

Bank of England

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Boris Chafwehé,⁽¹⁾ Andrea Colciago⁽²⁾ and Romanos Priftis⁽³⁾

Abstract

This paper introduces a New Keynesian multi-sector industry model that integrates firm heterogeneity, entry, and exit dynamics, while considering energy production from both fossil fuels and renewables. We investigate the effects of a sustained increase in fossil fuel prices on sectoral size, labour productivity, and inflation. A hike in the price of fossil resources results in higher energy prices. Due to ex-ante heterogeneity in energy intensity in production, the profitability of sectors is impacted asymmetrically. As production costs rise, less efficient firms leave the market, while new entrants must display higher idiosyncratic productivity. While this process enhances average labour productivity, it also results in a lasting decrease in the entry of new firms. A central bank with a strong anti-inflationary stance can circumvent the energy price increase and mitigate its inflationary effects by curbing rising production costs. This policy entails a higher impact cost in terms of output and lower average productivity, but leads to a faster recovery in business dynamism. Thus, our results suggest that monetary policy faces a trade-off between stabilising aggregate activity and business dynamism.

Key words: Energy, productivity, firm entry and exit, monetary policy.

JEL classification: E62, L16, O33, Q43.

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1 Introduction

In 2022 the global economy was shaken by a major energy crisis. Global fossil fuel prices started to soar in early 2022 upon Russia’s invasion of Ukraine. The latter, combined with the repercussions of the COVID-19 pandemic, the rise in food prices, and supply bottlenecks, led to an overall rise in consumer prices, with inflation rates increasing by more than 10% in many European countries.

Increases in the relative price of fossil fuels lead to higher energy prices, which can burden the economy, especially if firms and households are unable to switch to cheaper, less carbon-intensive energy alternatives. This situation may pose greater challenges for countries with significant energy-intensive sectors. Indeed, the pandemic underscored how supply shocks can have broad, persistent inflationary effects, with surprising speed. Strong upward price pressures in some industries may spread through supply chains, and to wages, or affect inflation expectations, influencing price and wage-setting mechanisms.

This has led some commentators to suggest that central banks should react more forcefully to an energy shock under certain initial conditions. Factors such as sectoral composition and labor market rigidities are likely to matter for the transmission of supply shocks and their persistence. Central banks may also need to be more aggressive in their responses in economies where workers are less willing to accept real wage declines. The central bank may also have to react more if the shocks are broad-based rather than concentrated in particular sectors.

In this paper, we provide a general equilibrium model to study these issues. We develop a multi-sector industry dynamic model with endogenous entry and exit of heterogeneous firms to analyze the short- to medium-term impact of a persistent increase in the price of an imported fossil resource on sectoral reallocation, productivity, and business dynamism. These aspects have received scant attention in the academic literature. This is surprising for at least three reasons. First, advanced countries on both sides of the Atlantic have witnessed both productivity and business dynamism slowdowns in recent decades as reported, *inter alia*, by [Akcigit and Ates \(2019, 2021\)](#) and [OECD \(2021\)](#). In such an environment, it is all the more important to understand the transmission of higher energy prices on productivity and business creation and destruction. Second, firm entry and exit are widely thought to be major drivers of productivity growth, as shown by [Foster et al. \(2019\)](#). Hence to the extent that higher energy prices affect business dynamism, this could impact productivity. Third, reallocation is likely to occur among sectors and firms within sectors in response to a persistent rise in the price of the energy input. We study these issues in a context characterized by imperfectly competitive goods and labor markets, and nominal frictions.

We develop a model of firm dynamics in the spirit of [Melitz \(2003\)](#) and [Clementi and Palazzo \(2016\)](#). The Industry dynamics model that we propose has the following features. First, it is characterized by two final good sectors, which we identify as manufacturing and services, that have ex-ante different energy intensities. More precisely, manufacturing

is energy-intensive, while the service sector is labor-intensive. Second, each sector is populated by ex-ante heterogeneous firms, which produce goods in different varieties and compete monopolistically. In each sector, firms face initial uncertainty concerning their future productivity when making an investment decision to enter the market. Following [Bilbiie et al. \(2012a\)](#), firm entry is subject to sunk product development costs, which investors pay in expectation of future profits. Firms join the market up to the point where the expected value of their newly created product equals its sunk development cost. After entry, firms' production depends on their productivity levels. As in [Colciago and Silvestrini \(2022\)](#), firms face fixed production costs. As a result, given aggregate conditions, firms with idiosyncratic productivity levels below a specific threshold will be forced to discontinue production and stay inactive until production becomes profitable again. Third, we add to our Dynamic Stochastic General Equilibrium (DSGE) model an energy block. The production of both manufacturing goods and services requires a composite energy bundle, together with labor. We describe the former as a CES aggregate of two sources of energy, produced respectively with fossil and renewable resources. The clean and fossil resources are imported from abroad at exogenously given prices, and energy shocks affect the price of the fossil resource. In line with empirical regularities, firms in the energy sector operate in a perfectly competitive environment, implying their prices are fully flexible. Finally, to analyze the role of monetary policy, our framework assumes nominal rigidities in the form of sticky wages and prices for intermediate goods producers.

We analyze the productivity and reallocation effects ignited by a persistent rise in the price of fossil resources. We study the role played by monetary policy in shaping the response of the economy to the shock. Next, we briefly summarize our findings.

Our results suggest that a higher price of energy will have positive effects on average productivity within the final sectors, but a negative impact on business creation and aggregate activity. The increase in the price of energy produced with a fossil resource entails a reallocation of activity across firms within each final sector, and across sectors. Both effects are absent in a standard one-sector, homogeneous-firms framework. The mechanism is as follows. The higher cost of energy resulting from the increase in the price of the fossil resource implies a surge in marginal costs of production in both manufacturing and services, but more so in manufacturing which is energy intensive. The increase in marginal costs results in both cleansing and selection within industries. Firms with low idiosyncratic productivity are forced into inactivity since they can no longer break even on their costs. Thus, the endogenous component of exit increases. New entrants must be more productive to be profitable. As a result the entry rate declines. This process leads to a surge in average firm productivity within each sector. The cleansing and selection process implies a shake-up in both industries, that promotes a reallocation of activity to more productive firms. Higher productivity does not, however, prevent a substantial reduction in output.

Notably, sectoral differences arise due to the asymmetry in energy intensity utilized for production in each sector. Overall, the increased exit and lower entry rates are more sizeable

for the manufacturing than service sector, and translate to a larger increase in manufacturing productivity; the manufacturing sector suffers a larger increase in marginal costs after the shock, due to its larger energy-intensive nature (5% vs. 1.7%). Taken together, a fossil price shock leads to temporary sectoral reallocation from manufacturing to services. Moreover, the increase in energy prices produced with fossil fuel implies that the final energy bundler will resort more heavily to clean energy for the production of the energy input for the final sectors, promoting a greener energy mix in both manufacturing and services.

A central bank with a strong anti-inflationary stance operates by limiting the increase in marginal costs of production for firms producing the final goods, and less so by mitigating energy prices. Tighter monetary policy produces a stronger negative effect on demand, and *ceteris paribus*, reduces the revenues of firms, lowering the demand for labour and thus wages. A fall in wages translates into a lower idiosyncratic productivity level required for survival and entry into the market. However, the monetary policy stance does little to affect aggregate energy prices as the reduction in wages due to the fall in labour demand compensates for the increase in the price of clean energy that occurs as a result of the substitution into less costly clean energy inputs.

Therefore, a monetary policy that fights inflation more actively leads to a weaker response of both headline and core inflation and a larger output loss. While this implies that the increase in productivity is less sizeable with respect to that observed under a looser policy rule, it speeds the recovery of the entry rate of new firms, sustaining business dynamism in the medium-term. Moreover, given that the manufacturing sector is characterized by less energy-intensive technology, output in manufacturing declines by more than output in services, enabling monetary policy to amplify the rate of sectoral reallocation that occurs in the short- to medium-term. Nevertheless, the initial output drop is significantly larger under a tighter policy. Our results suggest that monetary policy faces a trade-off between stabilizing aggregate activity (and productivity) and business dynamism.

Related literature Our work is related to the still scarce literature assessing the implications of monetary policy in mitigating increases in energy prices in DSGE model-based studies.

Papers closely related to ours are those that study the supply-side effect of energy price shocks. [Baqae and Farhi \(2019\)](#), [Baqae and Farhi \(2022\)](#) and [Bachmann et al. \(2022\)](#), find that rises in energy prices have a very limited effect on GDP, given realistic substitution elasticities. [Kharroubi and Smets \(2023\)](#) analyze how negative energy supply shocks can manifest as negative demand shocks, or a Keynesian supply shock. However, since these papers abstract from nominal rigidities, they do not feature a role for monetary policy. Instead, [Bodenstein et al. \(2012\)](#) and [Pataracchia et al. \(2023\)](#) incorporate a role for monetary policy through nominal rigidities. Our paper is complementary but considers heterogeneity in the supply side of the economy and therefore speaks to the impact of monetary policy in affecting business dynamism and sectoral reallocation.

Much less vast, but closely related to our paper, is the literature featuring microeconomic heterogeneity and energy-related issues. [Känzig \(2023\)](#) and [Auclert et al. \(2023\)](#) study the macroeconomic effects of energy price shocks in energy-importing economies using a heterogeneous-agent New Keynesian model.¹ In their framework, increases in energy prices depress real incomes and cause a recession, even if the central bank does not tighten monetary policy. Our paper is complementary to their analysis since it considers heterogeneity in the supply side of the economy combined with endogenous entry and exit dynamics.

Finally, our work relates to the broader literature assessing the interaction between climate outcomes and the economy, which also feature an energy sector of different granularity into the framework. [Golosov et al. \(2014\)](#) is one of the first contributions to add fossil fuel (oil and coal) inputs in an otherwise standard DSGE model. For additional contributions, see e.g., [Annicchiarico and Di Dio \(2015\)](#), [Bartocci et al. \(2022\)](#), [Airaud et al. \(2022\)](#), [Ferrari and Nispi Landi \(2022\)](#), [Varga et al. \(2022\)](#), [Coenen et al. \(2023\)](#), [Finkelstein Shapiro and Metcalf \(2023\)](#), among others. However, in all these studies, the primary focus is on the impact of carbon taxation in affecting macroeconomic outcomes.

2 The model

The economy features 4 sectors. Two of them produce energy and are indexed with (ι), the remaining two sectors produce, instead, sectorial goods and are indexed with (q). The two sectors generating energy adopt two distinct natural resources, besides labor, in the production process. In sector ($\iota = d$) energy is produced with a fossil input, which is exhaustible. In sector ($\iota = c$) the input is, instead, a renewable source of energy. For these reasons, we will refer to energy produced in sector ($\iota = d$) as dirty energy, while that produced in sector ($\iota = c$) as clean energy. Final goods sectors differ due to the production technology. Sector $q = g$, meant to represent the service sector, is characterized by a labor-intensive technology, while sector ($q = b$) identifies an energy-intensive sector, such as manufacturing. As a result, we will refer to the service sector as the green sector, while to manufacturing as the brown sector. Each final good sector is characterized by ex-ante heterogeneous firms, which produce a good in different varieties, compete monopolistically, and are subject to price adjustment costs. The mass of firms in each of the two final goods sectors is determined endogenously by firms' entry and exit, which are modeled at the sectoral level. Sectoral goods are then aggregated into sectoral bundles. The economy features a unitary continuum of homogeneous households, which use the final good for consumption and investment purposes. Wages are subject to Calvo (1983) price stickiness. Finally, the Central Bank sets the nominal interest rate using a Taylor rule.

¹Also see [Chan et al. \(2022\)](#); [Bayer et al. \(2023\)](#); [Langot et al. \(2023\)](#); [Pieroni \(2023\)](#) for recent works on this topic.

2.1 The final consumption good and its composition

The final consumption good is assumed to be a composite between a ‘core’ good, denoted by Y_t^{core} , and energy, denoted by E_t^H . We assume that these two goods are bundled together using the following CES aggregator:

$$Y_t = \left(\omega^{\frac{1}{\tilde{\eta}}}(Y_t^{core})^{\frac{\tilde{\eta}-1}{\tilde{\eta}}} + (1-\omega)^{\frac{1}{\tilde{\eta}}}(E_t^H)^{\frac{\tilde{\eta}-1}{\tilde{\eta}}} \right)^{\frac{\tilde{\eta}}{\tilde{\eta}-1}}, \quad (1)$$

where the parameter ω captures the relative importance of the core good compared to energy in the consumption bundle, and the parameter $\tilde{\eta}$ measures the elasticity of substitution between the two goods. The price of the final good, denoted P_t , satisfies $P_t Y_t = P_t^{core} Y_t^{core} + P_t^E E_t^H$, where P_t^{core} and P_t^E denote the price of the core and the energy good, respectively. Headline inflation measures the growth rate of P_t , that is $\pi_t = \frac{P_t}{P_{t-1}}$. Demand functions for the core and the energy goods read as:

$$Y_t^{core} = \omega \left(\frac{P_t^{core}}{P_t} \right)^{-\tilde{\eta}} Y_t \quad (2)$$

$$E_t^H = (1-\omega) \left(\frac{P_t^E}{P_t} \right)^{-\tilde{\eta}} Y_t \quad (3)$$

The core good is an aggregate of the two sectorial goods $Y_t(g)$ and $Y_t(b)$, denoting aggregate output in the services (green) and manufacturing (brown) sector, respectively:

$$Y_t^{core} = \left(\chi^{\frac{1}{\eta}} Y_t(g)^{\frac{\eta-1}{\eta}} + (1-\chi)^{\frac{1}{\eta}} Y_t(b)^{\frac{\eta-1}{\eta}} \right)^{\frac{\eta}{\eta-1}} \quad (4)$$

Both $Y_t(g)$ and $Y_t(b)$ are aggregators of goods produced in the green and brown sector, respectively. The parameter χ captures the relative importance of services in the consumption basket and determines the steady state size of the sector, while the parameter $\eta > 1$ measures the elasticity of substitution between services and manufactured goods. The deflator on the price of core goods is denoted P_t^{core} , and its growth rate $\pi_t^{core} = \frac{P_t^{core}}{P_{t-1}^{core}}$ is our measure of core inflation.

The demand for the green good and brown goods read as:

$$Y_t(g) = \chi \left(\frac{P_t(g)}{P_t^{core}} \right)^{-\eta} Y_t^{core} \quad (5)$$

$$Y_t(b) = (1-\chi) \left(\frac{P_t(b)}{P_t^{core}} \right)^{-\eta} Y_t^{core}. \quad (6)$$

2.2 Households

The expected lifetime utility of the representative household at time $t = 0$ is:

$$E_0 \sum_{t=0}^{\infty} \beta^t \left(\log(c_t) - \chi \frac{(l_t^s)^{1+\phi}}{1+\phi} \right) \quad (7)$$

where c_t denotes household's consumption of the final good Y_t , and l_t^s the amount of labor (hours) supplied by the representative agent.

In each time period t , agents can purchase any desired state-contingent nominal payment A_{t+1} in period $t+1$ at the dollar cost $E_t \Lambda_{t,t+1} A_{t+1} / \pi_{t+1}$, where $\Lambda_{t,t+1}$ denotes the stochastic discount factor between period $t+1$ and t , and π_{t+1} denotes the inflation rate over the same period. Households choose consumption, hours of work, and how much to invest in state-contingent assets and in risky stocks $k_{t+1}(q)$. Stock ownership ensures to households a flow of dividend distributed by operative firms. The timing of investment in the stock market is as in [Bilbiie et al. \(2012a\)](#) and [Chugh and Ghironi \(2011\)](#). At the beginning of period t , the household owns $k_t(q)$ shares of a sector mutual fund that represents the ownership of the $N_t(q)$ incumbents in sector (q) in period t , with $(q) = \{(g), (b)\}$. The period- t asset value of the portfolio of firms held in sector (q) is the total firms' value in sector (q) , given by the product between the average value of a firm $\tilde{v}_t(q)$ and the existing mass of firms $N_t(q)$ in the same sector. To obtain the total value of the portfolio held by households, one needs to sum over the two sectoral funds. During period t , the household purchases $k_{t+1}(q)$ shares in new sectoral funds to be carried to period $t+1$. Since the household does not know which firms will disappear from the market, it finances the continued operations of all incumbent firms as well as those of the new entrants, $N_t^e(q)$, although at the very end of period t a fraction of these firms disappears. The value of total stock market purchases is thus $\sum_{q=g,b} \tilde{v}_t(q) (N_t(q) + N_t^e(q)) k_{t+1}(q)$. Households derive income from two sources: labor and dividend. We assume a continuum of differentiated labor inputs indexed by $j \in [0, 1]$. Wages are set by labor type specific unions, indexed by $j \in [0, 1]$. Given the nominal wage, W_t^j , set by union j , agents stand ready to supply as many hours to labor market j , L_t^j , as required by firms, that is:

$$L_t^j = (W_t^j / W_t)^{-\theta_w} L_t^d, \quad (8)$$

where θ_w is the elasticity of substitution between labor types, W_t is an aggregate nominal wage index, and L_t^d is aggregate labor demand. Agents are distributed uniformly across unions, hence aggregate demand for labor type j is spread uniformly across households. The labor market structure rules out differences in labor income between households without the need to resort to contingent markets for hours. The common labor income is given by:

$$\int_0^1 (w_t^j L_t^j) dj = L_t^d \int_0^1 w_t^j (w_t^j / w_t)^{-\theta_w} dj. \quad (9)$$

Operative firms in the final good markets distribute dividends, following the production and sales of varieties in the imperfectly competitive goods markets. Operative firms in sector (q) , that we denote as $N_{o,t}(q)$ and formally define below, are the firms that are actively producing in the final goods sectors at time t . As shown in the Online Appendix, total dividends received by a household in a sector can be written as $N_{o,t}(q) \tilde{e}_t(q)$, where $\tilde{e}_t(q)$ denotes average sectoral dividends, that is the amount of dividends distributed by the firm

with average sectoral productivity. We can write the budget constraint of the representative household as:

$$\begin{aligned}
& c_t + a_{t+1} + \sum_{q=g,b} \tilde{v}_t(q)[N_t(q) + N_t^e(q)]k_{t+1}(q) \\
& = R_t \frac{a_t}{\pi_t} + L_t^d \int_0^1 w_t^j \left(\frac{w_t^j}{w_t} \right)^{-\theta_w} dj + \sum_{q=g,b} [\tilde{v}_t(q)N_t(q) + N_{o,t}(q)\tilde{e}_t(q)]k_t(q) + T_t \quad (10)
\end{aligned}$$

where a_t denotes holdings of a risk-free asset, which gives the households a nominal (gross) return R_t . T_t denotes real lump sum transfers from the Government.

For brevity, the first order conditions of the household's problem are reported in the Online Appendix.

2.3 The production of Energy

The production of energy requires labor and a natural resource. We distinguish between clean and dirty energy. The former is produced using a renewable natural resource as input, the latter instead requires a resource of fossil origin. The prices of clean and dirty resources are exogenous, and supply is perfectly elastic at the price. The price of fossil is subject to exogenous shocks. Clean and dirty energy are then bundled by an energy provider in order to produce the energy which is sold on the market. For simplicity, we assume that energy producers and energy providers work in a perfectly competitive environment.

The Energy Provider The energy provider bundles clean energy, $E_{C,t}$, and dirty energy, $E_{D,t}$, with the following CES production function:

$$E_t = \left[\xi^{\frac{1}{\rho}} E_{D,t}^{\frac{\rho-1}{\rho}} + (1-\xi)^{\frac{1}{\rho}} E_{C,t}^{\frac{\rho-1}{\rho}} \right]^{\frac{\rho}{\rho-1}} \quad (11)$$

where ρ is the elasticity of substitution between clean and dirty energy, and ξ their relative weight in the input bundle. The variable E_t is the quantity of energy produced at time t that is sold to the market. The demand functions of clean and dirty energy have the usual CES form:

$$E_{C,t} = (1-\xi) \left(\frac{P_t^{EC}}{P_t^E} \right)^{-\rho} E_t \quad (12)$$

$$E_{D,t} = (\xi) \left(\frac{P_t^{ED}}{P_t^E} \right)^{-\rho} E_t \quad (13)$$

where P_t^{ED} is the price of one unit of dirty energy, P_t^{EC} is that of clean energy, and P_t^E is the price of one unit of the energy provided to the market. All prices are defined below.

Dirty energy. The production function of dirty energy reads as:

$$E_{D,t} = \left[\xi_D^{\frac{1}{\rho_D}} (A_t L_{D,t})^{\frac{\rho_D-1}{\rho_D}} + (1 - \xi_D)^{\frac{1}{\rho_D}} (Z_t^D X_{D,t})^{\frac{\rho_D-1}{\rho_D}} \right]^{\frac{\rho_D}{\rho_D-1}} \quad (14)$$

where the variable $X_{D,t}$ denotes the quantity of the fossil resource used in period t production, while $L_{D,t}$ is the labor input. Notice that A_t denotes labor productivity, while Z_t^{FR} denotes the efficiency of the fossil resource at translating into energy. The fossil resource $X_{D,t}$ is assumed to be imported at the (exogenous) price P_t^{FR} per unit.

Profits can be expressed as $p_t^{ED} E_{D,t} - TC_t^{ED}$, where $TC_t^{ED} = W_t L_{D,t} + p_t^{FR} X_{D,t}$ denotes total costs of production. Under perfect competition, profit maximization implies:

$$P_t^{ED} = MC_{D,t} \quad (15)$$

where $MC_{D,t}$ are marginal costs of production in the dirty energy sector. In the Online Appendix, we show that marginal costs can be expressed as:

$$MC_{D,t} = \left(\xi_D \left(\frac{W_t}{A_t} \right)^{1-\rho_D} + (1 - \xi_D) \left(\frac{P_t^{FR}}{Z_t} \right)^{1-\rho_D} \right)^{\frac{1}{1-\rho_D}} \quad (16)$$

Clean Energy. As for the production of dirty energy, assume that clean energy is produced through the following CES production function:

$$E_{C,t} = \left[\xi_C^{\frac{1}{\rho_C}} (A_t L_{C,t})^{\frac{\rho_C-1}{\rho_C}} + (1 - \xi_C)^{\frac{1}{\rho_C}} (Z_t^{RR} X_{C,t})^{\frac{\rho_C-1}{\rho_C}} \right]^{\frac{\rho_C}{\rho_C-1}} \quad (17)$$

where the variable $X_{C,t}$ denotes period- t demand of the renewable resource, and Z_t^C is its productivity. We assume that the available quantity of the renewable source of energy is constant and equal to CR , and denote with P_t^{RR} its unitary price. The variable $L_{C,t}$ denotes labor demand in this sector, where we assumed that labor productivity is the same across sectors.

Assuming perfect competition, the price of clean energy is:

$$P_t^{EC} = MC_{C,t} \quad (18)$$

and (nominal) marginal costs of production can be expressed as:

$$MC_{C,t} = \left(\xi_C \left(\frac{W_t}{A_t} \right)^{1-\rho_C} + (1 - \xi_C) \left(\frac{P_t^{RR}}{Z_t} \right)^{1-\rho_C} \right)^{\frac{1}{1-\rho_C}} \quad (19)$$

Energy price index The price index for the aggregate energy good P_t^E must satisfy $P_t^E E_t = P_t^{ED} E_{D,t} + P_t^{EC} E_{C,t}$. Substituting in the demand functions of clean and dirty

energy, we can show that:

$$P_t^E = \left[\xi (P_t^{ED})^{1-\rho} + (1-\xi) (P_t^{EC})^{1-\rho} \right]^{\frac{1}{1-\rho}} \quad (20)$$

2.4 Sectorial goods and services

Each sector (q) is populated by a mass $N_t(q)$ of atomistic firms. Upon entry, firms draw a time-invariant idiosyncratic productivity level, denoted by z , from a known distribution function, $g(z)$, which is identical across sectors and has a positive support. Within their sector of operation, the only source of heterogeneity across firms is the idiosyncratic productivity level, so that we can index firms within a sector with z . Firms compete monopolistically within the sector and are subject to entry and exit. Each firm produces an imperfectly substitutable good $y_{z,t}(q)$, using the following constant return to scale production function:

$$y_{z,t}(q) = z Z_t(q) l_{z,t}(q)^{1-\alpha_q} E_{z,t}(q)^{\alpha_q}, \quad (21)$$

where the variable Z_t is an exogenous level of productivity, common to all firms in the sector. The two inputs are labor, $l_{z,t}(q)$, and energy, $E_{z,t}(q)$. The labor input is defined as a CES aggregator of differentiated labor inputs indexed by $j \in [0, 1]$, defined as:

$$l_{z,t} = \left(\int_0^1 (l_{z,t}^j)^{\frac{\theta_w-1}{\theta_w}} dj \right)^{\frac{\theta_w}{\theta_w-1}}, \quad (22)$$

where $\theta_w > 1$ is the degree of substitution between labor inputs. The goods-producing sector is assumed to be more energy intensive than the service sector. For this reason, we denote the service sector with the letter (q)=(g), where (g) stands for green, and the goods-producing sector with (q)=(b), where (b) stands for brown. Consistently with this assumption, we differentiate the production functions across the two sectors by assuming that $\alpha_g < \alpha_b$. The goods $y_{z,t}(q)$ are input to the production of a sectoral bundle, $Y_t(q)$, by a sectoral good producer that operates in perfect competition. The latter adopts a CES production function defined as:

$$Y_t(q) = \left(\int_0^\infty N_t(q) y_{z,t}(q)^{\frac{\theta-1}{\theta}} g(z) dz \right)^{\frac{\theta}{\theta-1}}, \quad (23)$$

where $\theta > 1$ is the degree of substitution between goods within a specific sector. Firms face fixed costs of production $f_{x,t}$, defined in terms of the final good. The demand for the individual good z in sector q is given by:

$$y_{z,t}(q) = \left(\frac{p_{z,t}(q)}{P_t(q)} \right)^{-\theta} Y_t(q) \quad (24)$$

2.5 Price setting

Following Rotemberg (1982), we assume that firms face quadratic adjustment costs when setting their price $P_{z,t}(q)$:

$$PAC_{z,t}(q) = \frac{\tau_q}{2} \left(\frac{P_{z,t}(q)}{P_{z,t-1}(q)} - 1 \right)^2 P_{z,t}(q) y_{z,t}(q) \quad (25)$$

where the parameter $\tau_q \geq 0$ governs the degree of price rigidities in sector q . The Online Appendix provides the technical derivations concerning the cost minimization and the profit maximization problem of firm z . The equilibrium real price, $\rho_{z,t}(q) = \frac{P_{z,t}(q)}{P_t}$, reads as:

$$\rho_{z,t}(q) = \mu_{z,t}(q) mc_{z,t}(q), \quad (26)$$

where the price markup, $\mu_{z,t}(q)$, is a function of the desired markup $\frac{\theta}{\theta-1}$ and price adjustment costs. To lighten the analysis, the price markup is defined in the Appendix. Real marginal costs, $mc_{z,t}(q)$, are given by:

$$mc_{z,t}(q) = \frac{1}{z Z_t(q)} \left(\frac{w_t}{1 - \alpha_q} \right)^{1 - \alpha_q} \left(\frac{\rho_t^E}{\alpha_q} \right)^{\alpha_q}, \quad (27)$$

where $\rho_t^E = \frac{P_t^E}{P_t}$ is the relative price of energy.

2.6 Entry and exit

Entry costs take the form:

$$f_{i,t}(q) = \psi_o + \psi_1 N_t^e(q)^\gamma \quad (28)$$

Upon entry, firms draw their idiosyncratic productivity level z_i from a distribution with p.d.f $g(z)$. Firms in a sector become inactive when their profits become lower than zero. The idiosyncratic productivity that makes profits equal to zero is thus the productivity cut-off, defined as $z_t^c(q)$. Firms with idiosyncratic productivity below the cut-off turn inactive until production becomes profitable again. The cut-off productivity level in sector (q) is identified by setting profits of firm z to zero. We define it formally in the Online Appendix.

The number of operative firms in sector q is given by:

$$N_{o,t}(q) = N_t(q) \mathbb{P}[z > z^c(q)] \quad (29)$$

Firms permanently exit the market when they are hit by an exit shock, which takes place with probability $0 \leq \delta \leq 1$. Assuming a one period time to build, the evolution of the

number of firms in each sector is given by:

$$N_t(q) = (1 - \delta)[N_{t-1}(q) + N_{t-1}^e(q)] \quad (30)$$

2.7 Labor Unions and Monetary Policy

Nominal wage rigidities are modeled according to the Calvo (1983) mechanism. In each period a union faces a constant probability $(1 - \alpha^*)$ of reoptimizing the wage. The optimal nominal wage in sector j set at time t , that we denote with W_t^* , is chosen to maximize agents' lifetime utilities.² Due to symmetry, the newly reset wage is identical across labor markets. The Central Bank sets the nominal interest rate, R_t , according to the following Taylor rule with smoothing:

$$\left(\frac{R_t}{R}\right) = \left[\left(\frac{\pi_t}{\pi}\right)^{\phi_\pi} \left(\frac{Y_t}{Y}\right)^{\phi_Y} \right]^{1-\phi_R} \left(\frac{R_{t-1}}{R}\right)^{\phi_R}, \quad (31)$$

where variables without time subscript denote steady state values. For simplicity, we assume that the steady state gross inflation rate equals one.

2.8 Sectoral Average Productivity and Aggregation

To obtain tractable results, a Pareto distribution is assumed for the p.d.f. $g(z)$ with minimum $zmin$ and tail parameter κ . This assumption simplifies considerably several equilibrium conditions and allows us to compute analytical solutions. Following Melitz (2003), a special average productivity is defined over operating firms. In our case, however, the special average productivity is sector-specific and it is defined as $\tilde{z}_t(q)$. The special average productivity allows representing both final goods sectors as populated by a mass of homogeneous firms $N_{o,t}(q)$, each of which is endowed with idiosyncratic productivity $\tilde{z}_t(q)$, as we show in the Online Appendix. Thanks to the properties of the Pareto distribution, we can write $\tilde{z}_t(q)$ as a function of the cut-off productivity, $z_t^c(q)$, as follows:

$$\tilde{z}_t(q) = \left[\frac{1}{1 - G(z_t^c(q))} \int_{z_t^c(q)}^{\infty} z^{\theta-1} g(z) dz \right]^{\frac{1}{\theta-1}} = \Gamma z_t^c(q), \quad (32)$$

where $\Gamma = \left[\frac{\kappa}{\kappa - (\theta - 1)} \right]^{\frac{1}{\theta-1}}$ and $1 - G(z_t^c(q)) = \left(\frac{zmin}{z_t^c(q)} \right)^\kappa$. The latter illustrates that changes in the cut-off productivity levels, due either to carbon taxation or to other exogenous disturbances, lead to changes in average sectoral productivities. In what follows tilded variables refer to the average firm, that is the firm characterized by the special average productivity defined above.

²See the Online Appendix for details.

2.9 Equilibrium

In equilibrium, the representative household holds the entire portfolio of firms and the trade of state-contingent asset trade is nil. As a result, $k_{t+1}(q) = k_t(q) = 1$, and $a_{t+1} = a_t = 0$. Considering the Government budget, the budget constraint of households implies:

$$C_t + N_t^e(b)\tilde{v}_t(b) + N_t^e(g)\tilde{v}_t(g) = w_t L_t^d + N_{o,t}(b)\tilde{e}_t(b) + N_{o,t}(g)\tilde{e}_t(g). \quad (33)$$

Equilibrium in the energy market implies that aggregate energy production needs to be equal to the energy used in both production sectors, and the consumption of energy in the final consumption bundle:

$$E_t = E_t^H + E_t(g) + E_t(b) \quad (34)$$

Equilibrium in the labor market implies that total labor demand by firms must be equal to the sum of labor demand in the two production sectors and the two energy sectors:

$$L_t^d = L_t(g) + L_t(b) + L_{C,t} + L_{D,t} \quad (35)$$

In equilibrium, the value of a firm is equal to the values of entry costs in the sector, implying that $\tilde{v}_t(q) = f_{e,t}(q)$ for $q = g, b$. Combining the above equations and using the definition of firms' profits, we can show that the aggregate resource constraint in the economy is: , thus:

$$Y_t = C_t + (N_{o,t}(g) + N_{o,t}(b)) f_{x,t} + N_t^e(g) f_{e,t}(g) + N_t^e(b) f_{e,t}(b) + PAC_t + \rho_t^{FR} X_t^{FR} + \rho_t^{CR} X_t^{CR}, \quad (36)$$

stating that Y_t is either consumed, used to cover fixed costs of production, adjustment costs and entry costs, or to finance imports of clean and dirty energy resources.

3 Calibration

Table 1 reports calibrated parameters. The time unit is a quarter. Preference parameters are as follows. The discount factor, β , is set to the standard value of 0.98 for quarterly data. The parameter χ_l , denoting the relative utility cost of hours worked, is normalized to unity, and it is held constant across the analysis. The same applies to the Frisch elasticity of labor, ϕ , that we set to 4.

We set the steady state level of the common component of productivity, Z , to 1, while the rate of business destruction, δ , equals 0.025 to match the US empirical level of 10% job destruction per year. The elasticity of substitution between goods is set to $\theta = 3.8$ from Bernard et al. (2003), which is calibrated to fit US plant and macro trade data. The elasticity of substitution across labor types, θ_w , equals 4, which implies a 33% steady state

Table 1: Calibration

Parameter	Description	Value
<i>Households and wage setting</i>		
β	discount factor	0.98
ϕ	inverse Frisch elasticity	4
χ	share of good g in core good bundle	0.7
η	elasticity of substitution between good g and b	1.5
θ_w	elasticity of substitution between labour inputs	4
<i>Firms</i>		
α_g	production share of energy in sector g	0.017
α_b	production share of energy in sector b	0.051
θ	elasticity of substitution between sectoral goods	3.8
ψ_0	entry cost parameter 1	1
ψ_1	entry cost parameter 2	
γ	elasticity of entry cost to number of entrants	1.5
δ	exit rate	2.5%
$zmin$	minimum value of idiosyncratic productivity	1
κ	Pareto distribution parameter	6
<i>Energy sector</i>		
$\tilde{\eta}$	eos between energy and non-energy in consumption	0.94
ω	share of energy in consumption	0.04
ρ	eos between energy inputs	1.8
ξ	share of dirty energy in energy bundle	0.59
ξ_D	labour share, dirty energy sector	0.5
ρ_D	eos between labour and fossil resource	0.25
ξ_C	labour share, clean energy sector	0.5
ρ_C	eos between labour and clean resource	0.25
<i>Monetary policy</i>		
ϕ_π	inflation coefficient	1.5
ϕ_Y	output coefficient	0.25
ϕ_R	interest rate inertia	0.8
<i>Price and wage stickiness</i>		
α^*	Calvo wage stickiness	0.75
τ	Rotemberg price adjustment	77

Notes: Sector g reflects Services. Sector b reflects manufacturing

wage markup.

Turning to the parameters affecting the entry cost function, we set the elasticity of entry rates to $\gamma = 1.5$, in line with the estimate of [Gutierrez Gallardo et al. \(2019\)](#), who exploit the comovements between industry-level entry rates and stock prices to pin down this parameter. The parameters ψ_0 and ψ_1 affect the average firms value.

We normalize ψ_0 to 1, as in [Bilbiie et al. \(2012b\)](#), and then we set the value of ψ_1 such that the entry costs represent 5% of the GDP, in line with the parameterization adopted by [Boar and Midrigan \(2020\)](#).

As for the fixed costs of production, we follow [Ghironi and Kim \(2019\)](#). We calibrate the ratio $\frac{\psi_0}{f}$ to match the evidence reported by [Collard-Wexler \(2013\)](#), who finds that the ratio of entry costs to fixed production costs is approximately 4.5. Changing the entry costs while maintaining the same ratio $\frac{\psi_0}{f}$ does not alter any of the impulse responses.

The parameterization of the productivity distribution is as follows. We normalize, with no loss of generality, z_{min} to 1. In the spirit of [Gabaix \(2011\)](#) and [Di Giovanni and Levchenko \(2012\)](#), our sectors can be defined as granular when $1 < \frac{\kappa}{\theta-1} < 2$. Given the value assigned to θ , we set the baseline value of the Pareto tail parameter $\kappa = 6$. Under this calibration, the benchmark economy is just short of being granular, but the HHI is well defined.³

We assume that the share of the services sector in the total consumption bundle is $\chi = 0.7$, to match the share of services in total production in the US economy, which is approximately 70%. Available evidence suggests that in the US, the energy intensity of the manufacturing sector is about 3 times as large as in the service sector (see e.g. [Gutowski, 2007](#)). Following [Goloso et al. \(2014\)](#) and [Kotlikoff et al. \(2021\)](#), we assume that in the aggregate economy, the energy share in production is about 4%. Given the assumed relative size of the two sectors, we therefore obtain $\alpha_g = 0.017$ and $\alpha_b = 0.051$.

We calibrate the share of labor in dirty and clean energy production to $\xi_D = \xi_C = 0.5$, consistently with US data from the KLEMS database. The elasticity of substitution between labour and fossil/green resources is set following [Coenen et al. \(2023\)](#) who, consistently with values of the Rest-of-the-world block in the ECB-NAWM-E model implies imperfect complementarity between the two inputs; we set $\rho_D = 0.25$ and $\rho_C = 0.25$.

To determine the elasticity of substitution between energy inputs we follow [Papageorgiou et al. \(2017\)](#), and set $\rho = 1.8$, so that clean and dirty energy goods are imperfect substitutes. For the the share of dirty energy, ξ , we again follow [Goloso et al. \(2014\)](#), who report that in the US the share of the coal and oil sectors are respectively 0.5008 and 0.08916. We sum the two shares to define the share of dirty energy, obtaining $\xi = 0.59$. In line with [Bodenstein et al. \(2011\)](#) and [Coenen et al. \(2023\)](#) we assume that energy and non-energy goods in the consumption bundle are imperfect complements and calibrate ω to 0.94. The share of energy in consumption, given by $\tilde{\eta}$ is set to 0.04, reflecting that the contribution of

³Indeed, the HHI is not properly defined if the economy is granular.

energy to the US CPI is approximately 4%.

Regarding nominal rigidities, we assume that wages are reset every three quarters by setting $\alpha^* = 0.75$, while the price adjustment cost parameter for intermediate good production is set to $\tau = 77$ as in [Bilbiie et al. \(2007\)](#).

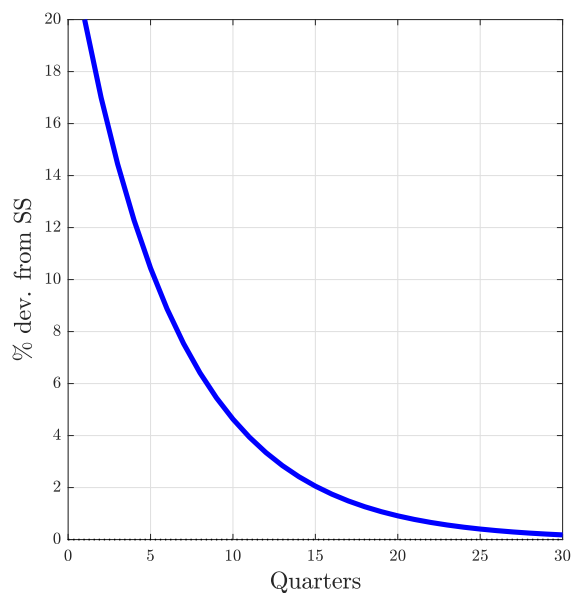
Finally, the baseline values of the parameters of the interest rate rule are set to customary values of $\phi_\pi = 1.5$, $\phi_Y = 0.25$ and $\phi_R = 0.8$. In the section dedicated to studying the role of monetary policy, we will specify the parameterizations of the alternative policy rules that we consider.

4 Quantitative Analysis

4.1 Benchmark case

Our exercise consists of studying the impact of an increase in the price of fossil resources on aggregate macroeconomic variables as well as its differential effects across the manufacturing and services sectors through changes in sectoral productivity and the entry and exit of firms. We then assess the potential of alternative monetary policy rules in mitigating these consequences.

Figure 1: Price of fossil resource



Notes: The price of fossil resources is expressed in percentage deviations from the initial steady state. Time on the horizontal axes is in quarters.

While the global commodity price increased by more than 100% over 2021-2022, we illustrate the channels through a one-period increase in the price of the fossil resource by 20% with persistence equal to 0.85. We select this benchmark to compare against similar studies in the literature; e.g., [Coenen et al. \(2023\)](#) undertake a similar exercise assuming

a 20% permanent increase in the price of fossil resources, while [Pataracchia et al. \(2023\)](#) employ an estimated model of the euro area with energy and assume a 10 USD increase in the Brent oil price with estimated persistence of 0.85. Figure 1 displays the imposed dynamics in the price of the fossil resource, which remains unchanged across all the experiments we run.

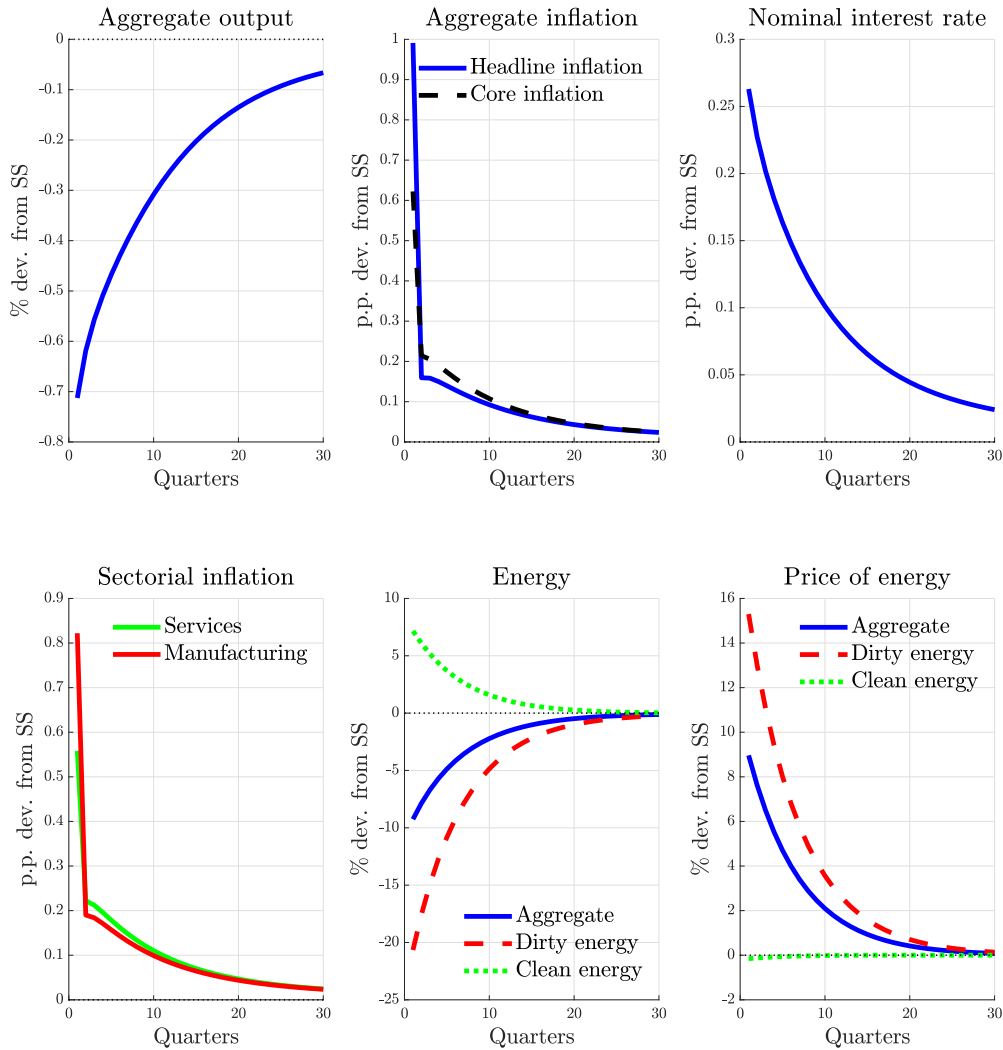
Figure 2 reports the dynamics of aggregate variables under our scenario. The shock to the price of the fossil resource leads to a spike in headline and core inflation, and a decline in aggregate output. Headline inflation increases by 1 percentage point as the increase in the price of the fossil resource leads to an increase in the price of energy, the latter which features directly into consumption. Core inflation increases by 0.6 percentage point as intermediate good producers utilize energy for production and experience an increase in their marginal costs, which they subsequently pass on to final goods producers. While energy prices are fully flexible, the increase in both inflation rates persists over several quarters due to the nominal rigidities in prices at the level of intermediate good producers. Since both the manufacturing and services sectors utilize energy for production, sectoral inflation rates increase. Our calibration, which is consistent with the share of energy in manufacturing being higher than that of services, implies that sectoral inflation in the manufacturing sector increases by relatively more, reaching 0.8 percentage point instead of 0.55 percentage point.

On the real side, aggregate output declines by 0.7% as households experience a fall in their current income due to an increase in the price of energy, while firms cut back on production due to their reduced profitability. The response of aggregate energy production as well as its price however masks heterogeneity in the response of the production of different energy types. The increase in the price of the fossil resources puts upward pressure on the price of dirty energy incentivizing firms to substitute into less costly clean energy, as dictated by the calibration of the elasticity of substitution across energy types. As a result, the production of clean energy increases. While increase in the demand for clean energy puts upward pressure on its price, the reduction in wages due to lower demand for labour by firms, implies that in equilibrium the price of clean energy remains little affected. Nevertheless, the results suggest that the fossil price increase promotes a greener mix in the production of energy.

The monetary authority in our benchmark scenario is a strict inflation targeter and reacts to deviations of headline inflation from target. Consistent with the strong increase in headline inflation, it increases the nominal interest rate by 0.25 percentage point.

Quantitatively, our results on aggregate variables remain within the varying range of estimates in the comparable literature. For example, [Peersman and Van Robays \(2012\)](#) find that a permanent 10% increase in the price of oil raises the level of consumer prices by about 0.4% and lowers GDP by about 0.3% in the long run. While, [Pataracchia et al. \(2023\)](#) find that a 10 USD increase in the Brent price increases CPI inflation by 0.4 percentage points and lowers output by 0.25%. Instead, [Coenen et al. \(2023\)](#) find that a 20% permanent increase in the price of fossil resources increases headline inflation by 0.4 percentage points and lowers output by 1.5% in the medium term.

Figure 2: Effects of fossil price shock on aggregate variables



Notes: Variables are expressed in percentage deviations from the initial steady state. Inflation rates and the nominal interest rate are in percentage point deviations from the initial steady state. Time on the horizontal axes is in quarters.

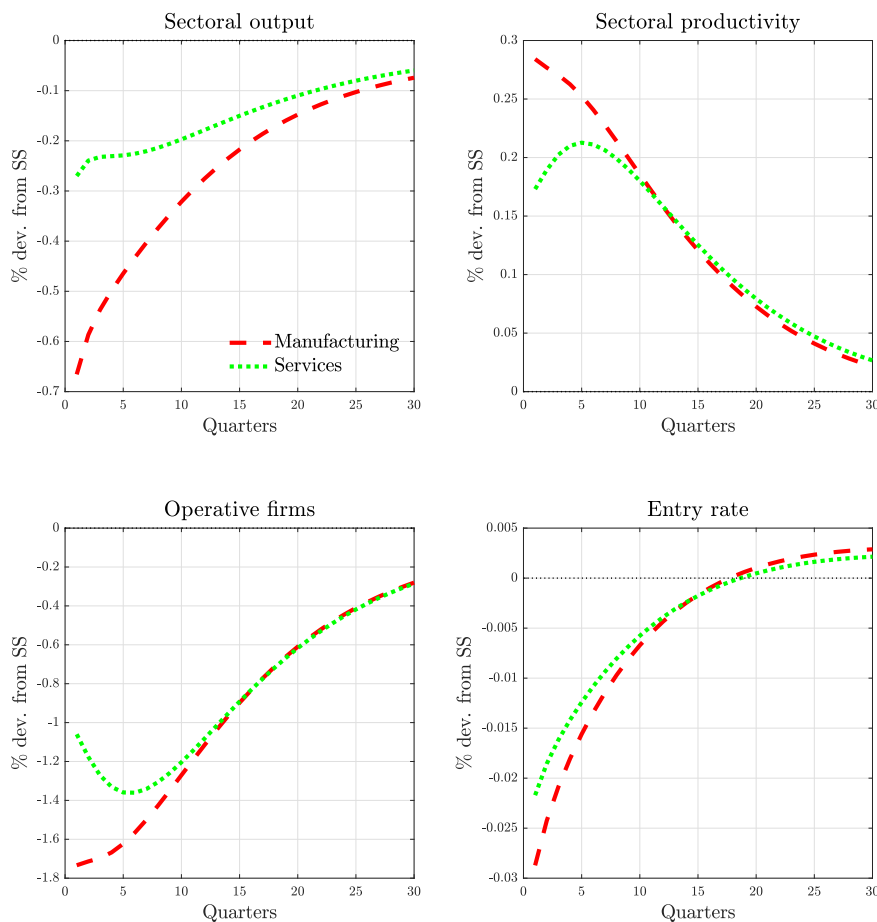
Figure 3 displays the response of sectoral variables. Since fossil resources are used for the production of energy, which is utilized in both sectors, the ensuing higher price of energy affects their profitability, leading to a reduction in output. However, productivity in both manufacturing and services sectors increases in response to the shock. A higher price of energy leads to an increase in the marginal costs of production, which in turn, induce the exit of the least productive businesses. Exit then translates to a persistent reduction in the number of operative firms in the market. At the same time, entry into the market is reduced as following the increase in energy prices a higher idiosyncratic productivity level is required for initiating production. In equilibrium, the smaller pool of active firms in the market are reflected with a higher level of productivity.

Notably, sectoral differences arise due to the asymmetry in energy intensity utilized for production in each sector. Overall, the increased exit and lower entry rates are more sizeable

for the manufacturing than service sector, and translate to a larger increase in manufacturing productivity; the manufacturing sector suffers a larger increase in marginal costs after the shock, due to its energy-intensive nature (5% vs. 1.7%).

Taken together, a fossil price shock leads to temporary sectoral reallocation from manufacturing to services and contributes to structural transformation in the economy. Given however the temporary nature of the shock, all variables return to their initial steady states in the longer-term.

Figure 3: Effects of fossil price shock on sectoral variables



Notes: Variables are expressed in percentage deviations from the initial steady state. Time on the horizontal axes is in quarters.

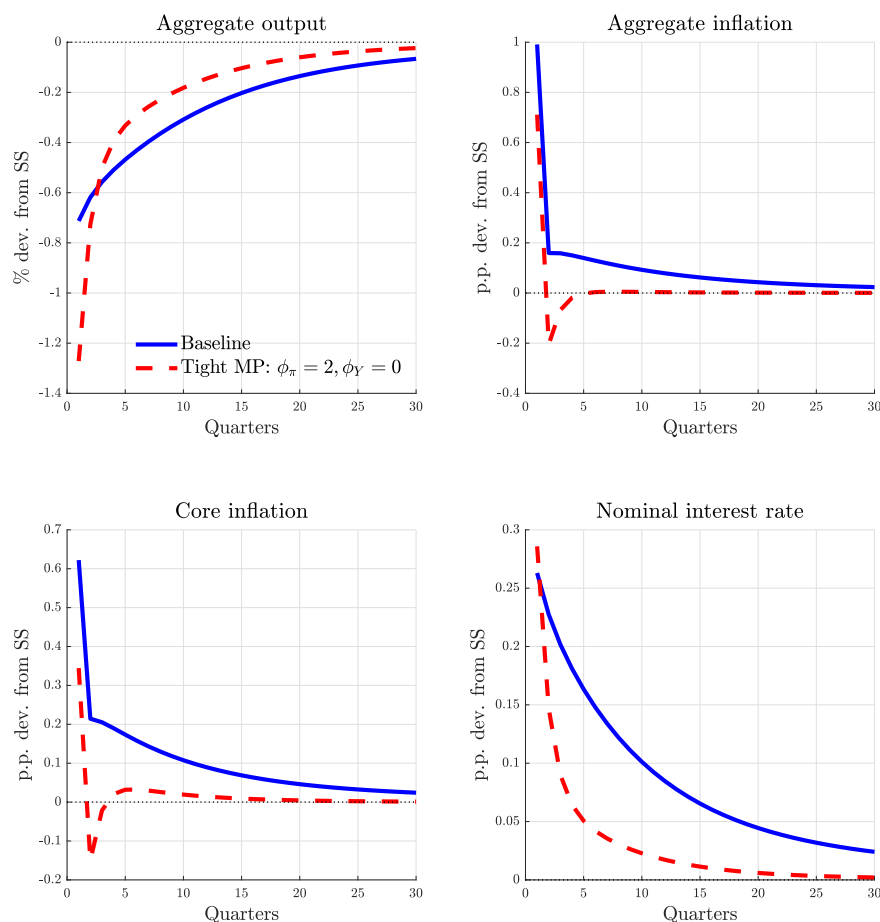
4.2 The role of monetary policy

Our analysis so far has concentrated on the Taylor rule specified in equation (31), where we set the inflation coefficient ϕ_π to the standard value of 1.5 and the coefficient on output, ϕ_Y , to 0.25. A natural question is then to what extent a more active monetary policy stance can meaningfully bring down inflation or mitigate the contraction in real activity. Importantly, we evaluate whether an alternative monetary policy could help prevent the decline in business dynamism and the associated structural transformation of a temporary

nature identified in our baseline scenario.

Figures (4) to (6) compare the responses of aggregate and sectoral variables of interest to the fossil fuel price shock, respectively, under alternative monetary policies. Blue solid lines refer to the baseline monetary policy rule, while dashed red lines refer to a tighter monetary policy featuring a stronger response to inflation and without a response to output, where $\phi_\pi = 2$ and $\phi_Y = 0$.

Figure 4: Effects of fossil price shock on aggregate variables under alternative monetary policies



Notes: Variables are expressed in percentage deviations from the initial steady state. Inflation rates and the nominal interest rate are in percentage point deviations from the initial steady state. Time on the horizontal axes is in quarters.

Figure (4) reports the responses on aggregate variables. It illustrates how a monetary policy that fights inflation more actively (dashed red lines) leads to a weaker response of both headline and core inflation and a larger output loss. In equilibrium, this is associated with the nominal interest rate increasing by more than in the benchmark case. Notably, tighter monetary policy contributes to causing both inflation rates to temporarily undershoot their steady state in the short term due to its stronger effect on marginal costs.⁴

⁴While this may suggest that the optimal monetary policy response of a central bank that gives a strong

While this result may carry over to other frameworks without business dynamism and firm heterogeneity, our model with firm entry and exit features an additional set of channels through which tighter monetary policy affects productivity. This occurs through its impact on the costs and revenues of firms and the interaction with energy prices. These channels are illustrated in Figure (5), which reports the responses of the components of marginal costs, $\tilde{m}c$, across the services and manufacturing sectors, g and b , respectively, for baseline (left panels) and tight monetary policy (right panels). Marginal costs are defined as the sum of wages, $(1 - \alpha)w$, the price of aggregate energy, $\alpha\rho_E$, minus productivity $-\tilde{z}$.

On the one hand, a tighter monetary policy produces a stronger negative effect on demand (as shown in Figure (4)), and *ceteris paribus*, reduces the revenues of firms. Lower firm revenues then imply that the idiosyncratic productivity level that is required for survival and entry into the market increases. However, lower firm revenues lower the demand for labour and thus through a cost channel also wages. A fall in wages instead lowers the idiosyncratic productivity level required to both break even on costs and for firms to enter the market.⁵ While the relative price of dirty to clean energy also contributes to increasing marginal costs, the results suggest that the monetary policy stance does little to affect aggregate energy prices. This occurs because the fossil resource price increases exogenously in our experiment (motivated by a global fossil price increase) and the fact that the reduction in wages due to the fall in labour demand compensates for the increase in the price of clean energy that occurs as a result of the substitution into less costly clean energy inputs. The latter is partially affected by the calibration of the technology of the clean energy sector whereby the share of labour in the production of clean energy is significant (5%), as implied by empirical evidence.

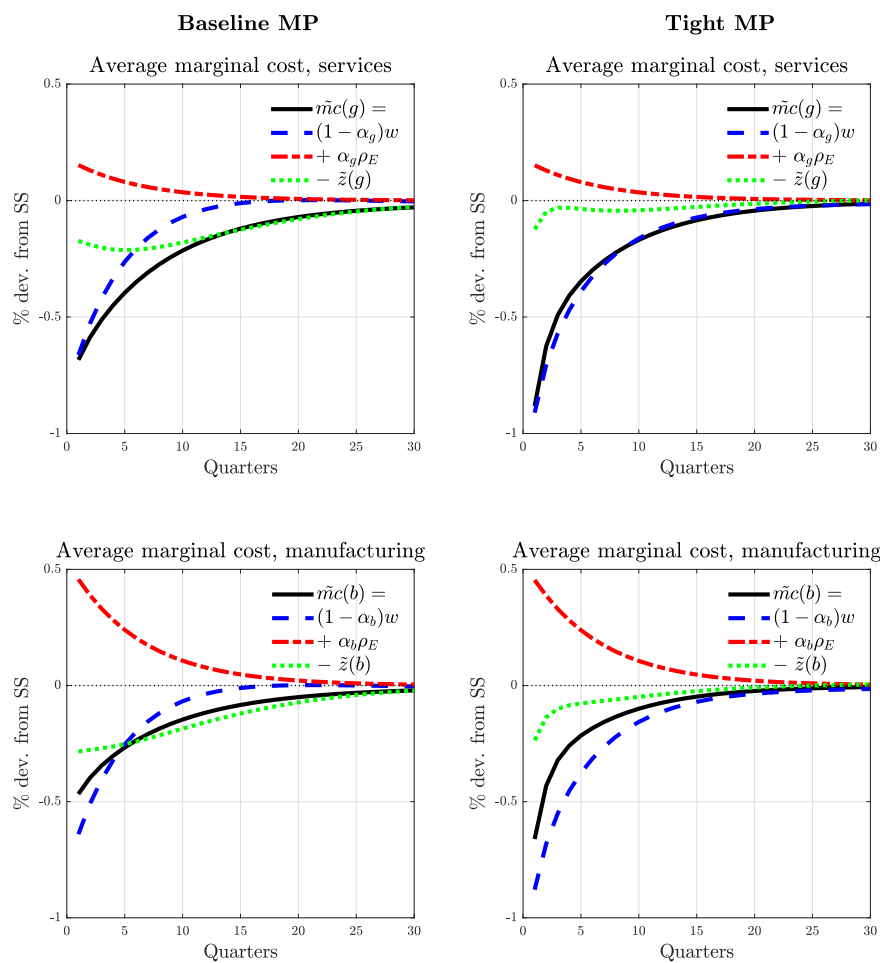
Overall, our model predicts that the revenue channel dominates the cost channel (as shown in Figure (3)), implying an increase in average productivity, however, a tighter monetary policy amplifies the cost channel resulting in lower average productivity under tighter monetary policy. This is illustrated in Figure (6), which reports the responses of different sectoral variables under alternative monetary policies.

Lower average productivity under tighter monetary policy comes at the benefit of lower and less persistent inflation but a more sizeable decrease in the entry rate of firms into the market. Given that the manufacturing sector is characterized by less energy-intensive technology, output in manufacturing declines by more than output in services, giving rise to a role for monetary policy in temporarily altering sectoral size and transforming the

weight to inflation stabilisation should respond aggressively to energy shocks, it should be acknowledged that in practice, lags in the monetary transmission mechanism may imply that the effects of the energy shock pass through quicker than the effects of monetary policy, making the optimal policy response less clear (see e.g. [Bandera et al., 2023](#))

⁵Note that we do not consider financial frictions or physical capital in our analysis, so the policy rate does not affect the marginal costs of firms. If that was the case, the cost channel through which the monetary tightening operates would be milder, as higher interest rates would mitigate the effect of lower wages on marginal costs. However, for reasonable calibrations we still expect the wage channel to dominate in that case.

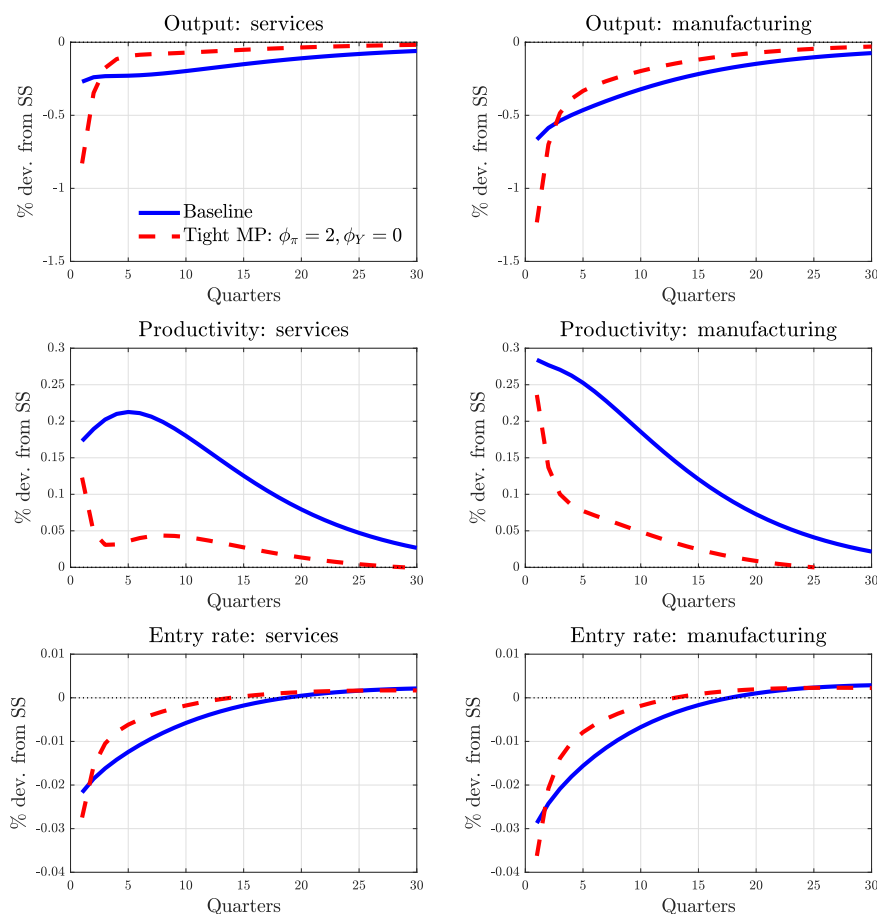
Figure 5: Effects of fossil price shock on components of marginal costs under alternative monetary policies



Notes: Components of marginal costs, ($\tilde{m}c$), across sectors given as the sum of wages, $(1 - \alpha)w$, the price of aggregate energy, $\alpha\rho_E$, minus productivity $-\tilde{z}$. Variables are expressed in percentage deviations from the initial steady state. Left panels: baseline monetary policy with $\phi_\pi = 1.5$ and $\phi_Y = 0.25$. Right panels: tight monetary policy with $\phi_\pi = 2$ and $\phi_Y = 0$. Time on the horizontal axes is in quarters.

economy in the medium-term. However, while a tighter policy implies a stronger negative impact reaction of output, it leads to a faster recovery, in both aggregate output and across sectors. Nevertheless, the initial output drop is significantly larger under a tighter policy, suggesting that a more balance monetary stance may comes with overall better stabilization on average.

Figure 6: Effects of fossil price shock on sectoral variables under alternative monetary policies

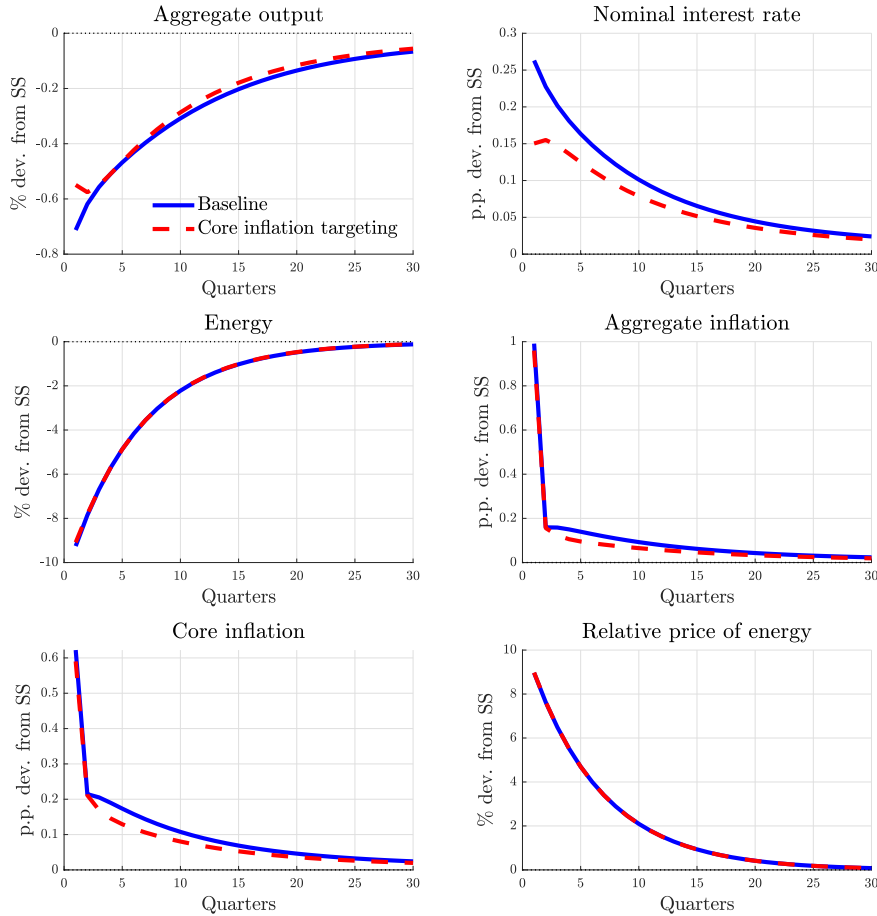


Notes: Variables are expressed in percentage deviations from the initial steady state. Time on the horizontal axes is in quarters.

Finally, Figure (7) illustrates the effects under a monetary policy which targets core inflation rather than headline inflation. The results suggest that targeting core inflation comes with lower output losses, and lower headline inflation in period $t > 1$. This is because headline inflation is heavily driven by energy inflation, which turns negative after one period given the one-period nature of the exogenous fossil price increase and the fact prices of energy producers are typically flexible as suggested by empirical evidence. Instead, intermediate good producers face nominal rigidities and adjust their price in a more staggered fashion, leading to core inflation remaining more persistent, while lower on impact. As a result, the nominal interest rate under core inflation targeting—while lower on impact—will also be more persistent which, in our forward looking model, helps to lower the inflationary impact

of the energy price increase.

Figure 7: Effects of targeting core inflation



Notes: Variables are expressed in percentage deviations from the initial steady state. Inflation rates and the nominal interest rate are in percentage point deviations from the initial steady state. Time on the horizontal axes is in quarters.

5 Conclusion

We have studied the macroeconomic effects of shocks to the price of fossil resources in economies that import natural resources using a multi-sector model with heterogeneous firms. We evaluated the effects of a persistent increase in the price of fossil resources on sectoral reallocation, productivity, and business dynamism, specifically the entry and exit of firms. Sectors are ex-ante heterogeneous in the intensity of their energy use. The rise in the price of energy produced with fossil resources affects the profitability of sectors asymmetrically and leads to a temporarily lower relative size of manufacturing with respect to services. Additionally, it promotes a greener mix of resources in energy production.

We find that the energy price shock triggers a selection and cleansing process that reshapes the competitive landscape of the economy, ultimately leading to higher average pro-

ductivity in both the manufacturing and service sectors. A central bank with a strong anti-inflationary stance can counter-balance the energy price increase and mitigate the inflationary effects of energy shocks by curbing rising production costs while promoting sectoral reallocation. Although this approach entails a higher impact cost in terms of output and lower average productivity, it leads to a faster recovery in business dynamism in the medium term. Our model results suggest that monetary policy can face a trade-off between stabilizing aggregate activity and fostering business dynamism.

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Online Appendix

A Model derivations

A.1 Households and utility maximization

The economy features a continuum of homogeneous households of mass one, and markets are complete. For these reasons we consider a representative household from the outset. The time- t utility of the representative household is:

$$\log(c_t) - \nu \left(\frac{(J_t^s)^{1+\phi}}{1+\phi} \right) \quad (\text{A.1})$$

where c_t is consumption of the final good, and l_t^s denotes labor supply. The parameter χ captures the relative importance of the social good in the consumption basket, while the parameter $\eta > 1$ measures the elasticity of substitution between the social and the non-social goods.

Income and Investment In each time period t , agents can purchase any desired state-contingent nominal payment A_{t+1} in period $t+1$ at the dollar cost $E_t \Lambda_{t,t+1} A_{t+1} / \pi_{t+1}$, where $\Lambda_{t,t+1}$ denotes the stochastic discount factor between period $t+1$ and t , and π_{t+1} denotes the inflation rate over the same period. Households choose consumption, hours of work, and how much to invest in state-contingent assets and in risky stocks $k_{t+1}(q)$. Stock ownership ensures to households a flow of dividend distributed by operative firms. We assume a continuum of differentiated labor inputs indexed by $j \in [0, 1]$. Wages are set by labor type specific unions, indexed by $j \in [0, 1]$. Given the nominal wage, W_t^j , set by union j , agents stand ready to supply as many hours to labor market j , L_t^j , as required by firms, that is

$$L_t^j = (W_t^j / W_t)^{-\theta_w} L_t^d, \quad (\text{A.2})$$

where W_t is an aggregate wage index, and L_t^d is aggregate labor demand. The latter can be obtained by integrating firms' individual labor demand over the distribution of idiosyncratic productivities. Formal definitions of the labor demand and of the wage index can be found in the sections devoted to firms. Agents are distributed uniformly across unions, hence aggregate demand for labor type j is spread uniformly across households. Total hours must satisfy the time resource constraint $L_t^s = \int_0^1 L_t^j dj$. Combining the latter with equation (A.2) we obtain

$$L_t^s = L_t^d \int_0^1 (w_t^j / w_t)^{-\theta_w} dj \quad (\text{A.3})$$

where lower case letters denote wages in real terms. The labor market structure rules out differences in labor income between households without the need to resort to contingent markets for hours. The common labor income is given by

$$\int_0^1 (w_t^j L_t^j) = L_t^d \int_0^1 w_t^j (w_t^j / w_t)^{-\theta_w} dj. \quad (\text{A.4})$$

Besides labor income, households enjoy dividend income through stock ownership. The timing of investment in the stock market is as in [Bilbiie et al. \(2012b\)](#) and [Chugh and Ghironi \(2011\)](#). At the beginning of period t , the household owns $k_t(q)$ shares of a sector mutual fund that represents the ownership of the $N_t(q)$ incumbents in sector (q) in period t , with $(q) = \{(g), (b)\}$.

The period- t asset value of the portfolio of firms held in sector (q) can be expressed as the sum of two components. First, the total firms' value in sector (q) , which is the product between the average value of a firm $\tilde{v}_t(q)$ and the existing mass of firms $N_t(q)$ in the same sector. Second component is total firms' dividends, distributed only by operative firms. Operative firms in sector (q) , that we denote as $N_{o,t}(q)$ and formally define below, are the set of firms that are actively producing in each sector at time t . As shown in Appendix ??, total dividends received by a household in a sector can be written as $N_{o,t}(q)\tilde{e}_t(q)$, where $\tilde{e}_t(q)$ denotes average sectoral dividends, that is the amount of dividends distributed by the firm with average sectoral productivity. To obtain the total value of the portfolio held by households, one needs to sum both components over the two sectoral funds.

During period t , the household purchases $k_{t+1}(q)$ shares in new sectoral funds to be carried to period $t + 1$. Since the household does not know which firms will disappear from the market, it finances the continued operations of all incumbent firms as well as those of the new entrants, $N_t^e(q)$, although at the very end of period t a fraction of these firms disappears. The value of total stock market purchases is thus $\sum_{q=d,b} \tilde{v}_t(q) (N_t(q) + N_t^e(q)) k_{t+1}(q)$.

We can finally write the flow budget constraint of the representative household as:

$$\begin{aligned} & c_t + a_{t+1} + \sum_{q=g,b} \tilde{v}_t(q) (N_t(q) + N_t^e(q)) k_{t+1}(q) \\ &= L_t^d \int_0^1 w_t^j \left(\frac{w_t^j}{w_t} \right)^{-\theta_w} dj + R_t \frac{a_t}{\pi_t} + \sum_{q=g,b} (N_t(q)\tilde{v}_t(q) + N_{o,t}(q)\tilde{e}_t(q)) k_t(q) + T_t \end{aligned} \quad (\text{A.5})$$

Utility Maximization Denoting with V_t the household's value at time t , utility can be written in recursive form as:

$$V_t(\cdot) = \log(c_t) - \nu \left(\frac{(l_t^s)^{1+\phi}}{1+\phi} \right) + \beta E_t V_{t+1}(\cdot) \quad (\text{A.6})$$

The household maximizes (A.6) with respect to c_t , l_t^s , $k_{t+1}(g)$ and $k_{t+1}(b)$ at any t . Constraints to the problem are the household's budget constraint presented above, and the time resource constraint $L_t^s = L_t^d \int_0^1 (w_t^j/w_t)^{-\theta_w} dj$. The recursive utility maximization problem reads as:

$$\begin{aligned}
V_t(a_t, k_t(g), k_t(b)) = & \log(c_t) - \nu \left(\frac{(l_t^s)^{1+\phi}}{1+\phi} \right) + \beta E_t V_{t+1}(a_{t+1}, k_{t+1}(g), k_{t+1}(b)) + \\
& + \lambda_t \left[L_t^d \int_0^1 w_t^j \left(\frac{w_t^j}{w_t} \right)^{-\theta_w} dj + R_t \frac{a_t}{\pi_t} + \sum_{q=g,b} (N_t(q) \tilde{v}_t(q) + N_{o,t}(q) \tilde{e}_t(q)) k_t(q) + \right. \\
& + w_t L_t^d + T_t - c_t - a_{t+1} - \left. \sum_{q=g,b} \tilde{v}_t(q) (N_t(q) + N_t^e(q)) k_{t+1}(q) \right] + \\
& + \frac{\lambda_t w_t}{\tilde{\mu}_t} \left[l_t^s - L_t^d \int_0^1 \left(\frac{w_t^j}{w_t} \right)^{-\theta_w} dj \right] \tag{A.7}
\end{aligned}$$

The First Order Conditions (FOCs) are the following:

$$c_t : \quad \lambda_t = \frac{1}{c_t} \tag{A.8}$$

$$l_t^s : \quad \nu (l_t^s)^\phi = \frac{\lambda_t w_t}{\tilde{\mu}_t} \tag{A.9}$$

$$a_{t+1} : \quad \beta E_t V_{a,t+1} - \lambda_t = 0 \tag{A.10}$$

$$k_{t+1}(g) : \quad \beta E_t V_{b(g),t+1} - \lambda_t \tilde{v}_t(g) (N_t(g) + N_t^e(g)) = 0 \tag{A.11}$$

$$k_{t+1}(b) : \quad \beta E_t V_{b(b),t+1} - \lambda_t \tilde{v}_t(b) (N_t(b) + N_t^e(b)) = 0 \tag{A.12}$$

Finally, the envelope conditions are:

$$V_{a,t} = \lambda_t \frac{R_t}{\pi_t} \tag{A.13}$$

$$V_{k(g),t} = \lambda_t [N_t(g) \tilde{v}_t(g) + N_{o,t}(g) \tilde{e}_t(g)] \tag{A.14}$$

$$V_{k(b),t} = \lambda_t [N_t(b) \tilde{v}_t(b) + N_{o,t}(b) \tilde{e}_t(b)] \tag{A.15}$$

A.2 Labor Unions and wage setting

Nominal wage rigidities are modeled according to the [Calvo \(1983\)](#) mechanism. In each period, a union faces a constant probability $(1 - \alpha^*)$ of re-optimizing the wage. Due to symmetry, we denote the optimal real wage chosen at time t , as w_t^* . This wage is chosen to maximize the relevant part of agents' lifetime utilities. We assume that wages that are not re-optimized are not index to inflation, i.e. $w_{t+s} = \frac{w_t^*}{\prod_{k=1}^s \pi_{t+k}}$

Then, the maximization problem of the union can be written as follows:

$$E_t \sum_{s=0}^{\infty} (\beta\alpha^*)^s \left(\prod_{k=1}^s \pi_{t+k} \right)^{\theta_w} L_{t+s}^d (w_{t+s})^{\theta_w} \lambda_{t+s} \left\{ (w_t^*)^{1-\theta_w} \left(\prod_{k=1}^s \frac{1}{\pi_{t+k}} \right) - \left(\frac{w_{t+s}}{\tilde{\mu}_{t+s}} \right) (w_t^*)^{-\theta_w} \right\} \quad (\text{A.16})$$

The FOCs with respect to w_t^* reads as: -

$$E_t \sum_{s=0}^{\infty} (\beta\alpha^*)^s \left(\prod_{k=1}^s \pi_{t+k} \right)^{\theta_w} L_{t+s}^d (w_{t+s})^{\theta_w} \lambda_{t+s} \left[(1 - \theta_w) (w_t^*)^{-\theta_w} \left(\prod_{k=1}^s \frac{1}{\pi_{t+k}} \right) + \theta_w \left(\frac{w_{t+s}}{\tilde{\mu}_{t+s}} \right) (w_t^*)^{-\theta_w-1} \right] = 0 \quad (\text{A.17})$$

or

$$E_t \sum_{s=0}^{\infty} (\beta\alpha^*)^s \left(\prod_{k=1}^s \pi_{t+k} \right)^{\theta_w} L_{t+s}^d \left(\frac{w_t^*}{w_{t+s}} \right)^{-\theta_w} \lambda_{t+s} \left[w_t^* \frac{(\theta_w-1)}{\theta_w} \left(\prod_{k=1}^s \frac{1}{\pi_{t+k}} \right) + \left(\frac{w_{t+s}}{\tilde{\mu}_{t+s}} \right) \right] = 0 \quad (\text{A.18})$$

For simplicity, define:

$$\left(\frac{w_{t+s}}{\tilde{\mu}_{t+s}} \right) = \chi_{t+s}^* \quad (\text{A.19})$$

such that:

$$E_t \sum_{s=0}^{\infty} (\beta\alpha^*)^s \left(\prod_{k=1}^s \pi_{t+k} \right)^{\theta_w} L_{t+s}^d (w_{t+s})^{\theta_w} \lambda_{t+s} \left[w_t^* \frac{(\theta_w-1)}{\theta_w} \left(\prod_{k=1}^s \frac{1}{\pi_{t+k}} \right) - \chi_{t+s}^* \right] = 0 \quad (\text{A.20})$$

The latter is equivalent to:

$$\frac{(\theta_w-1)}{\theta_w} w_t^* E_t \sum_{s=0}^{\infty} (\beta\alpha^*)^s \left(\prod_{k=1}^s \pi_{t+k} \right)^{\theta_w-1} L_{t+s}^d (w_{t+s})^{\theta_w} \lambda_{t+s} \quad (\text{A.21})$$

$$= E_t \sum_{s=0}^{\infty} (\beta\alpha^*)^s \left(\prod_{k=1}^s \pi_{t+k} \right)^{\theta_w} L_{t+s}^d (w_{t+s})^{\theta_w} \lambda_{t+s} \chi_{t+s}^* \quad (\text{A.22})$$

Define, following [Schmitt-Grohé and Uribe \(2005\)](#):

$$f_t^1 = E_t \sum_{s=0}^{\infty} (\beta\alpha^*)^s \left(\prod_{k=1}^s \pi_{t+k} \right)^{\theta_w-1} L_{t+s}^d (w_{t+s})^{\theta_w} \lambda_{t+s} \quad (\text{A.23})$$

and

$$f_t^2 = E_t \sum_{s=0}^{\infty} (\beta\alpha^*)^s \left(\prod_{k=1}^s \pi_{t+k} \right)^{\theta_w} L_{t+s}^d (w_{t+s})^{\theta_w} \lambda_{t+s} \chi_{t+s}^* \quad (\text{A.24})$$

The first order condition for wage setting is thus:

$$w_t^* = \frac{\theta_w}{\theta_w-1} \frac{f_t^2}{f_t^1} \quad (\text{A.25})$$

where w_t^* is the newly reset wage in real terms, and f_t^1 and f_t^s are recursively defined as:

$$f_t^1 = L_t^d (w_t)^{\theta_w} \lambda_t + \alpha^* \beta E_t \pi_{t+1}^{\theta_w - 1} f_{t+1}^1 \quad (\text{A.26})$$

and

$$f_t^2 = L_t^d (w_t)^{\theta_w} \left(\nu (l_t^s)^\phi \right) + \alpha^* \beta E_t \pi_{t+1}^{\theta_w} f_{t+1}^2 \quad (\text{A.27})$$

since $\lambda_t \chi_t^* = \lambda_t \frac{w_t}{\mu_t} = \nu (l_t^s)^\phi$.

Aggregation For the law of large number, in each period t the wage is optimally reset in a fraction $1 - \alpha^*$ of the labor markets. Demand of hours in each of those markets is:

$$l_t^* = \left(\frac{W_t^*}{W_t} \right)^{-\theta_w} L_t^d \quad (\text{A.28})$$

As a result, total demand of hours in market where the wage has been newly reset is : $L_t^* = (1 - \alpha^*) l_t^*$. In markets where the wage was last reset τ periods ago the demand of hours is:

$$L_{t,\tau} = (1 - \alpha^*) (\alpha^*)^\tau \left(\frac{W_{t,t-\tau}^*}{W_t} \right)^{-\theta_w} L_t^d \quad (\text{A.29})$$

Summing across all possible τ we obtain:

$$L_{t,t-\tau} = (1 - \alpha^*) \sum_{\tau=1}^{\infty} (\alpha^*)^\tau \left(\frac{W_{t,t-\tau}^*}{W_t} \right)^{-\theta_w} L_t^d \quad (\text{A.30})$$

Combining these definitions we can write:

$$L_t^s = L_t^* + L_{t,t-\tau} = (1 - \alpha^*) \sum_{\tau=0}^{\infty} (\alpha^*)^\tau \left(\frac{W_{t,t-\tau}^*}{W_t} \right)^{-\theta_w} L_t^d = \tau_t^* L_t^d \quad (\text{A.31})$$

where τ_t^* measures the resource cost due to wage dispersion. The latter entails an inefficiently large labor supply with respect to the one that is required for production. The variable τ_t^* can be written recursively as:

$$\tau_t^* = (1 - \alpha^*) \left(\frac{w_t^*}{w_t} \right)^{-\theta_w} + \alpha^* \left(\frac{w_{t-1}}{w_t} \right)^{-\theta_w} \pi_t^{\theta_w} \tau_{t-1}^* \quad (\text{A.32})$$

Using the wage index $W_t = \left[\int_0^1 (W_t^j)^{1-\theta_w} dj \right]^{1/(1-\theta_w)}$ one can show that:

$$w_t^{1-\theta_w} = (1 - \alpha^*) (w_t^*)^{1-\theta_w} + \alpha^* \left(\frac{w_{t-1}}{\pi_t} \right)^{1-\theta_w} \quad (\text{A.33})$$

A.3 Energy markets

Energy Provider The energy provider bundles clean energy, $E_{C,t}$, and dirty energy, $E_{D,t}$, with the following CES production function:

$$E_t = \left[\xi^{\frac{1}{\rho}} E_{D,t}^{\frac{\rho-1}{\rho}} + (1-\xi)^{\frac{1}{\rho}} E_{C,t}^{\frac{\rho-1}{\rho}} \right]^{\frac{\rho}{\rho-1}} \quad (\text{A.34})$$

where ρ is the elasticity of substitution between clean and dirty energy, and ξ their relative weight in the input bundle. The variable E_t is the quantity of energy produced at time t that is sold to the market. The demand functions of clean and dirty energy have the usual CES form:

$$E_{C,t} = (1-\xi) \left(\frac{P_t^{EC}}{P_t^E} \right)^{-\rho} E_t \quad (\text{A.35})$$

$$E_{D,t} = (\xi) \left(\frac{P_t^{ED}}{P_t^E} \right)^{-\rho} E_t \quad (\text{A.36})$$

where P_t^{ED} is the price of one unit of dirty energy, P_t^{EC} is that of clean energy, and P_t^E is the price of one unit of the energy provided to the market. All prices are defined below.

Dirty energy. The production function of dirty energy reads as:

$$E_{D,t} = \left[\xi_D^{\frac{1}{\rho_D}} (A_t L_{D,t})^{\frac{\rho_D-1}{\rho_D}} + (1-\xi_D)^{\frac{1}{\rho_D}} (Z_t^D X_{D,t})^{\frac{\rho_D-1}{\rho_D}} \right]^{\frac{\rho_D}{\rho_D-1}} \quad (\text{A.37})$$

where the variable $X_{D,t}$ denotes the quantity of the fossil resource used in period t production, while $L_{D,t}$ is the labor input. Notice that A_t denotes labor productivity, while Z_t^{FR} denotes the efficiency of the fossil resource at translating into energy. The fossil energy is assumed to be imported at the (exogenous) price P_t^{FR} per unit. Further denoting the nominal wage W_t , the Lagrangian for cost minimization reads as:

$$L_t = W_t L_{D,t} + P_t^{FR} X_{D,t} + MC_{D,t} \left(E_{D,t} - \left[\xi_D^{\frac{1}{\rho_D}} N_{D,t}^{\frac{\rho_D-1}{\rho_D}} + (1-\xi_D)^{\frac{1}{\rho_D}} (Z_t^D X_{D,t})^{\frac{\rho_D-1}{\rho_D}} \right]^{\frac{\rho_D}{\rho_D-1}} \right), \quad (\text{A.38})$$

where the variable $MC_{D,t}$ denotes the nominal marginal cost of production of dirty energy. The first-order condition with respect to $L_{D,t}$ reads as:

$$W_t = MC_{D,t} \left(\xi_D^{\frac{1}{\rho_D}} \left(\frac{(A_t L_{D,t})}{E_{D,t}} \right)^{\frac{-1}{\rho_D}} A_t \right), \quad (\text{A.39})$$

while that with respect to $X_{D,t}$ is:

$$P_t^{FR} = MC_{D,t} \left((1 - \xi_D)^{\frac{1}{\rho_D}} \left(\frac{Z_t X_{D,t}}{E_{D,t}} \right)^{\frac{-1}{\rho_D}} Z_t \right) \quad (\text{A.40})$$

From the former we can obtain an expression for labor demanded to produce dirty energy as follows

$$L_{D,t} = W_t^{-\rho_D} MC_{D,t}^{\rho_D} \xi_D E_{D,t} A_t^{\rho_D - 1} \quad (\text{A.41})$$

Similarly, the demand of fossil fuel reads as:

$$X_{D,t} = (P_t^{FR})^{-\rho_D} MC_{D,t}^{\rho_D} (1 - \xi_D) E_{D,t} Z_t^{\rho_D - 1} \quad (\text{A.42})$$

Nominal marginal costs of production of dirty energy must be such that:

$$MC_{D,t} E_{D,t} = W_t L_{D,t} + P_t^{FR} X_{D,t} \quad (\text{A.43})$$

Substituting for factors' demand

$$MC_{D,t} = \left(\xi_D \left(\frac{W_t}{A_t} \right)^{1 - \rho_D} + (1 - \xi_D) \left(\frac{P_t^{FR}}{Z_t} \right)^{1 - \rho_D} \right)^{\frac{1}{1 - \rho_D}} \quad (\text{A.44})$$

Profits can be expressed as $p_t^{ED} E_{D,t} - TC_t^{eD}$, where $TC_t^{eD} = W_t L_{D,t} + p_t^{FR} X_{D,t}$ denotes total costs of production. Under perfect competition, profit maximization implies:

$$P_t^{ED} = MC_t^{ED} \quad (\text{A.45})$$

Clean Energy. As for the production of dirty energy, assume that clean energy is produced through the following CES production function:

$$E_{C,t} = \left[\xi_C^{\frac{1}{\rho_C}} (A_t L_{C,t})^{\frac{\rho_C - 1}{\rho_C}} + (1 - \xi_C)^{\frac{1}{\rho_C}} (Z_t^{RR} X_{C,t})^{\frac{\rho_C - 1}{\rho_C}} \right]^{\frac{\rho_C}{\rho_C - 1}} \quad (\text{A.46})$$

where the variable $X_{C,t}$ denotes period-t demand of the renewable resource, and Z_t^C is its productivity. We assume that the available quantity of the renewable source of energy is constant and equal to CR , and denote with P_t^{RR} its unitary price. The variable $L_{C,t}$ denotes labor demand in this sector, where we assumed that labor productivity is the same across sectors. Taking the same steps as above, cost minimization delivers an expression labor demand as:

$$L_{C,t} = W_t^{-\rho_C} MC_{C,t}^{\rho_C} \xi_C E_{C,t} A_t^{\rho_C - 1}. \quad (\text{A.47})$$

The demand of renewable energy reads as:

$$X_{C,t} = (P_t^{CR})^{-\rho_C} MC_{C,t}^{\rho_C} (1 - \xi_C) E_{C,t} Z_t^{\rho_C - 1}. \quad (\text{A.48})$$

Nominal marginal costs of production of clean energy are:

$$MC_{C,t} = \left(\xi_C \left(\frac{W_t}{A_t} \right)^{1-\rho_C} + (1 - \xi_C) \left(\frac{P_t^{RR}}{Z_t} \right)^{1-\rho_C} \right)^{\frac{1}{1-\rho_C}} \quad (\text{A.49})$$

Assuming perfect competition, the price of clean energy is:

$$P_t^{EC} = MC_t^{EC} \quad (\text{A.50})$$

A.4 Demand for individual goods

The demand for individual goods $y_{z,t}(q)$ can be obtained as the solution to the optimization program of an aggregate sectorial firm that buys goods from individual producers, bundles them, and sell them as a final good $Y_t(q)$ at price $P_t(q)$:

$$\max_{y_{z,t}(q)} \left\{ P_t(q) Y_t(q) - \int_0^\infty N_t(q) p_{z,t}(q) y_{z,t}(q) g(z) dz \right\} \quad (\text{A.51})$$

$$\text{s.t.} \quad Y_t(q) = \left(\int_0^\infty N_t(q) y_{z,t}(q)^{\frac{\theta-1}{\theta}} g(z) dz \right)^{\frac{\theta}{\theta-1}} \quad (\text{A.52})$$

The first order condition gives:

$$P_t(q) \frac{\partial Y_t(q)}{\partial y_{z,t}(q)} - N_t(q) p_{z,t}(q) g(z) = 0 \quad (\text{A.53})$$

where

$$\begin{aligned} \frac{\partial Y_t(q)}{\partial y_{z,t}(q)} &= N_t(q) y_{z,t}(q)^{-\frac{1}{\theta}} g(z) \left(\int_0^\infty N_t(q) y_{z,t}(q)^{\frac{\theta-1}{\theta}} g(z) dz \right)^{\frac{\theta}{\theta-1} - 1} \\ &= N_t(q) y_{z,t}(q)^{-\frac{1}{\theta}} Y_t(q)^{\frac{1}{\theta}} g(z) \end{aligned} \quad (\text{A.54})$$

Using (A.54), equation (A.53) can be rewritten as:

$$P_t(q) y_{z,t}(q)^{-\frac{1}{\theta}} Y_t(q)^{\frac{1}{\theta}} = p_{z,t}(q) \quad (\text{A.55})$$

Re-arranging, we get the demand for good z :

$$y_{z,t}(q) = \left(\frac{p_{z,t}(q)}{P_t(q)} \right)^{-\theta} Y_t(q) \quad (\text{A.56})$$

A.5 Cost minimization

Each sector (q) is populated by a mass $N_t(q)$ of atomistic firms. Once upon entry, firms draw a time invariant idiosyncratic productivity level, denoted by z , from a known distribution function, $g(z)$, which is identical across sectors and has a positive support. Within their sector of operation, the only source of heterogeneity across firms is the idiosyncratic productivity level, so that we can index firms within a sector with z . Firms compete monopolistically within the sector and are subject to entry and exit. Each firm produces an imperfectly substitutable good $y_{z,t}(q)$, which is an input to the production of a sectoral bundle $Y_t(q)$ by a sectoral good producer. The latter adopts a CES production function defined as:

$$Y_t(q) = \left(\int_0^\infty N_t(q) y_{z,t}(q)^{\frac{\theta-1}{\theta}} g(z) dz \right)^{\frac{\theta}{\theta-1}} \quad (\text{A.57})$$

where $\theta > 1$ is the degree of substitution between sectoral goods. The production function of individual goods producers is a constant return to scale Cobb-Douglas function, with parameter $0 \leq \alpha \leq 1$. The two inputs are labor, $l_{z,t}(q)$, and energy, $E_{z,t}(q)$. The individual production function reads as:

$$y_{z,t}(q) = Z_t z l_{z,t}(q)^{1-\alpha} E_{z,t}(q)^\alpha \quad (\text{A.58})$$

where the variable Z_t is an exogenous, and common to all firms, level of productivity. The labor input is defined as a CES aggregator of differentiated labor inputs indexed by $j \in [0, 1]$, defined as:

$$l_{z,t}(q) = \left(\int_0^1 (l_{z,t}^j(q))^{\frac{\theta_w-1}{\theta_w}} dj \right)^{\frac{\theta_w}{\theta_w-1}} \quad (\text{A.59})$$

where $\theta_w > 1$ is the degree of substitution between labor inputs. The minimization of total labor costs, $\int_0^1 W_t^j l_{z,t}^j(q) dj$, delivers firm z 's demand of labor input j and the definition of the aggregate nominal wage index, which are respectively:

$$l_{z,t}^j(q) = \left(\frac{w_t^j}{w_t} \right)^{-\theta_w} l_{z,t}(q) \quad (\text{A.60})$$

and

$$w_t = \left(\int_0^1 (w_t^j)^{1-\theta_w} dj \right)^{\frac{1}{\theta_w-1}} \quad (\text{A.61})$$

where W_t^j (w_t^j) is the nominal (real) wage of labor input j , and $l_{z,t}(q)$ denotes the demand of the labor bundle by firm z . Firms face fixed costs of production $f_{x,t}$, defined in terms of the final good.

Before maximizing profits, firms choose the optimal levels of labor and energy to minimize the costs of production for a given level of idiosyncratic output. The minimization of the

costs of production for a firm is:

$$\min_{l_{z,t}(q), X_{z,t}(q)} W_t l_{z,t}(q) + P_t^E E_{z,t}(q) + P_t f_{x,t} \quad (\text{A.62})$$

subject to the definition of the production function:

$$y_{z,t}(q) = Z_t z l_{z,t}(q)^{1-\alpha_q} X_{z,t}(q)^{\alpha_q} \quad (\text{A.63})$$

Note that $f_{x,t} P_t$ are the nominal fixed costs of production (since $f_{x,t}$ are the real fixed production costs). The Lagrangian is:

$$\mathcal{L} = W_t l_{z,t}(q) + P_t^E E_{z,t}(q) + P_t f_{x,t} + \lambda_{z,t}(q) [y_{z,t}(q) - Z_t z l_{z,t}(q)^{1-\alpha_q} E_{z,t}(q)^{\alpha_q}] \quad (\text{A.64})$$

The F.O.C. with respect to $l_{z,t}(q)$ is:

$$W_t = \lambda_{z,t}(q) (1 - \alpha_q) Z_t z l_{z,t}(q)^{-\alpha_q} E_{z,t}(q)^{\alpha_q} \quad (\text{A.65})$$

while the F.O.C. with respect to $E_{z,t}(q)$ is:

$$P_t^E = \lambda_{z,t}(q) \alpha_q Z_t z l_{z,t}(q)^{1-\alpha_q} E_{z,t}(q)^{\alpha_q-1} \quad (\text{A.66})$$

Combining the two F.O.C.s we get the optimal ratio between the two inputs, which does not depend on idiosyncratic variables nor on sectoral quantities, but it does depend on sectoral technology:

$$\frac{E_{z,t}(q)}{l_{z,t}(q)} = \frac{\alpha_q}{1 - \alpha_q} \frac{W_t}{P_t^E} \quad (\text{A.67})$$

this condition holds in both sectors and for all firms.

Moreover, it is easy to show that $\lambda_{z,t}(q)$ is the marginal cost. First, substitute (A.65) and (A.66) in the cost function

$$\begin{aligned} W_t l_{z,t}(q) + P_t^E E_{z,t}(q) + f_{x,t} P_t &= \lambda_{z,t}(q) (1 - \alpha_q) Z_t z l_{z,t}(q)^{-\alpha_q} E_{z,t}(q)^{\alpha_q} l_{z,t}(q) + \\ &= + \lambda_{z,t}(q) \alpha_q Z_t z l_{z,t}(q)^{1-\alpha_q} X_{z,t}(q)^{\alpha_q-1} E_{z,t}(q) + f_{x,t} P_t = \\ &= \lambda_{z,t}(q) Z_t z l_{z,t}(q)^{1-\alpha_q} X_{z,t}(q)^{\alpha_q} + f_{x,t} P_t = \lambda_{z,t}(q) y_{z,t}(q) + f_{x,t} P_t \end{aligned} \quad (\text{A.68})$$

Hence the cost function is linear in output (CRTS) and $\frac{\partial TC_{z,t}(q)}{\partial y_{z,t}(q)} = MC_{z,t}(q) = \lambda_{z,t}(q)$.

Second, note that from (A.65) and (A.66) we can get the expression for the marginal cost which is given by $\lambda_{z,t}(q)$

$$W_t = \lambda_{z,t}(q) (1 - \alpha_q) Z_t z \left(\frac{E_{z,t}(q)}{l_{z,t}(q)} \right)^{\alpha_q} \quad (\text{A.69})$$

or

$$\begin{aligned} W_t &= \lambda_{z,t}(q) (1 - \alpha_q) Z_t z \left(\frac{\lambda_{z,t}(q) \alpha_q Z_t z}{P^E} \right)^{\frac{\alpha}{1-\alpha}} \\ &= \lambda_{z,t}(q)^{\frac{1}{1-\alpha_q}} (1 - \alpha_q) (Z_t z)^{\frac{1}{1-\alpha_q}} \left(\frac{\alpha_q}{P_t^E} \right)^{\frac{\alpha_q}{1-\alpha_q}} \end{aligned} \quad (\text{A.70})$$

$$\frac{W_t}{1 - \alpha_q} \left(\frac{P_t^E}{\alpha_q} \right)^{\frac{\alpha_q}{1-\alpha_q}} = [\lambda_{z,t}(q) (Z_t z)]^{\frac{1}{1-\alpha_q}} \quad (\text{A.71})$$

Thus:

$$MC_{z,t}(q) = \frac{1}{Z_t z} \left[\frac{W_t}{1 - \alpha_q} \right]^{1-\alpha_q} \left(\frac{P_t^E}{\alpha_q} \right)^{\alpha_q}. \quad (\text{A.72})$$

Marginal costs are affected by both the idiosyncratic productivity level, z , and by aggregate productivity, Z_t . Notice that marginal costs are sector-specific due to the different energy intensity across sectors. Real profits of firm z in sector (q) read as:

$$e_{z,t}(q) = \rho_{z,t}(q) y_{z,t}(q) - w_t l_{z,t}(q) - p_t^E E_{z,t}(q) - PAC_{z,t}(q) - f_{x,t} \quad (\text{A.73})$$

A.6 Price setting

Firms maximise their discounted sum of profits, given by:

$$E_t \sum_{j=0}^{\infty} \Lambda_{t,t+j} (p_{z,t+j}(q) y_{z,t+j}(q) - MC_{z,t+j}(q) y_{z,t+j}(q) - PAC_{z,t}(q) - f_{x,t+j}(q)) \quad (\text{A.74})$$

subject to:

$$y_{z,t+j}(q) = \left(\frac{p_{z,t+j}(q)}{P_{t+j}(q)} \right)^{-\theta} Y_{t+j}(q) \quad (\text{A.75})$$

$$PAC_{z,t}(q) = \frac{\tau_q}{2} \left(\frac{p_{z,t+j}(q)}{p_{z,t+j-1}(q)} - 1 \right)^2 p_{z,t+j}(q) y_{z,t+j}(q) \quad (\text{A.76})$$

The first order condition with respect to $p_{z,t}(q)$ gives:

$$\begin{aligned} 0 &= 1 - \theta + \theta MC_{z,t}(q) \frac{1}{p_{z,t}(q)} - (1 - \theta) \frac{\tau_q}{2} \left(\frac{P_{z,t}(q)}{P_{z,t-1}(q)} - 1 \right)^2 \\ &- \tau_q \left(\frac{P_{z,t}(q)}{P_{z,t-1}(q)} - 1 \right) \frac{P_{z,t}(q)}{P_{z,t-1}(q)} + \tau_q E_t \Lambda_{t,t+1} \left(\frac{P_{z,t+1}(q)}{P_{z,t}(q)} - 1 \right) \left(\frac{P_{z,t+1}(q)}{P_{z,t}(q)} \right)^2 \frac{y_{z,t+1}(q)}{y_{z,t}(q)} \end{aligned} \quad (\text{A.77})$$

which can be re-expressed as:

$$\begin{aligned} \frac{\theta}{\theta-1} MC_{z,t}(q) &= P_{z,t} \left[1 - \frac{\tau_q}{2} \left(\frac{P_{z,t}(q)}{P_{z,t-1}(q)} - 1 \right)^2 + \frac{\tau_q}{\theta-1} \left\{ \frac{P_{z,t}(q)}{P_{z,t-1}(q)} \left(\frac{P_{z,t}(q)}{P_{z,t-1}(q)} - 1 \right) \right. \right. \\ &\quad \left. \left. - E_t \Lambda_{t,t+1} \left(\frac{P_{z,t+1}(q)}{P_{z,t}(q)} - 1 \right) \left(\frac{P_{z,t+1}(q)}{P_{z,t}(q)} \right)^2 \frac{y_{z,t+1}(q)}{y_{z,t}(q)} \right\} \right] \end{aligned} \quad (\text{A.78})$$

Dividing both sides by P_t and using $\frac{P_{z,t}(q)}{P_{z,t-1}(q)} = \frac{\rho_{z,t}(q)}{\rho_{z,t-1}(q)} \pi_t$ we get:

$$\begin{aligned} \frac{\theta}{\theta-1} mc_{z,t}(q) &= \rho_{z,t} \left[1 - \frac{\tau_q}{2} \left(\frac{\rho_{z,t}(q)}{\rho_{z,t-1}(q)} \pi_t - 1 \right)^2 + \frac{\tau_q}{\theta-1} \left\{ \frac{\rho_{z,t}(q)}{\rho_{z,t-1}(q)} \pi_t \left(\frac{\rho_{z,t}(q)}{\rho_{z,t-1}(q)} \pi_t - 1 \right) \right. \right. \\ &\quad \left. \left. - E_t \Lambda_{t,t+1} \left(\frac{\rho_{z,t+1}(q)}{\rho_{z,t}(q)} \pi_{t+1} - 1 \right) \left(\frac{\rho_{z,t+1}(q)}{\rho_{z,t}(q)} \pi_{t+1} \right)^2 \frac{y_{z,t+1}(q)}{y_{z,t}(q)} \right\} \right] \end{aligned} \quad (\text{A.79})$$

Defining:

$$\begin{aligned} \Gamma_{z,t}(q) &\equiv \frac{\rho_{z,t}(q)}{\rho_{z,t-1}(q)} \pi_t \left(\frac{\rho_{z,t}(q)}{\rho_{z,t-1}(q)} \pi_t - 1 \right) \\ -E_t \Lambda_{t,t+1} &\left(\frac{\rho_{z,t+1}(q)}{\rho_{z,t}(q)} \pi_{t+1} - 1 \right) \left(\frac{\rho_{z,t+1}(q)}{\rho_{z,t}(q)} \pi_{t+1} \right)^2 \frac{y_{z,t+1}(q)}{y_{z,t}(q)} \end{aligned} \quad (\text{A.80})$$

and

$$\mu_{z,t}(q) \equiv \frac{\theta}{\theta-1} \left[1 - \frac{\tau_q}{2} \left(\frac{\rho_{z,t}(q)}{\rho_{z,t-1}(q)} \pi_t - 1 \right)^2 + \frac{\tau_q}{\theta-1} \Gamma_{z,t}(q) \right]^{-1} \quad (\text{A.81})$$

we get the pricing equation:

$$\rho_{z,t}(q) = \mu_{z,t}(q) mc_t(q) \quad (\text{A.82})$$

A.7 Productivity cut-off

Real profits of firm z in sector (q) read as:

$$e_{z,t}(q) = [\rho_{z,t}(q) - mc_{z,t}(q)] y_{z,t}(q) - pac_{z,t}(q) - f_{x,t} \quad (\text{A.83})$$

Using the pricing equation (A.82) and the definition of price adjustment costs (A.76), this can be rewritten as:

$$e_{z,t}(q) = \left[1 - \frac{1}{\mu_{z,t}(q)} - \frac{\tau_q}{2} \left(\frac{\rho_{z,t}(q)}{\rho_{z,t-1}(q)} \pi_t - 1 \right)^2 \right] \rho_{z,t}(q) y_{z,t}(q) - f_{x,t} \quad (\text{A.84})$$

Using equation (A.56) and the CES demand function for sectorial goods we can write

the demand for good z as:

$$y_{z,t}(q) = \left(\frac{\rho_{z,t}(q)}{\rho_t(q)} \right)^{-\theta} Y_t(q) \quad (\text{A.85})$$

$$= \Theta \left(\frac{\rho_{z,t}(q)}{\rho_t(q)} \right)^{-\theta} \left(\frac{\rho_t(q)}{\rho_t^C} \right)^{-\eta} (\rho_t^{core})^{-\bar{\eta}} Y_t \quad (\text{A.86})$$

$$= \Theta \rho_{z,t}(q)^{-\theta} \rho_t(q)^{\theta-\eta} (\rho_t^{core})^{\eta-\bar{\eta}} Y_t \quad (\text{A.87})$$

where $\Theta = \omega \chi$ if $q = g$ and $\Theta = \omega (1 - \chi)$ if $q = b$.

Firms turn inactive when, by producing, they would make negative profits. Using this, we can define the cut-off productivity level in sector q as $z_t^c(q)$, below which firms become idle. Setting equilibrium real profits equal to zero we get:

$$f_{x,t} = \Theta \left[1 - \frac{1}{\mu_t^c(q)} - \frac{\tau_q}{2} \left(\frac{\rho_t^c(q)}{\rho_{t-1}^c(q)} \pi_t - 1 \right)^2 \right] \rho_t^c(q) y_t^c(q) \quad (\text{A.88})$$

And the productivity cut-off $z_t^c(q)$, together with variables $\rho_t^c(q)$, $y_t^c(q)$, $mc_t^c(q)$, $\mu_t^c(q)$, $\Gamma_t^c(q)$ are pinned down by (A.88) and the following equations:

$$\rho_t^c(q) = \mu_t^c(q) mc_t^c(q) \quad (\text{A.89})$$

$$mc_t^c(q) = \frac{1}{Z_t z_t^c(q)} \left[\frac{w_t}{1 - \alpha_q} \right]^{1 - \alpha_q} \left(\frac{\rho_t^E}{\alpha_q} \right)^{\alpha_q} \quad (\text{A.90})$$

$$\mu_t^c(q) = \frac{\theta}{\theta - 1} \left[1 - \frac{\tau_q}{2} \left(\frac{\rho_t^c(q)}{\rho_{z,t-1}^c(q)} \pi_t - 1 \right)^2 + \frac{\tau_q}{\theta - 1} \Gamma_t^c(q) \right]^{-1} \quad (\text{A.91})$$

$$\Gamma_t^c(q) = \frac{\rho_t^c(q)}{\rho_{t-1}^c(q)} \pi_t \left(\frac{\rho_t^c(q)}{\rho_{t-1}^c(q)} \pi_t - 1 \right) - E_t \Lambda_{t,t+1} \left(\frac{\rho_{t+1}^c(q)}{\rho_t^c(q)} \pi_{t+1} - 1 \right) \left(\frac{\rho_{t+1}^c(q)}{\rho_t^c(q)} \pi_{t+1} \right)^2 \frac{y_{t+1}^c(q)}{y_t^c(q)} \quad (\text{A.92})$$

$$y_t^c(q) = \Theta \rho_t^c(q)^{-\theta} \rho_t(q)^{\theta-\eta} (\rho_t^{core})^{\eta-\bar{\eta}} Y_t \quad (\text{A.93})$$

A.8 Aggregation

Following Melitz (2003), we assume that the distribution function $g(z)$ is Pareto with parameters $zmin$ (minimum) and κ (tail). We then define $\tilde{z}_t(q)$ as a special average productivity in each sector (q). This productivity summarizes all the relevant information within a sector, as the industry is isomorphic to one populated by identical $N_{o,t}(q)$ firms endowed with productivity $\tilde{z}_t(q)$, as we show below.

Thanks to the properties of the Pareto distribution, we can write $\tilde{z}_t(q)$ as a function of the cut-off productivity, $z_t^c(q)$, as follows:

$$\tilde{z}_t(q) = \left[\frac{1}{1 - G(z_t^c(q))} \int_{z_t^c(q)}^{\infty} z^{\theta-1} g(z) dz \right]^{\frac{1}{\theta-1}} = \Gamma z_t^c(q) \quad (\text{A.94})$$

where $\Gamma = \left[\frac{\kappa}{\kappa - (\theta - 1)} \right]^{\frac{1}{\theta-1}}$ and, again due to the properties of the Pareto distribution, $1 -$

$$G(z_t^c(q)) = \left(\frac{z_{min}}{z_t^c(q)} \right)^\kappa.$$

In what follows, tilded variables refer to firms characterized by the special average productivity. Given that only some firms are active in each sector, the sectoral price $P_t(q)$ can be written as:

$$\begin{aligned} P_t(q) &= \left[\frac{1}{1 - G(z_t^c(q))} \int_{z_t^c(q)}^{\infty} p_{z,t}(q)^{1-\theta} N_{o,t}(q) g(z) dz \right]^{\frac{1}{1-\theta}} \\ &= N_{o,t}(q)^{\frac{1}{1-\theta}} \tilde{p}_t(q) \end{aligned} \quad (\text{A.95})$$

The latter implies that the ratio $\rho_t(q) = \frac{P_t(q)}{P_t}$ equals

$$\rho_t(q) = N_{o,t}(q)^{\frac{1}{1-\theta}} \tilde{\rho}_t(q) \quad (\text{A.96})$$

It also provides the following relationship between producer price inflation and inflation in sector q :

$$\begin{aligned} \pi_t^P(q) &= \frac{\tilde{p}_t(q)}{\tilde{p}_{t-1}(q)} \\ &= \left(\frac{N_{o,t}}{N_{o,t-1}} \right)^{\frac{1}{\theta-1}} \pi_t(q) \end{aligned} \quad (\text{A.97})$$

We can use this result to substitute out $\rho_t(q)$ from the equilibrium conditions regarding profits and cut-off productivities.

Moreover, by definition $w_t L_t(q) = \frac{1}{1-G(z_t^c(q))} \int_{z_t^c(q)}^{\infty} w_t l_{t,z}(q) N_{o,t}(q) g(z) dz$ and the same holds for $E_t(q)$ and $\Omega_t(q)$, where $L_t(q)$ is the total labor demanded in sector (q) , $E_t(q)$ is the total quantity of energy demanded in sector (q) and $\Omega_t^e(q)$ are the total dividends of sector (q) . Following the steps above, namely by substituting for $l_{t,z}(q)$, $E_{t,z}(q)$ and $e_{t,z}(q)$ as function of z only, one can show that:⁶

$$L_t(q) = N_{o,t}(q) \tilde{l}_t(q), \quad E_t(q) = N_{o,t}(q) \tilde{E}_t(q) \quad \text{and} \quad \Omega_t(q) = N_{o,t}(q) \tilde{\Omega}_t(q) \quad (\text{A.98})$$

To pin down $\tilde{\rho}_t(q)$, $\tilde{y}_t(q)$, $\tilde{m}c_t(q)$, $\tilde{\mu}_t(q)$, $\tilde{\Gamma}_t(q)$ we can make use of the following equations:

$$\tilde{\rho}_t(q) = \tilde{\mu}_t(q) \tilde{m}c_t(q) \quad (\text{A.99})$$

$$\tilde{m}c_t(q) = \frac{1}{Z_t \tilde{z}_t(q)} \left[\frac{w_t}{1 - \alpha_q} \right]^{1-\alpha_q} \left(\frac{\rho_t^E}{\alpha_q} \right)^{\alpha_q} \quad (\text{A.100})$$

$$\tilde{\mu}_t(q) = \frac{\theta}{\theta - 1} \left[1 - \frac{\tau_q}{2} \left(\frac{\tilde{\rho}_t(q)}{\tilde{\rho}_{t-1}(q)} \pi_t - 1 \right)^2 + \frac{\tau_q}{\theta - 1} \tilde{\Gamma}_t(q) \right]^{-1} \quad (\text{A.101})$$

$$\tilde{\Gamma}_t(q) = \frac{\tilde{\rho}_t(q)}{\tilde{\rho}_{t-1}(q)} \pi_t \left(\frac{\tilde{\rho}_t(q)}{\tilde{\rho}_{t-1}(q)} \pi_t - 1 \right) - E_t \Lambda_{t,t+1} \left(\frac{\tilde{\rho}_{t+1}(q)}{\tilde{\rho}_t(q)} \pi_{t+1} - 1 \right) \left(\frac{\tilde{\rho}_{t+1}(q)}{\tilde{\rho}_t(q)} \pi_{t+1} \right)^2 \frac{\tilde{y}_{t+1}(q)}{\tilde{y}_t(q)} \quad (\text{A.102})$$

$$\tilde{y}_t(q) = \Theta \tilde{\rho}_t(q)^{-\theta} \rho_t(q)^{\theta-\eta} (\rho_t^{core})^{\eta-\bar{\eta}} Y_t \quad (\text{A.103})$$

⁶For a more detailed derivation see Colciago and Silvestrini (2020).

And the profits of the firm with average special productivity are given by:

$$\tilde{e}_{z,t}(q) = \left[1 - \frac{1}{\tilde{\mu}_t(q)} - \frac{\tau_q}{2} \left(\frac{\tilde{\rho}_t(q)}{\tilde{\rho}_{t-1}(q)} \pi_t - 1 \right)^2 \right] \tilde{\rho}_t(q) \tilde{y}_t(q) - f_{x,t} \quad (\text{A.104})$$

B Summary of model equations

76 equations in 76 variables:

$$\begin{aligned} & \rho_t(g), \tilde{\rho}_t(g), N_{o,t}(g), N_t(g), N_t^e(g), E_t(g), \tilde{E}_t(g), L_t(g), \tilde{l}_t(g), \tilde{z}_t(g), z_t^c(g), \tilde{y}_t(g), Y_t(g), f_{e,t}(g), \tilde{e}_t(g), \\ & \tilde{\mu}_t(g), \tilde{\Gamma}_t(g), \tilde{m}c_t(g), \mu_t^c(g), \Gamma_t^c(g), mc_t^c(g), y_t^c(g), \rho_t^c(g), \pi_t^P(g), \\ & \rho_t(b), \tilde{\rho}_t(b), N_{o,t}(b), N_t(b), N_t^e(b), E_t(b), \tilde{E}_t(b), L_t(b), \tilde{l}_t(b), \tilde{z}_t(b), z_t^c(b), \tilde{y}_t(b), Y_t(b), f_{e,t}(b), \tilde{e}_t(b), \\ & \tilde{\mu}_t(b), \tilde{\Gamma}_t(b), \tilde{m}c_t(b), \mu_t^c(b), \Gamma_t^c(b), mc_t^c(b), y_t^c(b), \rho_t^c(b), \pi_t^P(b), \\ & Y_t^{core}, \rho_t^{core}, Y_t, E_t^H, \rho_t^E, E_t, E_{C,t}, E_{D,t}, L_{E,t}, L_{D,t}, X_{C,t}, X_{D,t}, MC_{D,t}, MC_{C,t}, \rho_t^{E_c}, \rho_t^{E_d}, \\ & \lambda_t, l_t^s, w_t, \tilde{\mu}_t, c_t, R_t, \pi_t, L_t^d, w_t^*, f_t^1, f_t^2, \tau_t^* \end{aligned}$$

Useful expressions (from aggregation)

$$\rho_t(g) = N_{o,t}(g)^{\frac{1}{1-\theta}} \tilde{\rho}_t(g) \quad (\text{B.1})$$

$$\rho_t(b) = N_{o,t}(b)^{\frac{1}{1-\theta}} \tilde{\rho}_t(b) \quad (\text{B.2})$$

$$E_t(g) = N_{o,t}(g) \tilde{E}_t(g) \quad (\text{B.3})$$

$$E_t(b) = N_{o,t}(b) \tilde{E}_t(b) \quad (\text{B.4})$$

$$L_t(g) = N_{o,t}(g) \tilde{l}_t(g) \quad (\text{B.5})$$

$$L_t(b) = N_{o,t}(b) \tilde{l}_t(b) \quad (\text{B.6})$$

$$N_{o,t}(g) = \left(\frac{zmin}{z_t^c(g)} \right)^\kappa N_t(g) \quad (\text{B.7})$$

$$N_{o,t}(b) = \left(\frac{zmin}{z_t^c(b)} \right)^\kappa N_t(b) \quad (\text{B.8})$$

$$\tilde{z}_t(g) = \Gamma z_t^c(g) \quad (\text{B.9})$$

$$\tilde{z}_t(b) = \Gamma z_t^c(b) \quad (\text{B.10})$$

$$\tilde{y}_t(g) = N_{o,t}(g)^{\frac{\theta}{1-\theta}} Y_t(g) \quad (\text{B.11})$$

$$\tilde{y}_t(b) = N_{o,t}(b)^{\frac{\theta}{1-\theta}} Y_t(b) \quad (\text{B.12})$$

Households

$$\nu (l_t^s)^\phi = \frac{\lambda_t w_t}{\tilde{\mu}_t} \quad (\text{B.13})$$

$$\lambda_t = \frac{1}{c_t} \quad (\text{B.14})$$

$$1 = \beta E_t \left[\left(\frac{\lambda_{t+1}}{\lambda_t} \right) \frac{R_t}{\pi_{t+1}} \right] \quad (\text{B.15})$$

$$f_{e,t}(g) = \beta (1 - \delta) E_t \left[\left(\frac{\lambda_{t+1}}{\lambda_t} \right) \left(f_{e,t+1}(g) + \left(\frac{zmin}{z_{t+1}^c(g)} \right)^\kappa \tilde{e}_{t+1}(g) \right) \right] \quad (\text{B.16})$$

$$f_{e,t}(b) = \beta (1 - \delta) E_t \left[\left(\frac{\lambda_{t+1}}{\lambda_t} \right) \left(f_{e,t+1}(b) + \left(\frac{zmin}{z_{t+1}^c(b)} \right)^\kappa \tilde{e}_{t+1}(b) \right) \right] \quad (\text{B.17})$$

Wage setting

$$w_t^* = \frac{\theta_w}{(\theta_w - 1)} \frac{f_t^2}{f_t^1} \quad (\text{B.18})$$

$$f_t^1 = L_t^d w_t^{\theta_w} \lambda_t + \alpha^* \beta E_t \pi_{t+1}^{\theta_w - 1} f_{t+1}^1 \quad (\text{B.19})$$

$$f_t^2 = L_t^d w_t^{\theta_w} \left(\nu (l_t^s)^\phi \right) + \alpha^* \beta E_t \pi_{t+1}^{\theta_w} f_{t+1}^2 \quad (\text{B.20})$$

$$w_t^{1-\theta_w} = (1 - \alpha^*) (w_t^*)^{1-\theta_w} + \alpha^* \left(\frac{w_{t-1}}{\pi_t} \right)^{1-\theta_w} \quad (\text{B.21})$$

$$l_t^s = \tau_t^* L_t^d \quad (\text{B.22})$$

$$\tau_t^* = (1 - \alpha^*) \left(\frac{w_t^*}{w_t} \right)^{-\theta_w} + \alpha^* \left(\frac{w_t}{w_{t-1}} \right)^{\theta_w} \pi_t^{\theta_w} \tau_{t-1}^* \quad (\text{B.23})$$

Sectorial demands

$$Y_t^{core} = \omega (\rho_t^{core})^{-\tilde{\eta}} Y_t \quad (\text{B.24})$$

$$E_t^H = (1 - \omega) (\rho_t^E)^{-\tilde{\eta}} Y_t \quad (\text{B.25})$$

$$Y_t(g) = \chi \left(\frac{\rho_t(g)}{\rho_t^{core}} \right)^{-\eta} Y_t^{core} \quad (\text{B.26})$$

$$Y_t(b) = (1 - \chi) \left(\frac{\rho_t(b)}{\rho_t^{core}} \right)^{-\eta} Y_t^{core} \quad (\text{B.27})$$

Firms

$$\tilde{e}_t(g) = \left[1 - \frac{1}{\tilde{\mu}_t(g)} - \frac{\tau_g}{2} \left(\frac{\tilde{\rho}_t(g)}{\tilde{\rho}_{t-1}(g)} \pi_t - 1 \right)^2 \right] \tilde{\rho}_t(g) \tilde{y}_t(g) - f_{x,t} \quad (\text{B.28})$$

$$\tilde{e}_t(b) = \left[1 - \frac{1}{\tilde{\mu}_t(b)} - \frac{\tau_b}{2} \left(\frac{\tilde{\rho}_t(b)}{\tilde{\rho}_{t-1}(b)} \pi_t - 1 \right)^2 \right] \tilde{\rho}_t(b) \tilde{y}_t(b) - f_{x,t} \quad (\text{B.29})$$

$$\tilde{m}c_t(g) = \frac{1}{Z_t(g) \Gamma z_t^e(g)} \left(\frac{w_t}{1 - \alpha_g} \right)^{1 - \alpha_g} \left(\frac{\rho_t^E}{\alpha_g} \right)^{\alpha_g} \quad (\text{B.30})$$

$$\tilde{m}c_t(b) = \frac{1}{Z_t(b) \Gamma z_t^e(b)} \left(\frac{w_t}{1 - \alpha_b} \right)^{1 - \alpha_b} \left(\frac{\rho_t^E}{\alpha_b} \right)^{\alpha_b} \quad (\text{B.31})$$

$$\tilde{y}_t(g) = Z_t(g) \tilde{z}_t(g) \tilde{E}_t(g)^{\alpha_g} \tilde{l}_t(g)^{1 - \alpha_g} \quad (\text{B.32})$$

$$\tilde{y}_t(b) = Z_t(b) \tilde{z}_t(b) \tilde{E}_t(b)^{\alpha_b} \tilde{l}_t(b)^{1 - \alpha_b} \quad (\text{B.33})$$

$$\frac{\tilde{E}_t(g)}{\tilde{l}_t(g)} = \frac{\alpha_g}{1 - \alpha_g} \frac{w_t}{p_t^E} \quad (\text{B.34})$$

$$\frac{\tilde{E}_t(b)}{\tilde{l}_t(b)} = \frac{\alpha_b}{1 - \alpha_b} \frac{w_t}{p_t^E} \quad (\text{B.35})$$

$$\tilde{\rho}_t(g) = \tilde{\mu}_t(g) \tilde{m}c_t(g) \quad (\text{B.36})$$

$$\tilde{\rho}_t(b) = \tilde{\mu}_t(b) \tilde{m}c_t(b) \quad (\text{B.37})$$

$$\tilde{\mu}_t(g) = \frac{\theta}{\theta - 1} \left[1 - \frac{\tau_q}{2} (\pi_t^p(g) - 1)^2 + \frac{\tau_q}{\theta - 1} \tilde{\Gamma}_t(g) \right]^{-1} \quad (\text{B.38})$$

$$\tilde{\mu}_t(b) = \frac{\theta}{\theta - 1} \left[1 - \frac{\tau_q}{2} (\pi_t^p(b) - 1)^2 + \frac{\tau_q}{\theta - 1} \tilde{\Gamma}_t(b) \right]^{-1} \quad (\text{B.39})$$

$$\tilde{\Gamma}_t(g) = \pi_t^p(g) (\pi_t^p(g) - 1) - E_t \Lambda_{t,t+1} (\pi_{t+1}^p(g) - 1) (\pi_{t+1}^p(g))^2 \frac{\tilde{y}_{t+1}(g)}{\tilde{y}_t(g)} \quad (\text{B.40})$$

$$\tilde{\Gamma}_t(b) = \pi_t^p(b) (\pi_t^p(b) - 1) - E_t \Lambda_{t,t+1} (\pi_{t+1}^p(b) - 1) (\pi_{t+1}^p(b))^2 \frac{\tilde{y}_{t+1}(b)}{\tilde{y}_t(b)} \quad (\text{B.41})$$

$$\pi_t^p(g) = \frac{\tilde{\rho}_t(g)}{\tilde{\rho}_{t-1}(g)} \pi_t \quad (\text{B.42})$$

$$\pi_t^p(b) = \frac{\tilde{\rho}_t(b)}{\tilde{\rho}_{t-1}(b)} \pi_t \quad (\text{B.43})$$

Entry & exit and productivity cutoff

$$f_{e,t}(g) = \psi_0 + \psi_1 (N_t^e(g))^\gamma \quad (\text{B.44})$$

$$f_{e,t}(b) = \psi_0 + \psi_1 (N_t^e(b))^\gamma \quad (\text{B.45})$$

$$N_{t+1}(g) = (1 - \delta) (N_t(g) + N_t^e(g)) \quad (\text{B.46})$$

$$N_{t+1}(b) = (1 - \delta) (N_t(b) + N_t^e(b)) \quad (\text{B.47})$$

$$f_{x,t} = \omega \chi \left[1 - \frac{1}{\mu_t^c(g)} - \frac{\tau_g}{2} \left(\frac{\rho_t^c(g)}{\rho_{t-1}^c(g)} \pi_t - 1 \right)^2 \right] \rho_t^c(g) y_t^c(g) \quad (\text{B.48})$$

$$\rho_t^c(g) = \mu_t^c(g) m c_t^c(g) \quad (\text{B.49})$$

$$m c_t^c(g) = \frac{1}{Z_t z_t^c(g)} \left[\frac{w_t}{1 - \alpha_g} \right]^{1 - \alpha_g} \left(\frac{\rho_t^E}{\alpha_g} \right)^{\alpha_g} \quad (\text{B.50})$$

$$\mu_t^c(g) = \frac{\theta}{\theta - 1} \left[1 - \frac{\tau_q}{2} \left(\frac{\rho_t^c(g)}{\rho_{z,t-1}^c(g)} \pi_t - 1 \right)^2 + \frac{\tau_q}{\theta - 1} \Gamma_t^c(g) \right]^{-1} \quad (\text{B.51})$$

$$\Gamma_t^c(g) = \frac{\rho_t^c(g)}{\rho_{t-1}^c(g)} \pi_t \left(\frac{\rho_t^c(g)}{\rho_{t-1}^c(g)} \pi_t - 1 \right) - E_t \Lambda_{t,t+1} \left(\frac{\rho_{t+1}^c(g)}{\rho_t^c(g)} \pi_{t+1} - 1 \right) \left(\frac{\rho_{t+1}^c(g)}{\rho_t^c(g)} \pi_{t+1} \right)^2 \frac{y_{t+1}^c(g)}{y_t^c(g)} \quad (\text{B.52})$$

$$y_t^c(g) = \omega \chi \rho_t^c(g)^{-\theta} \rho_t(g)^{\theta - \eta} (\rho_t^{\text{core}})^{\eta - \bar{\eta}} Y_t \quad (\text{B.53})$$

$$f_{x,t} = \omega (1 - \chi) \left[1 - \frac{1}{\mu_t^c(b)} - \frac{\tau_b}{2} \left(\frac{\rho_t^c(b)}{\rho_{t-1}^c(b)} \pi_t - 1 \right)^2 \right] \rho_t^c(b) y_t^c(b) \quad (\text{B.54})$$

$$\rho_t^c(b) = \mu_t^c(b) m c_t^c(b) \quad (\text{B.55})$$

$$m c_t^c(b) = \frac{1}{Z_t z_t^c(b)} \left[\frac{w_t}{1 - \alpha_b} \right]^{1 - \alpha_b} \left(\frac{\rho_t^E}{\alpha_b} \right)^{\alpha_b} \quad (\text{B.56})$$

$$\mu_t^c(b) = \frac{\theta}{\theta - 1} \left[1 - \frac{\tau_q}{2} \left(\frac{\rho_t^c(b)}{\rho_{z,t-1}^c(b)} \pi_t - 1 \right)^2 + \frac{\tau_q}{\theta - 1} \Gamma_t^c(b) \right]^{-1} \quad (\text{B.57})$$

$$\Gamma_t^c(b) = \frac{\rho_t^c(b)}{\rho_{t-1}^c(b)} \pi_t \left(\frac{\rho_t^c(b)}{\rho_{t-1}^c(b)} \pi_t - 1 \right) - E_t \Lambda_{t,t+1} \left(\frac{\rho_{t+1}^c(b)}{\rho_t^c(b)} \pi_{t+1} - 1 \right) \left(\frac{\rho_{t+1}^c(b)}{\rho_t^c(b)} \pi_{t+1} \right)^2 \frac{y_{t+1}^c(b)}{y_t^c(b)} \quad (\text{B.58})$$

$$y_t^c(b) = \omega (1 - \chi) \rho_t^c(b)^{-\theta} \rho_t(b)^{\theta - \eta} (\rho_t^{\text{core}})^{\eta - \bar{\eta}} Y_t \quad (\text{B.59})$$

Energy sectors

$$E_t = \left[\xi^{\frac{1}{\rho}} E_{D,t}^{\frac{\rho-1}{\rho}} + (1-\xi)^{\frac{1}{\rho}} E_{C,t}^{\frac{\rho-1}{\rho}} \right]^{\frac{\rho}{\rho-1}} \quad (\text{B.60})$$

$$E_{C,t} = (1-\xi) \left(\frac{\rho_t^{E_c}}{\rho_t^E} \right)^{-\rho} E_t \quad (\text{B.61})$$

$$E_{D,t} = (\xi) \left(\frac{\rho_t^{E_d}}{\rho_t^E} \right)^{-\rho} E_t \quad (\text{B.62})$$

$$E_{D,t} = \left[\xi_D^{\frac{1}{\rho_D}} (A_t^D L_{D,t})^{\frac{\rho_D-1}{\rho_D}} + (1-\xi_D)^{\frac{1}{\rho_D}} (Z_t^D X_{D,t})^{\frac{\rho_D-1}{\rho_D}} \right]^{\frac{\rho_D}{\rho_D-1}} \quad (\text{B.63})$$

$$\frac{L_{D,t}}{X_{D,t}} = \frac{\xi_D}{1-\xi_D} \left[\frac{\rho_t^{FR}}{w_t} \right]^{\rho_D} \left[\frac{A_t^D}{Z_t^D} \right]^{\rho_D-1} \quad (\text{B.64})$$

$$m_{C,D,t} = \left(\xi_D \left(\frac{w_t}{A_t^D} \right)^{1-\rho_D} + (1-\xi_D) \left(\frac{\rho_t^{FR}}{Z_t^D} \right)^{1-\rho_D} \right)^{\frac{1}{1-\rho_D}} \quad (\text{B.65})$$

$$\rho_t^{E_d} = m_{C,D,t} \quad (\text{B.66})$$

$$E_{C,t} = \left[\xi_C^{\frac{1}{\rho_C}} (A_t^C L_{C,t})^{\frac{\rho_C-1}{\rho_C}} + (1-\xi_C)^{\frac{1}{\rho_C}} (Z_t^C X_{C,t})^{\frac{\rho_C-1}{\rho_C}} \right]^{\frac{\rho_C}{\rho_C-1}} \quad (\text{B.67})$$

$$\frac{L_{C,t}}{X_{C,t}} = \frac{\xi_C}{1-\xi_C} \left[\frac{\rho_t^{CR}}{w_t} \right]^{\rho_C} \left[\frac{A_t^C}{Z_t^C} \right]^{\rho_C-1} \quad (\text{B.68})$$

$$m_{C,C,t} = \left(\xi_C \left(\frac{w_t}{A_t^C} \right)^{1-\rho_C} + (1-\xi_C) \left(\frac{\rho_t^{CR}}{Z_t^C} \right)^{1-\rho_C} \right)^{\frac{1}{1-\rho_C}} \quad (\text{B.69})$$

$$\rho_t^{E_c} = m_{C,C,t} \quad (\text{B.70})$$

Market clearing in energy and labour market:

$$E_t = E_t(g) + E_t(b) + E_t^H \quad (\text{B.71})$$

$$L_t(g) + L_t(b) + L_{C,t} + L_{D,t} = L_t^d \quad (\text{B.72})$$

Aggregation

$$Y_t = \left(\omega^{\frac{1}{\tilde{\eta}}} (Y_t^{core})^{\frac{\tilde{\eta}-1}{\tilde{\eta}}} + (1-\omega)^{\frac{1}{\tilde{\eta}}} (E_t^H)^{\frac{\tilde{\eta}-1}{\tilde{\eta}}} \right)^{\frac{\tilde{\eta}}{\tilde{\eta}-1}} \quad (\text{B.73})$$

$$Y_t^{core} = \left[\chi^{\frac{1}{\eta}} Y_t(g)^{\frac{\eta-1}{\eta}} + (1-\chi)^{\frac{1}{\eta}} Y_t(b)^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}} \quad (\text{B.74})$$

$$\begin{aligned} Y_t = & C_t + (N_{o,t}(g) + N_{o,t}(b)) f_{x,t} + N_t^e(g) f_{e,t}(g) + N_t^e(b) f_{e,t}(b) \\ & + PAC_t + \rho_t^{FR} X_t^{FR} + \rho_t^{CR} X_t^{CR} \end{aligned} \quad (\text{B.75})$$

Taylor Rule

$$\left(\frac{R_t}{R} \right) = \left[\left(\frac{\pi_t}{\pi} \right)^{\varphi_\pi} \left(\frac{Y_t}{Y} \right)^{\varphi_Y} \right]^{1-\varphi_R} \left(\frac{R_{t-1}}{R} \right)^{\varphi_R} \quad (\text{B.76})$$