides with the Core (ensuring stability) and the unique outcome consistent with players receiving their Shapley Value. This provides a formal foundation for the possibility of non-Ricardian fiscal policies, central to the FTPL.

**Key words:** D51, E31, E62.

**JEL classification:** The Core, Shapley Value, the fiscal theory of the price level.

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# 1 Introduction

The Fiscal Theory of the Price Level (Leeper, 1991; Sims, 1994; Woodford, 1994; Cochrane, 2023), or FTPL for short, remains beset by a difficulty which it is ill-placed to solve: it is an equilibrium theory relying on an exogenous disequilibrium adjustment. Given a budget-infeasible fiscal policy, the price level adjusts to implement the only competitive equilibrium that is affordable, meaning that the government can violate its budget equation out of equilibrium. But no explicit mechanism is provided by which prices may be expected to adjust in this way.

Of course, a variant of this criticism may be leveled more generally at the Walrasian auctioneer who benevolently steers the economy to competitive equilibrium. A celebrated response to this contrivance is provided by the game-theoretic concepts of the Core and the Shapley Value, which uniquely select competitive equilibrium as the economy becomes large. This paper shows that these concepts can be marshaled to provide an equally compelling foundation for the FTPL.[1]

Edgeworth's conjecture—subsequently formalized in the Core Convergence Theorem (Debreu and Scarf, 1963) and the Core Equivalence Theorem (Aumann, 1964)—is the leading game-theoretic foundation for competitive equilibrium. Aumann's result in particular gives the identity of the Core with Walrasian allocations when an exchange economy is composed of an "atomless" set of traders. The atomless assumption here formalizes the idea, intrinsic to perfect competition, that each individual trader is negligible.

The Value Principle describes the parallel result that, in a perfectly competitive economy, every allocation consistent with the Shapley Value is a competitive equilibrium, and sufficiently differentiable utilities yield Value Equivalence with Walrasian equilibria (Shapley, 1964; Aumann and Shapley, 1974). In practice, however, markets are likely to include "large" traders who may not be treated as negligible, for instance those possessing a large fraction of the endowment of particular goods (e.g. monopolists), common interests (e.g. trade unions), or the power to levy taxes (government)—the latter being a key ingredient to the FTPL.

One advantage of both the Core and the Shapley Value is that their definition does not require the price-taking behavior characteristic of the Walrasian approach; rather, under perfect competition, they provide a foundation for it. But the two concepts also admit an imperfectly competitive formulation, where a large trader is represented by an atom

[1] In doing so, we also respond to a call by Dogra (2024), who points to limitations inherent to equilibrium models and argues in favor of "process models" that are more explicit in specifying how endogenous variables come about.

(i.e. a non-null subset of traders who act as an indivisible collective). Core Equivalence need not hold in such a "mixed" market, since a large trader may be able budgetarily to exploit other traders—in the sense that its consumption bundle may be worth more than its endowment (Shitovitz, 1973). If large traders are similar to one another, or to enough small traders, then Core Equivalence persists in a mixed market (Shitovitz, 1973; Gabszewicz and Mertens, 1971), but this fails if a single large trader is the only one of its type, as is the case for a government. Moreover, a competitive equilibrium need not even exist when modeling the government in a realistic way—moving away from treating it as maximizing its own consumption, and endowing it with a bliss point instead (Aumann and Drèze, 1986).[2]

Nonetheless, this paper shows that the allocation consistent with the Shapley Value (where all players are rewarded in line with their marginal contribution to the economy)[3] implies a price-driven revaluation of initial debt that—surprisingly—is of exactly the size envisioned in the FTPL. The unique equilibrium implied by the FTPL moreover satisfies the Core Property, in the sense that no coalition of agents is able to yield a superior allocation to each of its members—implying this equilibrium is "stable". No other allocation satisfies this Property.

The FTPL provides a natural application of the imperfectly competitive versions of the Core and Shapley Value: whilst traders may be atomistic, the government is inherently non-negligible and its market power is key to the Theory's logic. In particular, the government is assumed able to violate its budget in its choice of a "non-Ricardian" fiscal policy (Leeper, 1991; Woodford, 1995), with the price level then adjusting to restore budget balance; in this view, the government's budget "constraint" really ought to be viewed as an equilibrium condition (Cochrane, 2005, 2023).[4] The operation of the market forces supposed to bring about this adjustment has been criticized (Buiter, 2002; McCallum, 2001). Buiter (2002, 2023), in particular, has argued that there is nothing special about the government in this regard and that one might as well formulate a "Mrs Jones the-

[2]Modeling the government as non-satiable implies that it would strive to maximize its own consumption stream, with no regard whatsoever for private consumption. This is not a very accurate reflection of reality. There, even though governments differ in their ideal government spending share (depending on their political leaning, for example), this bliss point tends to lie well below 100%.

[3]This is the cooperative interpretation. There are also a number of non-cooperative foundations for the Value allocation: Gul (1989) and Hart and Mas-Colell (1996) offer bargaining models yielding each player her Shapley Value, whilst the same payoffs are obtained in collusive pre-auction "knockouts" (Graham et al., 1990) and under cooperative "conference structures" (Myerson, 1980). A prominent application of the Value is to the bargaining arising under incomplete contracts in the property rights theory of the firm (Hart and Moore, 1990; Acemoglu et al., 2007).

[4]The same applies, in varying degrees, to models that mix elements of the FTPL with the traditional monetary-led regime (e.g. Bianchi et al., 2023; Caramp and Silva, 2023; Smets and Wouters, 2024).

ory of the price level" (Buiter, 2023). In this paper, we point out that the government is in fact special along two crucial dimensions (having satiable preferences and being a large trader), the combination of which is shown to imply that it is reasonable to use the government's budget equation to pin down the price level (but not that of Mrs Jones).

The concept of competitive equilibrium is, however, inherently ill-suited to analyzing the mechanics of disequilibrium. In response to this, Bassetto (2002) constructs a game-theoretic market model with endogenous prices, where the FTPL is implemented as the unique outcome of a non-cooperative game. As Bassetto himself notes, however, his results rely on the very specific market microstructure of Shubik's (1973) "trading posts" model, with a further assumption of enough symmetry to yield the Walrasian outcome.[5]

In the general cooperative game-theoretic setting explored here, minimal institutional structure is imposed on trading, and yet the FTPL emerges as a natural consequence of the allocation implied by the Shapley Value and the Core. Whereas competitive equilibrium may fail to exist in markets where agents (in this case, the government) can be satiated, a natural concept implementing equilibrium after redistribution of unused resources (a "dividend equilibrium") does exist (Drèze and Müller, 1980; Aumann and Drèze, 1986; Mas-Colell, 1992; Cornet et al., 2003) and is implied by the Shapley Value allocation.[6] The Core permits such a redistribution, and indeed Core Equivalence with dividend equilibrium uniquely selects the equilibrium promoted by the FTPL, which is surprising given the departure from perfect competition.[7] Intuitively, it results from the government being able to exploit its market power to steer the economy towards its bliss point, which can then be used to pin down the price level. We thus provide a formal rationalization for the claim (popular with proponents of the FTPL) that the government might not be subject to a proper budget *constraint* thanks to its "large" status.[8] We show that the resulting redistribution of wealth (between the government and households, via the revaluation of the initial stock of government debt) compensates agents according

[5] Peck et al. (1992) find such market games to be generically indeterminate.

[6] Another instance of such an equilibrium is provided by Kajii (1996), where the value of a pre-existing stock of fiat money plays the redistributive role.

[7] Konovalov (2005) establishes Core Equivalence of his "rejective Core" (which is the same as the Core here) with dividend equilibrium under satiation, but in the context of an atomless economy.

[8] See, for example, Woodford (2001, p. 693) who writes: "the government is a large agent, whose actions can certainly change equilibrium prices, and an optimizing government surely should take account of this in choosing its actions. Such a government should also understand the advantages of committing itself to a rule (given the way that expected future government policy affects equilibrium), and should consider which rule is most desirable". While the competitive equilibrium framework, in which the FTPL is typically formulated, is not able to substantiate this claim (as it is ill-equipped to distinguish between equilibrium conditions and constraints—see Bassetto 2008), the game-theoretic route taken by this paper can do so.

to their Shapley Value, providing a solid game-theoretic foundation for the equilibrium proposed by the FTPL; it moreover satisfies the Core Property, implying the equilibrium is “stable” in the game-theoretic sense, and indeed uniquely so.

# 2 The Fiscal Theory of the Price Level

The FTPL is built on the notion that the government may violate its budget off-equilibrium, but prices will then adjust to render its spending plans affordable. Essentially, it is claimed that the government budget equation need only hold in equilibrium (Kocherlakota and Phelan, 1999; Woodford, 2001; Buiter, 2002), but it is difficult to give meaning to this claim in models that include no explicit mechanism for price formation, and in particular are silent on the prices prevailing in disequilibrium. For this reason, Bassetto (2002) abandons the dynamic competitive equilibrium framework in favor of a game-theoretic market model that determines prices under any strategy profile, equilibrium or disequilibrium.

The present paper offers a more minimal departure from the standard setting by allowing competitive equilibrium to fail, but maintaining the Core. This allows one to treat the government's budget “constraint” as an equilibrium condition, as supposed by the FTPL, but in a well-defined game with rational behavior and market power.

Consider the following two-period economy $\mathscr{E}$ (adapted from Bassetto, 2002) with a representative household $h$ and a government $g$. The household's pre-tax endowment consists of one unit of a single homogeneous good each period. It furthermore starts the first period with $B_1 > 0$ units of nominal one-period government bonds maturing in period 1. The government chooses the nominal interest rate $R_1$ (which applies to the timespan in between periods 1 and 2) and tax revenues $\tau_1, \tau_2$ in the two periods, which it uses to finance exogenous government spending $G_1$ and $G_2$ as well as debt repayment. It adopts a monetary policy rule $R_1 = R_1(p_1)$ as a function of the first-period price of consumption $p_1$, in a way that leaves monetary policy “passive” in the sense of Leeper (1991). The representative household has preferences ordered by

$$U_h(c_1) + U_h(c_2),$$

where $c_j$ is household consumption in period $j$. Whilst the government's preferences are often not explicitly modeled under the FTPL, we will assume that it has a utility function

$U_g$, and moreover one that is satiable with a bliss point at $(c_1, c_2) = (1 - G_1, 1 - G_2)$.[9]

The household's period budget constraints are given by

$$p_1 c_1 \leq p_1(1-\tau_1) + B_1 - \frac{B_2^d}{1+R_1}, \quad p_2 c_2 \leq p_2(1-\tau_2) + B_2^d, \tag{1}$$

which can be combined into its intertemporal version

$$c_1 = 1 - \tau_1 + (p_2/p_1)(1-\tau_2-c_2)/(1+R_1) + B_1/p_1, \tag{2}$$

where $B_2^d$ is the household demand for period-2 maturity bonds and $p_j$ is the price of period-$j$ consumption relative to the unit of account. If $B_2$ is the period-2 supply of bonds, the government's budget equations are given by

$$p_1 G_1 \leq p_1\tau_1 - B_1 + \frac{B_2}{1+R_1}, \quad p_2 G_2 \leq p_2\tau_2 - B_2, \tag{3}$$

which, intertemporally, implies that

$$\frac{B_1}{p_1} = (\tau_1 - G_1) + \frac{(\tau_2 - G_2)}{(1+R_1)p_1/p_2}. \tag{4}$$

Now, fixing consumption at $(c_1, c_2) = (1 - G_1, 1 - G_2)$ determines the intertemporal marginal rate of substitution $(1+R_1)p_1/p_2$ in equilibrium. The FTPL then dictates that, given the values of $\tau_1$ and $\tau_2$ chosen by the government, the initial price level $p_1$ adjusts to make (4) hold, ultimately putting both $h$ and $g$ on the budget line $BC$ at $\mathbf{a}'$ in Figure 1 (which illustrates the FTPL in an Edgeworth Box, with $I_h$ and $I_g$ representing the household and government indifference curves, respectively).

It does this by determining the real value of the inherited nominal debt $B_1 > 0$.[10] In this way, fiscal policy determines the initial price level $p_1'$. From $\mathbf{a}'$, the agents trade to equilibrium at $\mathbf{b}$, where the government's exogenous spending objective $(G_1, G_2)$ is met. The equilibrium price ratio $p_2'/p_1'$ (and hence inflation) is then determined in the household first-order condition by the monetary policy rule's prescribed $R_1(p_1')$. This outcome forms the *Fiscal Theory efficiency equilibrium*, for reasons that will become clear in Lemma 1

[9]This is entirely consistent with the government's actions under the FTPL, and indeed it is hard to see how to rationalize those actions otherwise—see footnote 2.

[10]The presence and essentiality of this initial debt stock to the FTPL has been criticized by Niepelt (2004) as being inconsistent with optimizing behavior of forward-looking households, who would anticipate the possibility of a surprise revaluation and not buy (as much) government liabilities to begin with. McMahon et al. (2018) explore the implications of a more complex asset structure.

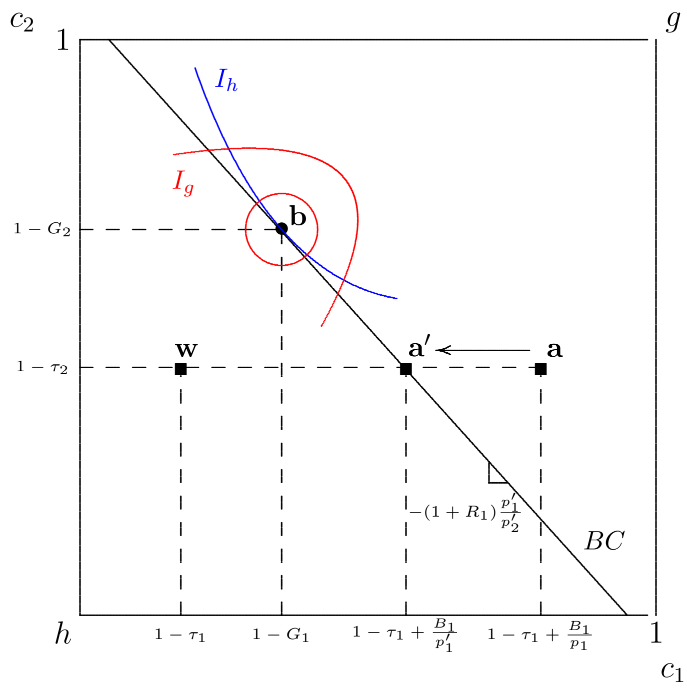


Figure 1: Edgeworth Box illustration of the FTPL; initial price level adjusts (from $p_1$ to $p_1'$) to satisfy the government budget equation (4) in equilibrium

below.

The operation of the FTPL requires the government to be able to choose values of $\tau_1$ and $\tau_2$ that are not conditional on the price level, and which are only market-clearing and budget-feasible for one particular price. Since this is then the equilibrium price, it is argued that the government need only satisfy its budget equation in equilibrium, with disequilibrium budget-infeasibility driving the economy to this outcome. However, since the concept of competitive equilibrium is silent on the prices and allocations prevailing out of equilibrium, it is inadequate to assess this argument. Instead, we use the FTPL economy to define a corresponding cooperative game played after the government has fixed its fiscal and monetary policies, which define a post-tax endowment from which households and the government may trade to their final consumption bundles, determining prices and the real value of inherited debt in the process.

**Sketch of proof.** Armed with the above understanding of the FTPL, we now proceed by providing a sketch of the crucial proof that is to follow. It demonstrates how the equilibrium consistent with the Shapley Value allocation coincides with the FTPL equilibrium, with this equilibrium moreover coinciding with the Core (implying this equilibrium is "stable" in the game-theoretic sense).

Suppose that the government's bliss point is not a competitive equilibrium under the original post-tax endowment.[11] Then a Shapley Value allocation $x$ of the economy maximizes some weighted average of household and government utility, $\lambda U_h(x) + (1 - \lambda)U_g(x)$, $\lambda \in [0, 1]$; for such a maximum, there must exist some $q$ such that

$$\lambda U_h'(x) = (1 - \lambda)U_g'(x) = q. \tag{5}$$

If $\lambda \in (0, 1)$, we establish in the proof of Theorem 1 that $x$ is a competitive equilibrium, as in the standard case without satiation (Champsaur, 1975). This contradicts our supposition that the government bliss point is not a competitive equilibrium under the original post-tax endowment; hence, we may suppose that $\lambda \in \{0, 1\}$. It follows from (5) and monotonicity of household preferences (which rules out $U_h'(x) = 0$) that $q = \lambda = U_g'(x) = 0$, i.e. the government is satiated at $x$ and the unique Shapley Value allocation is $x = (1 - G_1, 1 - G_2)$. This is the sense in which the government's bliss point pins down the only consumption bundle consistent with agents receiving their Shapley Value.

Moreover, a calculation of the household's expected marginal contribution upon joining a coalition (conducted in detail below, in the proof to Theorem 1) reveals that $x$ must be worth strictly more than the household's post-tax endowment $(1 - \tau_1, 1 - \tau_2)$; that is to say, there is a potential gain from trade at this endowment. Because we start from a situation in which there is a pre-existing debt stock $B_1 > 0$, this opens the door to the household being able to afford $x$ (as $B_1 > 0$ allows for the possibility of the household's budget constraint being relaxed by $B_1/p_1$ in real terms).[12] In particular, to give the household that Shapley-implied expected marginal contribution, $p_1$ must solve (2) at $(c_1, c_2) = (1 - G_1, 1 - G_2)$. But when substituting $(c_1, c_2) = (1 - G_1, 1 - G_2)$ into (2) it is easy to verify that one ends up with (4), the intertemporal version of the government's budget equation. This implies that $p_1$ *is determined in the exact way that is envisioned by the FTPL*, demonstrating how the associated equilibrium can be seen as resulting from the Shapley Value allocation.

[11] In the non-generic event where it is a competitive equilibrium, the FTPL is moot and $p_1$ must explode to drive the value of the inherited debt $B_1$ to zero (this will be proven in Theorem 1).

[12] Note how this step also illustrates that the presence of a positive initial government debt stock is crucial to the FTPL's viability, as described in footnote 10.

The Core is non-empty when such a Shapley Value allocation exists, and indeed coincides with the same point $x$: Pareto efficiency requires that the government allocation be worth strictly less than its post-tax endowment $(\tau_1, \tau_2)$. This is only compatible with the government's status as a "large" player if $x$ coincides with the government's bliss point (absent satiation, a large "atomic" player cannot be "exploited" in the sense of ending up with an allocation that is worth strictly less than its initial endowment; Shitovitz, 1973). At this satiation point, there may be some unused resources left over, which would then result in a finite first-period price level $p_1$—meaning that these "left-overs" get rebated to households (who then see their budget constraint expanded by $B_1/p_1 > 0$). This is the sense in which the FTPL equilibrium is a "dividend equilibrium" of the kind explored by Drèze and Müller (1980), Aumann and Drèze (1986), Mas-Colell (1992), and Cornet et al. (2003).

The remainder of this paper is primarily concerned with formalizing the above logic.

# 3 The environment

This section is concerned with defining a "market game" corresponding to the above economy, which we may then use to explore its game-theoretic foundations.[13]

Consider then the exchange economy $\mathscr{E}$ from the previous section, but with the disaggregated households and the government described by a measure space $(T, \mathscr{T}, \mu)$ of traders $T$ with $\sigma$-field $\mathscr{T}$ of possible *coalitions* and "weights" captured by a totally finite complete positive $\sigma$-additive measure $\mu$ on $\mathscr{T}$. An *atom* of $(T, \mathscr{T}, \mu)$ is a coalition $S$ with $\mu(S) > 0$ and for each subcoalition $R \subseteq S$ of which either $\mu(R) = 0$ or $\mu(S \backslash R) = 0$. In a *mixed market*, the set $T$ can be divided into a countable union of atoms $T_1$ and an atomless sector $T_0$. We will assume that the government is an atom, but that there are no atoms among the households; hence, $g = T_1$ and each household belongs to $T_0$. A *commodity bundle* $x = (c_1, c_2)$ is a point in $\Omega := \mathbb{R}^2_+$, whilst an *assignment* (of commodity bundles to traders) is an integrable function $\mathbf{x} \in \mathscr{X}$ from $T$ to $\Omega$.[14] The initial assignment is the *endowment* $\mathbf{w}$ arising after taxes have been collected by the government; specifically, households are (after paying their taxes) left with $\mathbf{w}(t) = (1 - \tau_1, 1 - \tau_2)$, $t \in T_0$,

[13] A "market game" is a cooperative game arising from a pure exchange economy, originally studied by Shapley and Shubik (1969) under the assumption of transferable utility (TU). Generally it is thought unreasonable to assume that utility may be transferred one-for-one between economic agents, and relaxing this assumption gives us non-transferable utility (NTU) games, which provide the setting for Core Equivalence.

[14] The notation $x > y$ will mean that $x_i \geq y_i$ for all $i$ but $x \neq y$, whilst $x \gg y$ will mean that $x_i > y_i$ for all $i$.

while the government has $\mathbf{w}(g) = (\tau_1, \tau_2)$. We assume that $\int_T \mathbf{w}(t)\, d\mu(t)$ has all elements strictly positive. Notice that the endowment does not include the inherited debt $B_1$, the real value of which is to be determined endogenously as a result of trade pinning down the initial price level.

Each household $t \in T_0$ has a (complete, transitive) *preference relation* $\succeq_t$ on $\Omega$ that satisfies the standard assumptions of: *strong desirability*, $x > y \Rightarrow x \succ_t y$; *continuity*, the sets $\{y \,|\, y \succ_t x\}$ and $\{y \,|\, x \succ_t y\}$ are open in $\Omega$ for all $x \in \Omega$; *strict convexity*, $y \in \Omega$ implies $\{x \in \Omega : x \succ_t y\}$ is a strictly convex set for each $t \in T$; and *measurability*, the set $\{t \,|\, \mathbf{x}(t) \succ_t \mathbf{y}(t)\}$ is in $\mathscr{T}$ for any assignments $\mathbf{x}$ and $\mathbf{y}$. Two traders $s$ and $t$ are of the same *type* $k$ if $\mathbf{w}(s) = \mathbf{w}(t)$ and, for all $x, y \in \Omega$, $x \succ_s y$ if and only if $x \succ_t y$. Under the assumptions on $\succeq_t$, each trader $t$ has a measurable, continuous, quasiconcave utility function $U_t$ on $\Omega$ (which we also suppose to be differentiable), and all traders of the same type $k$ share a common such function $U_k$. The government $g$ also has a preference relation $\succeq_g$ satisfying the same assumptions except strong desirability, in order to allow the possibility of its satiation; instead, we assume its (differentiable) utility function $U_g$ to be strictly concave, attaining a maximum at the unique *bliss point* $\mathbf{b}$ where $\mathbf{b}(g) = (G_1, G_2)$. After all, governments would generally be assumed to maximize the welfare of consumers in this context, rather than their own consumption. We assume $\mathbf{b}$ to be Pareto efficient and *individually rational*, i.e. preferred by each household $t \in T_0$ to the endowment $\mathbf{w}(t)$.

Fixing $G_1$, $G_2$, $\tau_1$, $\tau_2$, $R_1(p_1)$ and $B_1 > 0$ from the previous section then, we have a well-defined market game $\mathscr{G}$. An *allocation* of $\mathscr{G}$ is an assignment $\mathbf{x}$ for which $\int_T \mathbf{x}(t)\, d\mu(t) = \int_T \mathbf{w}(t)\, d\mu(t)$. An allocation $\mathbf{y}$ *dominates* an allocation $\mathbf{x}$ *via* a *blocking coalition* $S$ if: (a) for almost every $t \in S$, $\mathbf{y}(t) \succeq_t \mathbf{x}(t)$; (b) for a non-null $t$-set of traders in $S$, $\mathbf{y}(t) \succ_t \mathbf{x}(t)$; and (c) $\int_S \mathbf{y}(t)\, d\mu(t) = \int_S \mathbf{w}(t)\, d\mu(t)$. The *Core* $\mathscr{C}(\mathscr{T})$ is the set of all allocations that are not dominated via any non-null coalition $S \in \mathscr{T}$. If $\mathbf{x}(t) = x_0 =: \mathbf{x}(k)$, for all traders $t$ of the same type $k$, $\mathbf{x}$ is called an *equal-treatment* allocation. A *competitive equilibrium* is a pair $(p, \mathbf{x})$ consisting of a *price system* $p \in \mathbb{R}^2_+$ and an allocation $\mathbf{x}$ such that, for $\mu$-almost all traders $t$, $\mathbf{x}(t)$ is maximal with respect to $\succeq_t$ in $t$'s budget set $\{x \in \Omega : p \cdot x \leq p \cdot \mathbf{w}(t)\}$.[15] A *dividend* is a real-valued vector $\boldsymbol{\beta} = (\beta_t)_{t \in T}$; a *dividend equilibrium* is a price vector $p$, a dividend $\boldsymbol{\beta}$, and an allocation $\mathbf{x}$ such that, for all $t$, $\mathbf{x}(t)$ is maximal with respect to

[15] Whilst it is well known that the set $\mathscr{E}$ of competitive (equilibrium) allocations coincides with the Core in atomless economies (Aumann, 1964), it is less widely appreciated that such Core Equivalence applies in a mixed market if "large" traders number at least two and are all of the same type (Shitovitz, 1973), or if they are not "too large" (Gabszewicz and Mertens, 1971). The latter result has a simple application to the single-atom setting, where Core Equivalence holds if $\mu(A)/\mu(T_A) < 1$ and $T_A$ is the set of all traders who are of the same type as the atom $A$; thus, any non-null set of "small" traders of the same type as the atom nullifies the anti-competitive effect of the atom's size.

$\succeq_t$ in the *dividend budget set* $\{x \in \Omega : p \cdot (x - \mathbf{w}(t)) \leq \beta_t\}$. In a market with satiation, a dividend may be thought of as distributing the unused budget of any satiated agents among the unsatiated agents.

## 4 Efficiency equilibrium

An important result for the analysis of the Core in the presence of a large trader is Shitovitz' (1973) "budgetary exploitation" theorem, which shows that any core allocation is a competitive equilibrium after redistribution in favor of a large trader. This result provides a natural price system for Core allocations, but it requires modification in a market with satiation, which is the purpose of this section.

To begin with, an *efficiency equilibrium* (Shitovitz, 1973) is a pair $(p, \mathbf{x})$ consisting of a *price system* $p \in \mathbb{R}^2_+$ and an allocation $\mathbf{x}$ such that, for $\mu$-almost all traders $t$, $\mathbf{x}(t)$ is maximal with respect to $\succeq_t$ in $t$'s *efficiency budget set* $\{x \in \Omega : p \cdot x \leq p \cdot \mathbf{x}(t)\}$. Clearly every competitive equilibrium is an efficiency equilibrium, but not vice versa.

We then have the following consequence of Shitovitz' (1973) "budgetary exploitation" theorem on the subset $\mathscr{X}_\mathbf{b} := \{\mathbf{x} \in \mathscr{X} : \mathbf{x}(g) \in [0, G_1) \times [0, G_2)\}$ of allocations where the government spends component-wise less than at its bliss point $\mathbf{b}$.

**Lemma 1** *For every Core allocation $\mathbf{x} \in \mathscr{C}(\mathscr{T}) \cap \mathscr{X}_\mathbf{b}$, there exists a price system $p$ such that:*

1. *$(p, \mathbf{x})$ is an efficiency equilibrium;*
2. *$p \cdot \mathbf{x}(t) \leq p \cdot \mathbf{w}(t)$ for almost all $t \in T_0$.*

**Proof.** Every Core allocation $\mathbf{x} \in \mathscr{C}(\mathscr{T}) \cap \mathscr{X}_\mathbf{b}$ is also a Core allocation of the market game restricted to $\mathscr{X}_\mathbf{b} \cup \mathbf{w}$, in which strong desirability holds for all traders on $\mathscr{X}_\mathbf{b}$. By Shitovitz' (1973) "budgetary exploitation" theorem, there then exists a price system $p$ such that $(p, \mathbf{x})$ is a competitive equilibrium of the economy restricted to $\mathscr{X}_\mathbf{b} \cup \mathbf{w}$ with allocation $\mathbf{x}$, and $p$ for almost all $t \in T_0$. Under strict convexity of preferences, $(p, \mathbf{x})$ is also a competitive equilibrium of the original economy $\mathscr{E}$ under allocation $\mathbf{x}$. ∎

Thus, below its bliss point, the government cannot be "budgetarily exploited" at a Core allocation; it cannot spend less than the value of its endowment. In effect, Shitovitz' (1973) theorem applies over the region of the commodity space where the standard assumption of strong desirability holds. Combined with Pareto efficiency of the Core, it follows that if the government were to be budgetarily exploited at a Core allocation, it would of necessity be at its bliss point.

**Efficiency-equilibrium prices.** Whilst the concept of the Core features no explicit prices, the very notion of a government budget equation requires that some price system exist, and Lemma 1 provides a price system consistent with the Core. If different prices to these applied at a Core allocation, there would exist some trader with a profitable deviation and Pareto efficiency would be violated. However, prices are not the only feature of an efficiency equilibrium, a *transfer* $p \cdot (\mathbf{w}(t) - \mathbf{x}(t))$ also being required. In the FTPL, the presence of an initial debt stock $B_1$ provides the means for just such a transfer, and since the debt is nominal, its real value $B_1/p_1$ is determined by the initial price level $p_1$. Given an initial debt level $B_1$ and a Core allocation $\mathbf{x} \in \mathscr{C}(\mathscr{T}) \cap \mathscr{X}_{\mathbf{b}}$, efficiency equilibrium requires the initial price level to take on a value that effects a certain transfer $B_1/p_1$ implementing $\mathbf{x}$ as a competitive equilibrium. Prices satisfying these conditions constitute *efficiency-equilibrium prices.*

# 5 The Shapley Value allocation yields the FTPL equilibrium

Whilst the efficiency-equilibrium price ratio—as discussed in the previous section—is easily defensible, it is natural to ask: by what mechanism should the budgetary transfer be expected to occur, even given the presence of a stock of initial government debt? This is perhaps the central unanswered question that hangs over the FTPL.

This section proceeds by developing an answer. In particular, we show that if each trader in the model receives her Shapley Value, this implies not only the efficiency-equilibrium price ratio, but also a price level-driven redistribution of precisely the size required to effect efficiency equilibrium, and hence to yield the FTPL's initial price level. This establishes a (rather surprising and hitherto unnoticed) connection between Shapley Values and the FTPL.

Focusing on the Shapley Value-implied allocation carries considerable appeal. First, it represents an outcome in which each trader is rewarded in line with their marginal value to the economy, thus aligning it with the "marginalist" way in which prices are set in most (macro)economic models (Young, 1985). In addition, that allocation also satisfies intuitive notions of fairness, both in the appeal of its characteristic axioms (notably symmetry) and in its concrete applications (see Moulin, 2003, §5.2). We will show in Section 6 that this outcome furthermore belongs to the Core, implying that it is efficient and stable (as no trader will see any profitable opportunities to build deviating coalitions).

Consider then the following finite approximation of the economy $\mathscr{E}$:[16] Let $M^1$ be the market game with just one household of each of $K$ types $1, \dots, K$, with each household inheriting $B_1$ units of period-1 bonds, $\mu(\{k\}) > 0$ for all $k$, and $\sum_{k=1}^{K} \mu(\{k\}) = 1 = \mu(\{g\})$—i.e. including the government, there are $K+1$ atoms. The *n-fold replication* $M^n$ of $M^1$ is the market with trader set $T^n$ composed of $nk$ households ($n$ of each of the $K$ household types) but still just one government $g$, with $\sum_{i=1}^{n} \mu(\{i\}) = \mu(\{k\})$ for all $k$. A *(generalized) comparison vector* on $T^n$ is a vector $\boldsymbol{\lambda}^n = (\lambda_t^n)_{t \in T^n}$ of $(nk+1)$ non-negative real numbers. Given a comparison vector $\boldsymbol{\lambda}^n$ and a coalition $S \in \mathscr{T}$, define

$$v_{\boldsymbol{\lambda}^n}(S) := \max_{\mathbf{x} \in \mathscr{X}} \left\{ \sum_{t \in S} \boldsymbol{\lambda}_t^n U_t(\mathbf{x}(t)) : \sum_{t \in S} \mathbf{x}(t) = \sum_{t \in S} \mathbf{w}(t) \right\}. \tag{6}$$

The *Shapley Value* of the game $v_{\boldsymbol{\lambda}^n}$ is then

$$(\phi v_{\boldsymbol{\lambda}^n})(t) = \mathrm{E}(v_{\boldsymbol{\lambda}^n}(S \cup t) - v_{\boldsymbol{\lambda}^n}(S)), \tag{7}$$

where $S$ is the set of traders preceding $t$ in a random order on the set $T$. This embodies the classic motivation for the Shapley Value: that each agent should receive her expected contribution to total weighted utility, across all possible coalitions she might be the last agent to join. Shapley (1969) proves that every game has an NTU Value, provided that some (but not all) of the entries in the comparison vector are permitted to be zero (by contrast with the strictly positive non-generalized comparison vector). A *(generalized) Shapley Value allocation* is an allocation $\mathbf{x}^n$ for which there exists a (generalized) comparison vector $\boldsymbol{\lambda}^n$ such that for all traders $t \in T^n$,

$$(\phi v_{\boldsymbol{\lambda}^n})(t) = \lambda_t^n U_t(\mathbf{x}^n(t)), \tag{8}$$

in which case $\boldsymbol{\lambda}^n$ and $\mathbf{x}^n$ are *associated* with each other. If $\lambda_t^n = \lambda_k^n$ for all traders of type $k \in \{1, \dots, K\}$, $\boldsymbol{\lambda}^n$ is called an *equal-treatment comparison vector.*

In order fully to characterize Shapley Value allocations, we must distinguish between two cases: if the bliss point $\mathbf{b}$ is budget-feasible at its efficiency-equilibrium prices $p$, i.e. $p \cdot \mathbf{b}(g) \leq p \cdot \mathbf{w}(g)$, we will say that it is *affordable* (and *strictly* so if the inequality is strict); otherwise, it is *unaffordable.* Let $\mathbf{x}^\infty$ denote a limit point of $\{\mathbf{x}^n\}$.

[16]Here we employ the Shapley NTU Value, axiomatized by Aumann (1985). This Value is generally analyzed by calculating the limit of Values of approximating finite games (as in Acemoglu et al., 2007). Indeed, Aumann and Drèze (1986, §11.2) find the continuum approach inadequate for the analysis of the Shapley Value in non-atomic markets with satiation.

**Theorem 1** *There exists, for all $n$, an equal-treatment Value allocation $\mathbf{x}^n$ of $M^n$ with a finite $p_1$ if and only if $\mathbf{x}^\infty$ is a strictly affordable government bliss point, and efficiency-equilibrium prices prevail.*

**Proof.** In what follows, we utilize elements of Aumann and Drèze's (1986) analysis of the Value in markets with satiation.

"$\Rightarrow$": Since $\mathbf{x}^n$ is an equal-treatment Value allocation, it has an associated equal-treatment comparison vector $\boldsymbol{\lambda}^n$,[17] and hence a corresponding $(K+1)$-dimensional *type-comparison vector* $\lambda^n$. Normalize $\lambda^n$ so that $\lambda_g^n + \sum_{k=1}^K \lambda_k^n = 1$. Under strong desirability, each element of $\lambda^n$ tends to a positive limit (Champsaur, 1975), but with satiation some of these may tend to 0, in which case the corresponding type is called *lightweight* (as opposed to *heavyweight*). An allocation is *optimal* for a coalition $S$ if it achieves the maximum total weighted utility for $S$ defined in (6). The allocation $\mathbf{x}^n$ is optimal for the grand coalition $T^n$ by definition; hence, there exists $q^n$ such that $\lambda_g^n U_g'(\mathbf{x}^n(g)) = \lambda_k^n U_k'(\mathbf{x}^n(k)) = q^n$, $k = 1, \ldots, K$. Letting $n \to \infty$ and setting $\lambda_g^\infty := \lim_{n\to\infty} \lambda_g^n$, $\lambda_k^\infty := \lim_{n\to\infty} \lambda_k^n$, $k = 1, \ldots, K$, $q^\infty := \lim_{n\to\infty} q^n$, we have

$$\lambda_g^\infty U_g'(\mathbf{x}^\infty(g)) = \lambda_k^\infty U_k'(\mathbf{x}^\infty(k)) = q^\infty, \quad k = 1, \ldots, K. \tag{9}$$

The allocation that is optimal for a coalition $S$ containing $g$ approaches the allocation $\mathbf{x}^n$ that is optimal for the grand coalition $T^n$ as $|S|, n \to \infty$, by diagonality of $\phi v_{\boldsymbol{\lambda}^n}$ (Proposition 43.11, Aumann and Shapley, 1974; Neyman, 1977). Adding a household $t$ of type $k$ to $S$ will not change this optimal allocation by much, and hence $t$ will need to be allocated approximately $\mathbf{x}^n(k) - \mathbf{w}^n(k)$, decreasing the total weighted utility of $S$ by the product of $\mathbf{x}^n(k) - \mathbf{w}^n(k)$ with the common utility gradient $q^n$. Adding the weighted utility $\lambda_k^n U_k(\mathbf{x}^n(k))$ that $t$ now gets, we derive the approximate contribution that $t$ makes upon joining $S$,

$$\Delta := \lambda_k^n U_k(\mathbf{x}^n(k)) - q^n \cdot (\mathbf{x}^n(k) - \mathbf{w}^n(k)).$$

Letting $\delta$ be the conditional expectation of $t$'s contribution when $S$ is "small", which happens with probability $P^n \to 0$ as $n \to \infty$,

$$(\phi v_{\boldsymbol{\lambda}^n})(t) \approx (1 - P^n)\Delta + P^n \delta.$$

[17] Such a $\boldsymbol{\lambda}^n$ may be derived from any unequal-treatment comparison vector $\boldsymbol{\kappa}^n$ associated with $\mathbf{x}^n$ by taking $\boldsymbol{\lambda}_t^n$, for each trader $t$, to be the average of the weights $\kappa_s^n$ over traders $s$ of $t$'s type.

Since $(\phi v_{\boldsymbol{\lambda}^n})(t) = \lambda_t^n U_t(\mathbf{x}^n(t))$ by (8), it follows that

$$q^n \cdot (\mathbf{x}^n(k) - \mathbf{w}^n(k)) \approx \varepsilon^n(\delta - \lambda_k^n U_k(\mathbf{x}^n(k))), \tag{10}$$

where $\varepsilon^n := P^n/(1-P^n) \to 0$.

Consider first the case where each $\lambda_k^\infty, \lambda_g^\infty > 0$, which—since $U_k'(\mathbf{x}^\infty(k)) \neq 0$—implies by (9) that $\mathbf{x}^\infty$ does not satiate the government, that $q^\infty \neq 0$ and that the gradient of each $U_k$ at $\mathbf{x}^\infty(k)$ is in the direction of $q^\infty$. Letting $n \to \infty$ in (10), we have $q^\infty \cdot (\mathbf{x}^\infty(k) - \mathbf{w}^\infty(k)) = 0$; hence the trade $(\mathbf{x}^\infty(k) - \mathbf{w}^\infty(k))$ maximizes $U_k$ over the household's budget set $\{x \in \Omega : q \cdot x \leq q \cdot \mathbf{w}^\infty(k)\}$, and $\mathbf{x}^\infty$ satisfies (2). Given that a Value allocation must exhaust the endowment (by optimality), it follows that $\mathbf{x}^\infty$ also satisfies (4). But since $U_g'(\mathbf{x}^\infty(g)) \neq 0$ and a Value allocation cannot be Pareto inefficient, the bliss point $\mathbf{b}$ must be unaffordable, and $\mathbf{x}^\infty$ must also maximize government utility on its budget set (by strict concavity) and hence be a competitive equilibrium. Such a competitive equilibrium does not exist for $B_1 > 0$ and finite $p_1$, a contradiction.

Thus, consider now the possibility that $\lambda_t^\infty = 0$ for some $t \in T$, i.e. that some traders are lightweight (and others heavyweight). It then follows from (9) that $q^\infty = 0$, and hence $U_t'(\mathbf{x}^\infty(t)) = 0$ for heavyweight $t$, so that $\mathbf{x}^\infty$ satiates all heavyweights. Since $g$ is the only trader that may be satiated, it follows that $\mathbf{x}^\infty$ is the government bliss point, and each household must be lightweight.[18]

To establish that efficiency-equilibrium prices prevail, consider a lightweight household $t$ of type $k$ (which must exist in the absence of a competitive equilibrium). Since $q^\infty = 0$, if we took the limit $n \to \infty$ in (10) as before, we would simply obtain $0 = 0$. However, dividing (10) by $\|q^n\|$ (which also vanishes) and letting $n \to \infty$, second-order effects emerge and we obtain an expression for the size of the transfer implied by the Shapley Value:

$$p \cdot (\mathbf{x}^\infty(k) - \mathbf{w}^\infty(k)) = \lim_{n\to\infty} (\varepsilon^n/\|q^n\|)(\delta - \lambda_k^n U_k(\mathbf{x}^n(k))) =: \beta_k, \tag{11}$$

where $p$ is the limit of $q^n/\|q^n\|$. Here $\delta$ consists of: (i) $t$'s own utility after joining $S$; (ii) the change in the total utility of $S$'s lightweight traders when $t$ joins; and (iii) the change in the total utility of $S$'s heavyweight traders when $t$ joins. Since (i) and (ii) involve lightweight traders, their weights tend to 0, leaving just (iii); for the same reason, the remaining $U_t$ term vanishes. Given that $S$ is small when $\delta$ is earned, $g$ will not in general be satiated when $t$ joins, allowing $t$ to make a first-order improvement to $S$'s total

[18] The equal weighting of the households here accords with Aumann and Kurz (1977), whilst its lower order than the government weighting respects the population measure $\mu$ in the spirit of Hart (1980).

weighted utility under component (iii) of $\delta$. Hence, the limit $\beta_k$ of (11) is strictly positive, so that a *transfer* of $\beta_k$ must be made to the household in order for $\mathbf{b}$ to be a Value allocation. This must be the real value $B_1/p_1$ of the inherited debt, and since $B_1$ and $p_1$ take finite positive values, $\mathbf{b}$ must be strictly affordable.

Since $q^n$ is proportional to $U'_k(\mathbf{x}^n(k))$, its direction $q^n/\|q^n\|$ is the direction of $U'_k(\mathbf{x}^n(k))$, and hence $p$ is the direction of $U'_k(\mathbf{x}^\infty(k))$. Thus, $\mathbf{x}^\infty(k)$ maximizes $t$'s utility over the budget set defined by prices $p$, endowment $\mathbf{w}^\infty$ and transfer $\beta_k$. Since $p$ is proportional to $U'_k(\mathbf{x}^\infty(k))$, prices are proportional to households' marginal utilities and the Fiscal Theory's price ratio prevails. And since $\beta_k > 0$, it must be that $B_1 > 0$ (as we knew from Niepelt, 2004), in which case $p_1$ solves a linear equation and $\mathbf{x}^\infty = \mathbf{b}$ uniquely determines the Fiscal Theory's initial price level. The government is of course satiated at this point, so that we have the Fiscal Theory efficiency equilibrium.

Adding the government $g$ to a coalition $S$ will change its optimal allocation to approximate $\mathbf{b}$ as $|S|, n \to \infty$ (since only the government is heavyweight). Hence, $g$ will need to be allocated approximately $\mathbf{b}(g) - \mathbf{w}^n(g)$, decreasing the total weighted utility of $S$ by the product of $\mathbf{b}(g) - \mathbf{w}^n(g)$ with the common utility gradient $q^n$. Adding the weighted utility $\lambda_g^n U_g(\mathbf{b}(g))$ that $g$ now gets, we arrive at the approximate contribution that $g$ makes upon joining $S$,

$$\Gamma := \lambda_g^n U_g(\mathbf{b}(g)) - q^n \cdot (\mathbf{b}(g) - \mathbf{w}^n(g)).$$

Letting $\gamma$ be the conditional expectation of $g$'s contribution when $S$ is "small", which happens with probability $P^n \to 0$ as $n \to \infty$,

$$(\phi v_{\boldsymbol{\lambda}^n})(g) \approx (1 - P^n)\Gamma + P^n\gamma.$$

Since $(\phi v_{\boldsymbol{\lambda}^n})(g) = \lambda_g^n U_g(\mathbf{x}^n(g))$ by (8), it follows that

$$q^n \cdot (\mathbf{b}(g) - \mathbf{w}^n(g)) \approx \varepsilon^n(\gamma - \lambda_g^n U_g(\mathbf{b}(g))), \tag{12}$$

where $\varepsilon^n := P^n/(1 - P^n) \to 0$. Dividing by $\|q^n\|$ and letting $n \to \infty$, we obtain

$$p \cdot (\mathbf{b}(g) - \mathbf{w}^\infty(g)) = \lim_{n\to\infty} (\varepsilon^n/\|q^n\|)(\gamma - \lambda_g^n U_g(\mathbf{b}(g))) =: \beta_g. \tag{13}$$

Here $\gamma$ consists of: (i) $g$'s own utility after joining $S$; and (ii) the change in the total utility of $S$'s lightweight traders when $g$ joins, which vanishes with the households' weighting as $n \to \infty$. Since $\lambda_g^\infty U_g(\mathbf{b}(g))$ is the maximum that the government can contribute to total weighted utility, component (i) of $\gamma$ is more than cancelled out, so that the limit $\beta_g$ is strictly negative—a transfer must be made to households. This differs from the positive dividend received by heavyweights in Aumann and Drèze (1986), because the government is the only heavyweight here, and so cannot benefit any other heavyweights when joining a coalition.[19] Of course, since the government ends up being satiated, the transfer it makes comes at no cost to its utility (indeed, $g$ benefits).

"$\Leftarrow$": If $\mathbf{x}^\infty$ is a strictly affordable bliss point, then $M^n$ has such a point. For every $n$, there exists an equal-treatment Value allocation in $M^n$ by Aumann and Drèze's (1986) Proposition 5.4. Under efficiency-equilibrium prices, $p_1$ must then be finite. ■

Thus, the Shapley Value implements the Fiscal Theory efficiency equilibrium. Importantly, not only the price ratio, but also the balancing of the government budget equation via a unique $p_1$ is endogenous here, implied by the Shapley Value (we first saw this in the sketch of this part of the proof, provided at the end of Section 2).

# 6 The FTPL equilibrium coincides with the Core

In this section, we show that the Shapley Value allocation is stable in the sense that it belongs to the Core, and indeed no other allocation is stable in this way; Core Equivalence obtains and provides a strong foundation for the FTPL.

**Theorem 2 (Core equivalence)** *Suppose that $p_1$ is finite. If there is a strictly affordable bliss point, then it is identical to the Core with efficiency-equilibrium prices; if there is no strictly affordable bliss point, then the Core with efficiency-equilibrium prices is empty.*

**Proof.** Given the bliss point $\mathbf{b}$, any Pareto-efficient allocations must belong to $\mathscr{X}_{\mathbf{b}} \cup \mathbf{b}$. But since any efficiency-equilibrium allocation either involves budgetary exploitation of the government (which we know cannot happen in $\mathscr{X}_{\mathbf{b}}$ by Lemma 1) or $B_1 \leq 0$, no allocation in $\mathscr{X}_{\mathbf{b}}$ can belong to the Core with efficiency-equilibrium prices (and finite positive $B_1$ and $p_1$). The bliss point $\mathbf{b}$, meanwhile, belongs to the Core with efficiency-equilibrium prices if and only if it is strictly affordable: no subset of households can improve on a Pareto-efficient, individually rational allocation amongst themselves, and

[19] More generally, the market $M^n$ is not a special case of that in Aumann and Drèze (1986), for the government is not replicated in this paper's $M^n$. Whilst the proof of Theorem 1 employs similar ideas to that of Aumann and Drèze's Main Theorem, it is not a consequence of it. Indeed, their result has a non-negative budgetary expansion (or "dividend") for each trader, including the government. By contrast, the non-replication of the government in this paper's $M^n$ means that there are no other heavyweight traders to benefit from the government's presence in a coalition, reducing its dividend below zero.

hence any putative blocking coalition must contain $g$; but $\mathbf{b}$ achieves the government optimum; and $\mathbf{b}$ can be implemented in efficiency equilibrium with finite positive $B_1$ and $p_1$ if and only if it is strictly affordable. ■

**Can the government choose non-Ricardian policy rules?** A *Ricardian* policy rule is one that satisfies the government budget equation for any price vector. Hence, under a non-Ricardian policy rule, there exists some $(p_1, p_2)$ for which $\mathbf{b}$ does not satisfy the budget equation $BC$ in (4). Such budgetary violations have been the traditional focus of FTPL-critics, who argue that the government's intertemporal budget equation is a true constraint—also applying off-equilibrium. However, these violations clearly prevail in the current market game, under any prices inconsistent with the Shapley Value allocation. Figure 1, for example, illustrates a Fiscal Theory efficiency equilibrium that belongs to the Core under the (Value-implied) efficiency-equilibrium prices $(p'_1, p'_2)$ with $B_1 > 0$, and in which the government runs a primary surplus, but violates $BC$ for the putative prices $(p_1, p_2)$ prevailing at $\mathbf{a}$.

Since prices are fully endogenous to the model, it is not clear where the price vector $(p_1, p_2)$ would come from, but if it somehow were to prevail then the model would generate an initial price rise, from $p_1$ to $p'_1$, as in Figure 1. This would redistribute resources from consumers to the government via a diminished real worth of their initial nominal debt holding $B_1$, moving the economy from $\mathbf{a}$ to the efficiency-equilibrium budget line $BC$. Because there is a strictly affordable bliss point here, the Core is $\mathbf{b}$.

An intuition for the FTPL's price mechanism (explored thoroughly by Cochrane, 2023) is that households find themselves in possession of nominal debt that will not be honored in real terms, which they then seek to trade for the consumption good, driving up its price. This provides a reasonable narrative for the above "initial price rise": the non-Ricardian policy rule engenders a price adjustment that diminishes the worth of households' initial debt holdings, returning the economy to budget balance. However, this story is once again outside of the model: the account provided within the model is that the price vector $(p_1, p_2)$ is inconsistent with the Shapley Value allocation; a bliss point of $\mathbf{b}$ means that there is "too much" household consumption under prices $(p_1, p_2)$, in the sense that total weighted utility could be higher if it were reduced. Since prices are proportional to marginal utilities at a Value allocation, prices are hence too low (at least if we assume concave utility), and need to rise to $(p'_1, p'_2)$ in order to implement the grand coalition's optimal allocation $\mathbf{b}$.

By contrast, at the original post-tax endowment $\mathbf{w}$ in Figure 1, there is "too little" household consumption, and prices must fall to $(p_1', p_2')$ in order to implement $\mathbf{b}$. In other words, at $\mathbf{w}$, the unused resources that hamper the existence of competitive equilibrium may be redistributed to the utility gain of all agents if household consumption is expanded. By virtue of its size, the government (and no other trader) is essential to any such gains from trade, and the economy's resources are hence allocated entirely towards its satiation—namely, the realization of its ideal policy. This is the sense in which the government, as a large player, is able to exploit its market power.

# 7 Conclusion and directions for future work

This paper has provided a foundation for a crucial ingredient of the Fiscal Theory of the Price Level (FTPL), namely the notion that the government budget equation need not hold out of equilibrium. It is shown to arise in a game-theoretic setting featuring one large, satiable agent (in this case, the government) who—thanks to its "market power"—is able to move the economy away from its budget equation, provoking a price adjustment envisaged by the FTPL until its bliss point is reached. The resulting equilibrium, which satisfies the Core property, has the intuitive feature that all players are rewarded according to their Shapley Value and can be interpreted as being a "dividend equilibrium" of the type studied in Aumann and Drèze (1986).

While this overcomes one long-standing objection to the FTPL, the formalization offered in this paper also points to aspects that may warrant further attention. The proof to our main result exploits i) the government being the only large player with satiable preferences, and ii) the absence of distributional considerations in the government's objective function. If there was another large player with satiable preferences, it would be interesting to study how the game between the various large players would play out (and how that affects price level determination).

If one were to equip the government with preferences over the distribution of household consumption, the analysis of the Core would be complicated by externalities (as the payoffs of a potential blocking coalition may then be affected by the behavior of those outside it). Previous analyses in this space (e.g., Dufwenberg et al. (2011)) have pointed to the possibility of the Core being empty in such cases, requiring the study of modified concepts.

We hope that future work will be able to make progress on studying the viability of the FTPL and related theories (such as those in Bianchi et al., 2023; Caramp and Silva, 2023; Smets and Wouters, 2024) in these more general settings. Finally, it could also be interesting to mobilize the analysis offered in this paper to investigate whether bubbles can be expected to survive on government debt in this environment (Bassetto and Cui, 2018; Berentsen and Waller, 2018; Brunnermeier et al., 2022).

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