Recent advances in extracting policy-relevant information from market interest rates

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Market interest rates form an important part of the transmission mechanism of monetary policy. They also contain information about market expectations of future policy rates as well as attitudes to, and perceptions of, risk. Extracting and interpreting this policy-relevant information is not straightforward, however. This article describes recent advances in this field and how they can be used to shed light on the downward trend in long-term real forward interest rates and the upward trend in long-term inflation forward rates, both developments that have attracted the attention of policymakers.

Introduction

The Bank of England's Monetary Policy Committee (MPC) sets Bank Rate, and influences interest rates determined by financial markets through its effect on expectations about the likely future path of policy rates.

These market interest rates play an important part in how monetary policy is transmitted to the economy. This is because the expenditure decisions of households and businesses reflect the full spectrum of market interest rates at different horizons.(1) Market interest rates also contain useful information for policymakers. This includes financial market participants' expectations of future policy rates as well as their attitudes to, and perceptions of, risk. In turn, these embody expectations of the outlook for the economy, as well as perceptions of how policymakers will react to economic developments.

Forward interest rates — the interest rates available now, at which people can borrow or lend in the future — are one representation of market interest rates. Nominal forward rates can be estimated from conventional government bonds and real forward rates can be estimated from index-linked government bonds. The difference between nominal and real forward rates is given by so-called inflation forward rates (the box on page 165 explains the concept of forward rates and their estimation in more detail). The full spectrum of forward rates at different horizons, at a given point in time, is described by the so-called term structure of forward rates. **Charts 1–3** plot the term structures of UK nominal, real and RPI inflation forward rates since 1996. These charts show that over this period the level and the shape of these term structures have varied considerably, with particularly large changes occurring around May 1997 when the Bank of England was granted operational independence and the MPC was founded.

More recently, two features at longer horizons have attracted the attention of policymakers. For greater clarity, these features are illustrated in **Chart 4**, which slices through **Charts 1–3** at the ten-year horizon. The chart shows that since October 2003 the ten-year RPI inflation forward rate has risen to its highest level since the MPC was founded. And the ten-year real forward rate is close to its lowest level since UK index-linked bonds were first issued in 1981. The initial fall in long-term UK real forward rates was mirrored by similar falls in the United States and the euro area.(2) But movements in UK real forward rates since the start of 2006 have been less in line with international developments.

For policymakers these developments pose a number of questions. Does the fall in real forward rates reflect a fall in the 'equilibrium' real risk-free rate, perhaps reflecting a perception of weaker long-term growth prospects? Does the rise in inflation forward rates reflect a de-anchoring of inflation expectations away from the inflation target? Do both features reflect changes in attitudes to and/or perceptions about risk? Or do they reflect institutional factors specific to the UK bond market?

(1) See Monetary Policy Committee (1999) for a discussion of the transmission mechanism and Clews (2002) on the links between asset prices and monetary policy.
(2) Greenspan (2005) described the fall in international nominal and real long-term interest rates during 2004 and 2005 as a 'conundrum' because it accompanied rising US policy rates.

Chart 1 UK nominal forward rates

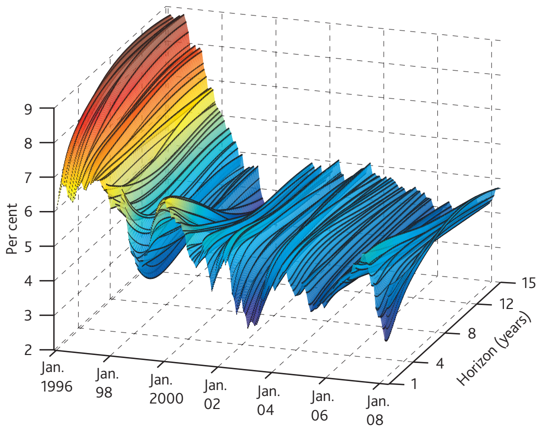


Chart 2 UK real forward rates

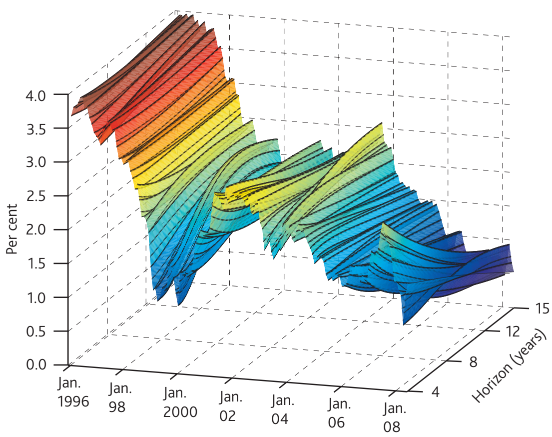
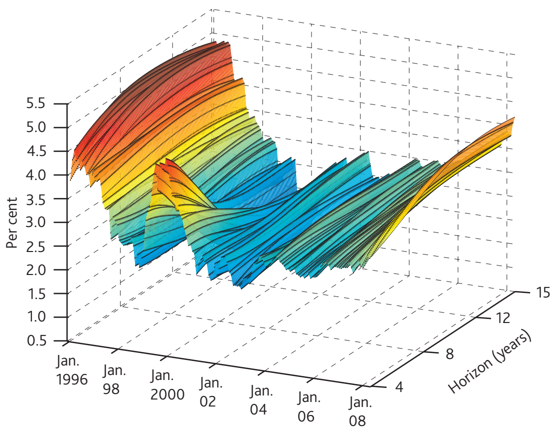


Chart 3 UK RPI inflation forward rates

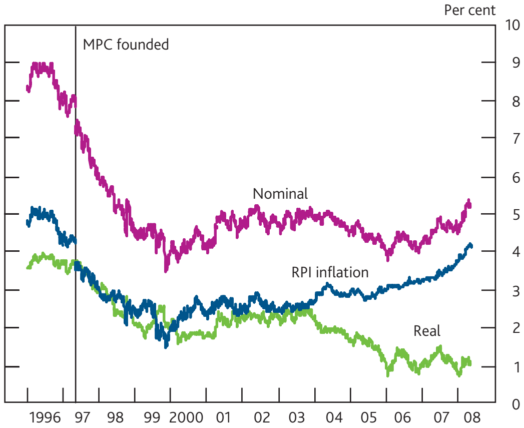


Notes: The level of nominal, real and inflation forward rates is colour coded. This ranges from dark red for periods of high forward rates to dark blue for periods of low forward rates. The shortest horizon for real and inflation forward rates available consistently over the sample is four years, reflecting the lack of index-linked bonds at short maturities.

Source: Bank calculations.

This article describes recent advances in extracting such policy-relevant information from market interest rates. It sets out how term premia are explained by economic theory and shows how both theoretical and empirical models can be applied to understanding the recent behaviour of the UK forward term structures.

Chart 4 UK ten-year forward rates



Source: Bank calculations.

Forward rates versus expected future rates

To begin with, it is important to understand how the forward rates shown in **Charts 1–4** are related to expectations of future interest rates and inflation. To describe this relationship in a tractable manner, much of this article assumes: that government bonds are default-free assets; that investors have no *a priori* preferences for investing in short-term or long-term bonds; and that financial markets operate efficiently and without frictions. The last two assumptions imply that investors do not face any restrictions when deciding whether to hold a long-term bond to maturity or whether to invest in a series of short-term bonds with the same total maturity as the long-term bond. The possible implications of such frictions are returned to later in the article.

Given this setting, a common starting point is to assume that investors require no compensation for risk. This would be the case if investors were not exposed to risk (ie if returns on their investments were certain) or if investors did not mind being exposed to risk (ie if they were 'risk-neutral'). Under these conditions, investors would be indifferent between holding a long-term bond to maturity and investing in a corresponding sequence of short-term bonds. The absence of compensation for risk means that the forward interest rates implied by this sequence of short-term bonds would only reflect investors' expectations about future interest rates. In such a case, the fall in real forward rates would solely reflect expectations of lower real risk-free rates. And the rise in inflation forward rates would solely reflect a rise in inflation expectations.

In reality, of course, investors face uncertain investment returns and tend to dislike being exposed to risk (ie they are 'risk-averse'). Under these conditions, investors are not indifferent between holding a long-term bond to maturity and investing in a corresponding series of short-term bonds. To make them indifferent would require some compensation for bearing risk. This compensation — a so-called risk premium — will drive a wedge between the forward rates shown in

Charts 1–4 and expectations of future policy rates, real risk-free interest rates and inflation rates.

There is a substantial literature which suggests that such risk premia can be large enough to matter, particularly when considering forward interest rates at longer horizons.(1) The presence of potentially large risk premia that may vary over time makes it important to understand the economic determinants of risk premia.

The economic determinants of risk premia

In general terms, a risk premium is the difference between the expected return from a risky asset and the return guaranteed by a corresponding risk-free asset. Risk premia will be related to how uncertain people are about asset returns and the economic outlook more generally. This will influence how exposed to risk people feel. They will also be related to how much people care about exposure to risk. These two factors should have a bearing on whether risk premia will be small or large in absolute terms. But the presence of risk and people caring about it does not imply that a risky asset would necessarily command a higher expected return than a risk-free asset. In other words, risk premia could be negative as well as positive. Indeed, one of the main insights from economic theory is that the sign of risk premia should depend on how asset returns tend to behave under different economic conditions.

Risk premia in the C-CAPM

One of the most popular models, the so-called consumption capital asset pricing model (C-CAPM), derives this insight from the premise that people want to smooth consumption over time. They do this by forgoing consumption and investing in assets during 'good times', when they do not value extra consumption very much, and by selling assets and using the proceeds to support consumption during 'bad times', when they value extra consumption more highly. Assets that deliver low returns during bad times are less useful in smoothing consumption than assets that deliver high returns during bad times. So investors require extra compensation (ie a positive risk premium) from assets with the former characteristics, and accept less compensation (ie a negative risk premium) from assets with the latter characteristics.

In addition to how exposed people are to consumption volatility and how much they care about smoothing consumption, the C-CAPM thus also links risk premia to how useful assets are in helping people to smooth consumption. What does this imply for the type of risk premia that may be present in the forward rates shown in **Charts 1–4**?

Risk premia on index-linked bonds

First, consider real forward rates derived from index-linked government bonds. If held to maturity, the real return on such a bond is certain, as the regular interest payments of the coupons and the final payment of the principal are both adjusted for inflation.(2) But if sold before maturity, the real return is uncertain as the price of the bond will fluctuate with economic conditions. It turns out that whether index-linked bonds carry a positive or a negative risk premium depends on whether people expect the effects of economic disturbances to diminish gradually or whether they expect the effects to increase. In other words, the sign of the risk premium depends on whether people expect bad times to be followed by better times or whether bad times are expected to be followed by even worse times.

Consider the case where bad times (when extra consumption would be valued highly) are expected to be followed by better times (when extra consumption would be valued less). In this case people would want to borrow to bring consumption forward. Such an increase in demand for borrowing would drive up real interest rates with a corresponding fall in the price of index-linked bonds. And these falls will be larger for long-dated index-linked bonds, as more of their cash flows accrue in the future and are thus more heavily discounted. In other words, the longer dated index-linked bonds are, the less useful they would be in helping to smooth consumption. Index-linked bonds would thus carry a positive risk premium, which would be larger the longer dated the bond.

In contrast, if times are expected to get worse before they get better, people would want to save even during bad times. Such an increase in the supply of savings would drive down real interest rates with a corresponding increase in the price of index-linked bonds. These increases would be largest for longer-dated bonds, which would thus be more useful in helping to smooth consumption. Index-linked bonds would thus carry a negative risk premium, which would be more negative the longer dated the bond.(3)

In reality, of course, people's expectations about the persistence of the effects of economic disturbances are likely to change over time. If, for example, in the past people had expected bad times to be followed by better times, but in recent years had expected times to get worse before they get better, the risk premium on index-linked bonds could have changed from positive to negative. This would be consistent with lower real forward rates, even if expectations of future real risk-free rates had remained unchanged.

(1) For a textbook review of empirical studies of risk premia in the term structure of interest rates, see Cuthbertson and Nitzsche (2005).
(2) In practice, index-linked bonds do not deliver a certain real return even if held to maturity because of indexation lags (see Deacon, Derry and Mirfendereski (2004)).
(3) See den Haan (1995) for a more formal discussion of the relationship between real term premia and the persistence of changes in the growth rate of so-called marginal utility.

Risk premia on conventional bonds

Second, consider nominal forward rates derived from conventional bonds. People investing in conventional government bonds will want to take account of the fact that inflation erodes real returns. So part of the difference between nominal and real forward rates will reflect the required compensation for what people expect inflation to be. However, because inflation is uncertain, part of the difference will also reflect a premium for inflation risk. From the previous discussion it follows that whether the inflation risk premium is positive or negative should depend on how useful conventional bonds are relative to index-linked bonds in helping to smooth consumption. It turns out that this is closely linked to the source of economic disturbances.

Suppose that people believed that economic disturbances only affected the amount of output the economy can supply. An example of such a supply disturbance would be an unexpected slowdown in total factor productivity — a measure of how efficiently output can be produced. Such a supply disturbance would reduce output and also lead to higher inflation for a period. So in this case high inflation would erode real returns on conventional bonds when times are already bad. In other words, relative to index-linked bonds, conventional bonds would not be very useful to help people smooth consumption and the inflation risk premium would thus be positive.

In contrast, suppose that people believed that economic disturbances only affected demand. An example of such a demand disturbance would be people becoming more pessimistic about their income prospects. This would tend to reduce spending and output growth and lower inflation. So in this case lower inflation would raise real returns on conventional bonds during bad times. In other words, relative to index-linked bonds, conventional bonds would be more useful in helping people to smooth consumption and the inflation risk premium would thus be negative.(1)

In reality, of course, people do not believe that economic disturbances affect only supply or only demand. Instead, their expectations about the type of disturbance that is most likely to occur in the future will change over time. If, for example, in the past people had expected the most likely disturbances to originate from demand, but in recent years expected them to originate from supply, the inflation risk premium could have changed from negative to positive. This would be consistent with higher forward inflation rates even if inflation expectations had remained unchanged.

Risk premia and term premia

One insight from the preceding section is that the horizon — or term — of the bond has a bearing on the size of the risk premium. As a result, the risk premium on index-linked bonds is also known as a *real term premium*. The same is true for conventional bonds, where the risk premium — which includes the compensation for inflation risk — is known as a *nominal term premium*.

Analysing the term structure in macroeconomic models

Much of the early literature on term premia — and on risk premia in general — focused on simple models that abstract from the fact that consumption goods need to be produced and that people have to work to be able to consume. In these models income arrives like fruit falling from a tree. The simplicity of these so-called endowment models makes them well suited to analysing the role of people's preferences — ie how much people care about smoothing consumption — in determining term premia.(2)

However, the models' simplicity means that they are not well suited to analysing the *interaction* between the macroeconomy and term premia. For example, rather than borrowing during bad times, people may try to smooth consumption by working more. Whether, and at what terms, they are able to do so will be influenced by how costly it is for businesses to adjust the amount of labour they employ. If such labour adjustment costs or other rigidities in the economy make it more difficult for people to smooth consumption in this way, it will effectively expose them to more risk than would otherwise be the case. In other words, term premia and the structure of the economy are not independent of each other.

It has only been in the past ten to fifteen years that advances in computational methods and increased processing power have enabled researchers to analyse term premia in models that allow for some of the interactions described above.(3) These so-called general equilibrium models reduce the complex interactions between households, businesses and policymakers observed in real life to a tractable, stylised description of an economy and have long been used to analyse many other aspects of the economy.

An application

These models can be applied to help think through some of the questions raised in the introduction.

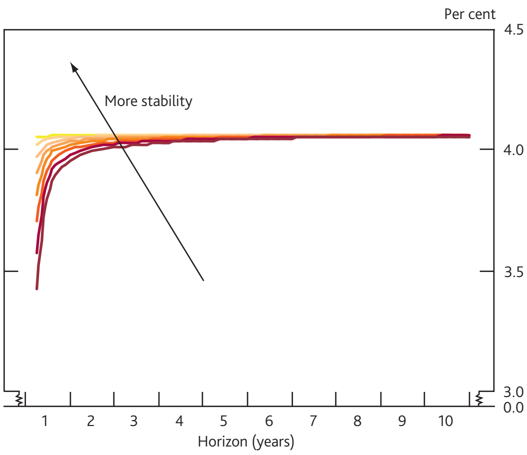
For example, the prolonged stability of output and inflation prior to the onset of the recent turbulence in financial markets — the so-called 'Great Moderation' — was cited at the time as

(1) See De Paoli *et al* (2007) for further discussion and simulation results. See also Campbell, Sunderam and Viceira (2008) and references therein.
(2) See Campbell (1999) for a survey of the implications of different types of preferences for risk premia. See Wachter (2006) for recent work on the implications of so-called habit formation for time variation in term premia.
(3) Jermann (1998) is an early paper considering the effect of capital adjustment costs on risk premia in a so-called real business cycle model. De Paoli *et al* (2007) consider capital adjustment costs and price adjustment costs in a so-called New Keynesian model. Rudebusch and Swanson (2007) consider a similar model, adding labour adjustment costs as well as adjustment costs to wages.

a candidate explanation for the fall in real forward rates that has attracted policymakers' attention.[1] In principle, greater economic stability should correspond to a reduction in uncertainty so that people should have felt less exposed to risk. So, the 'Great Moderation' would have been consistent with lower real term premia. However, in addition to reducing real term premia, a reduction in economic uncertainty should also have reduced people's incentives to invest in a risk-free asset to 'save for a rainy day'. Such a reduction in the supply of these so-called precautionary savings should in turn have led to an *increase* in the real risk-free rate both at the time and going forward. With real risk-free rates and real term premia moving in opposite directions, what would the likely effect of an increase in economic stability on real forward rates have been?

This can be illustrated by simulating models like the ones described above, under different degrees of economic stability.[2] **Chart 5** plots the average term structure of real forward rates for each set of simulations. The fainter lines correspond to the average term structure of real forward rates obtained from simulations that assume a high degree of economic stability. The darker lines correspond to the average term structure of real forward rates obtained from simulations with a low degree of economic stability. **Chart 5** shows that a low degree of economic stability is associated with a low level of the term structure of real forward rates, as the precautionary savings effect reduces real forward rates at all horizons. The effect is more pronounced at shorter horizons as people put more weight on economic uncertainty now relative to economic uncertainty in the future. As the degree of economic stability increases, the real forward curve shifts upwards. In the extreme, stability is so great that all uncertainty is effectively removed. This also removes the incentive for precautionary savings and the term structure of real forward rates is flat.

Chart 5 Illustrative relationship between economic uncertainty and the term structure of real forward rates[(a)]

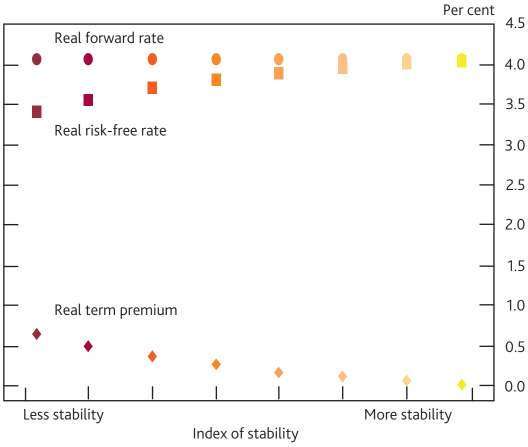


Source: Bank calculations.

(a) Each line shows the average term structure of real forward rates corresponding to one simulation of the model. The term structure is shown for annualised quarterly rates, starting from the real risk-free rate out to forward rates with a ten-year horizon.

Chart 6 slices through **Chart 5** to highlight the developments of the real risk-free rate and the real term premium that make up the real forward rate at the ten-year horizon. It shows that the stability of the ten-year real forward rate reflects offsetting movements in the real term premium and the real risk-free rate.

Chart 6 Illustrative relationship between economic uncertainty and ten-year real forward rates[(a)]



Source: Bank calculations.

(a) The term premium is defined as the difference between the real forward rate at the ten-year horizon and the real risk-free rate.

The result that greater economic stability tends to be accompanied by an increase, rather than a fall in real forward rates is robust across common specifications of preferences and for different economic disturbances.[3] So this analysis casts some doubt on the 'Great Moderation' as the sole explanation for low real forward rates.

Theory versus data

The discussion above shows that general equilibrium models provide a structured way for thinking through the fundamental linkages between the macroeconomy and the term structure of interest rates. However, these models have to date found it difficult to account quantitatively for movements in the term structure of interest rates. In particular, these models are unable to match the size and variability of term premia suggested by empirical studies.[4] One possibility is that the assumptions on which these models rest do not hold in the real world. For example, the models described usually make quite restrictive assumptions about the preferences of

(1) In the United Kingdom, this decline in inflation and output growth volatility is believed to have begun in the early 1990s. In the United States it is usually dated from around 1984. See Young (2008) for further details and Kohn (2005) for a discussion in the context of long-term interest rates.
(2) The simulations are based on the model described in De Paoli *et al* (2007). Perceptions of greater stability are proxied by reducing the variance of the disturbance to the economy.
(3) Haubrich (1999) and den Haan (1995) show mathematically that as long as shocks to the level of consumption are not permanent, the results shown here hold for standard specifications of preferences, ie 'power utility'.
(4) To the extent that general equilibrium models can match the size of term premia, it comes at the expense of counterfactually large variability in other economic variables such as consumption or wages. See Rudebusch and Swanson (2007) for further detail.

economic agents and assume that the financial markets they operate in are not subject to any frictions.

Empirical models of the term structure

Given the difficulties that general equilibrium models have in matching the size and variability of term premia, there can be advantages in also using approaches that impose less economic structure and rely more closely on the data.

One widely used approach is to model interest rates using so-called affine term structure models.(1) There are many variants of these models, but all are based on three main assumptions: first, that bond prices are set in such a way as to eliminate arbitrage opportunities, so that there are no risk-free profits to be made by trading combinations of bonds; second, that bond prices are driven by a small set of 'factors'; and third, that risk preferences are related to these factors.

The key difference to general equilibrium models is the last assumption, which means that affine models do not explicitly specify the underlying preferences of economic agents. Instead, these models posit a specific relationship between preferences and the factors in the model. These factors may be observed macroeconomic variables such as output or the unemployment rate, or unobservable variables extracted from the data as part of the model estimation, so-called 'latent factors'. The affine terminology stems from the fact that these models have the convenient property that their structure implies that interest rates themselves are a linear (ie affine) function of the factors.

The first assumption of no arbitrage is also made in the general equilibrium models just discussed and ensures that affine models price risk consistently for all bonds along the term structure. In turn this means that affine models can, for example, decompose nominal interest rates at any horizon into expectations about future policy rates and nominal term premia.(2) Expected policy rates in these models can be thought of as being generated as the forecasts from a regression, which includes the lagged values of the interest rate factors, where the coefficients in the regression have been restricted to be consistent with the assumption of no arbitrage. Nominal term premia estimates are derived from the difference between expected policy rates and fitted nominal forward rates.

Because of their simpler structure, affine models can be estimated using conventional econometric techniques. And because they embody fewer theoretical restrictions, and are therefore more flexible, they are able to match the term structure data much more closely than general equilibrium models do. The flipside of having less economic structure, however, is that affine models offer only a limited amount of economic interpretation. While the models can provide a decomposition of forward interest rates at any future horizon into expected policy rates and term premia, they do not generally allow movements in short rate expectations or term premia to be attributed to specific changes in the structure of the economy.(3) Affine models are therefore best viewed as complementary to general equilibrium models, rather than as an alternative to them.

An affine model-based decomposition of UK forward rates

Some recent research at the Bank has applied this affine modelling approach to understanding the UK nominal and real term structures.(4) In the model (which is used below) nominal and real interest rates are explained by four factors: RPI inflation and three latent factors. By modelling real and nominal interest rates jointly, the model ensures that investors price nominal and real bonds consistently, so that for example the real interest rates priced into index-linked bonds are the same as the real interest rates priced into nominal bonds. This means that risk is not only priced consistently along the term structure but also across the nominal and real term structures. As a result, it is possible to use the model to decompose expected future policy rates into inflation expectations and expectations of real risk-free interest rates and to decompose nominal term premia into real term premia and inflation risk premia.

Inflation expectations in this model can be thought of as being generated as the forecast from a regression of inflation on lags of the latent factors and inflation, where the coefficients are estimated subject to the restriction of no arbitrage. But because of the model's structure and the fact that it is estimated over a relatively short sample period (from October 1992 to May 2008), one potential problem is that the factors driving interest rates may be estimated to revert to their long-run values too quickly, so that long-horizon expectations exhibit insufficient variation. To help alleviate this problem, the model also incorporates Consensus survey expectations of inflation five to ten years ahead. The estimation method treats the survey measures as a noisy

(1) Strictly, the models described here are termed 'essentially affine' following Duffee (2002). Such models have been applied widely in academia and by central banks. Applications by central bank economists include Kim and Wright (2005) at the Federal Reserve Board and Hördahl, Tristani and Vestin (2007) at the European Central Bank.
(2) There will also be a 'convexity effect', which reflects the non-linear relationship between bond prices and interest rates. These effects are ignored here, as they are small at the horizons considered.
(3) Some affine models incorporate more economic structure and therefore permit more economic interpretation. In macro-factor models, for example, interest rate movements can be attributed to the included macroeconomic variables (see Ang and Piazzesi (2003) for the seminal paper in this area; for a more recent example see Kaminska (2008), who incorporates demand, supply and monetary policy disturbances into an affine model). Models based solely on macroeconomic factors tend to fit the term structure less well than latent factor models, however, particularly at longer horizons.
(4) See Joyce, Lildholdt and Sorensen (2008). This research builds on earlier work by Lildholdt, Panigirtzoglou and Peacock (2007), who apply a similar framework to model nominal interest rates, and Joyce, Kaminska and Lildholdt (2008), who use a similar approach to model real interest rates.

signal of expectations, putting greater or lesser weight on the surveys according to how closely they match the behaviour of the nominal and real interest rate data. So although the model-implied inflation expectations will equal the survey on average, the model does not constrain the two to be equal period by period.

Developments in long-term forward rates in 1997

Charts 7, 8 and **9** show the decomposition from this model of the ten-year nominal, real and inflation forward rates shown in **Chart 4**.(1) This decomposition sheds some further light on developments in forward rates around the time the MPC was founded in May 1997. For example, the model attributes the fall in the nominal forward rate to a fall in the nominal term premium and to a lesser extent to a lower expected future policy rate (**Chart 7**). **Chart 9** shows that the fall in the inflation forward rate can similarly be accounted for by both a fall in inflation expectations and a fall in the inflation risk premium. This is consistent with the view that the change in the monetary policy framework helped anchor people's expectations of inflation in the medium term and reduced uncertainty about future inflation.(2) In contrast, **Chart 8** shows that the fall in the real forward rate is predominantly attributed to a fall in the real term premium, with expectations of the real risk-free rate little changed.

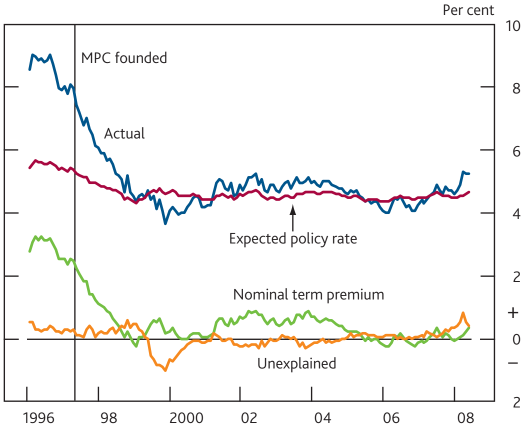
Developments in long-term forward rates since 2003

The model also provides some interesting insights into the concurrent fall in the real forward rate and the rise in the inflation forward rate since 2003, shown in **Chart 4**. The small and stable unexplained component in **Chart 7** shows that overall the model fits the long-horizon nominal forward rate well over this period. The model suggests little change in the expected future policy rate and attributes movements in the nominal forward rate predominantly to changes in the nominal term premium. **Chart 8** and **Chart 9** help to disentangle these movements.

Chart 8 shows that the model attributes a large part of the fall in the real forward rate since 2003 to a fall in the real term premium. In particular, the estimates suggest that the real term premium switched sign around the start of this period and then became increasingly negative. This might imply that investors in index-linked bonds at this time were prepared to pay a premium for the insurance-like characteristics of these bonds. As discussed earlier, this could in principle reflect changes in investors' expectations about the persistence of the effects of economic disturbances. However, the absolute increase in the unexplained component since about 2005 also indicates that more recently the model has found it more difficult to explain movements in the real forward rate.

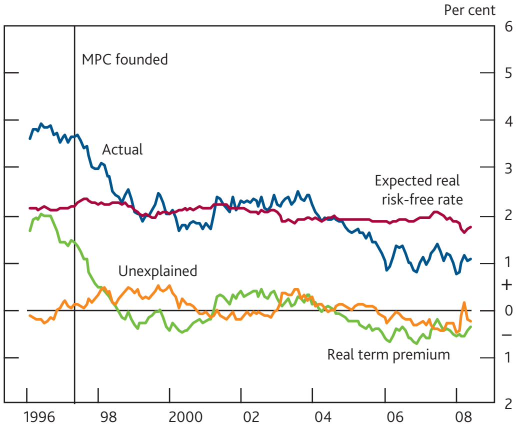
A similar development is apparent in **Chart 9**. Most of the increase in the inflation forward rate since 2003 is attributed to the unexplained component, which becomes positive after

Chart 7 Model-based decomposition of UK ten-year nominal forward rate



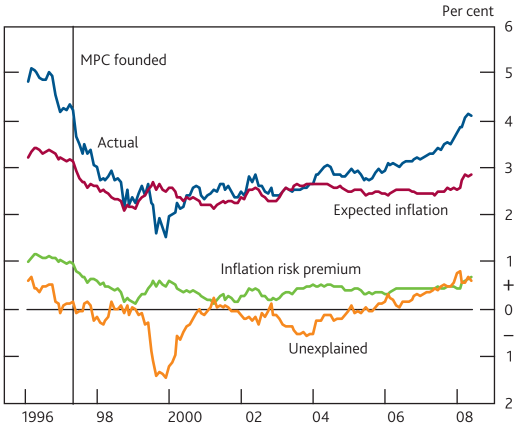
Source: Bank calculations.

Chart 8 Model-based decomposition of UK ten-year real forward rate



Source: Bank calculations.

Chart 9 Model-based decomposition of UK ten-year RPI inflation forward rate



Source: Bank calculations.

(1) The model is estimated using monthly data. So in contrast to **Chart 4**, which shows daily data, **Charts 7–9** show end-month data.
(2) See King (2007) for a further discussion.

2005. Since the start of this year, however, the model estimates do show some evidence of a pickup in long-term RPI inflation expectations and the inflation risk premium. However, despite this increase the level of RPI expectations implied by the model is still broadly consistent with CPI inflation being close to 2.0%, after allowing for the estimated long-run wedge between RPI and CPI inflation.

The model's inability to account fully for movements in real and inflation forward rates since 2003, while at the same time describing nominal forward rates well, could indicate the influence of factors that are specific to the market for index-linked bonds that the model does not adequately capture. In this context, it is worth noting that Bank of England market intelligence attributes lower real forward rates at longer horizons predominantly to the limited supply of long-term index-linked bonds in the face of exceptionally strong institutional demand for such assets. This demand may reflect several regulatory and accounting changes over the past few years that have encouraged pension funds to seek to match their liabilities more closely with inflation-linked assets.(1) If this development lies behind the model's failure to explain long-term real forward rates, it could also potentially account for part of the rise in long-term inflation forward rates over the same period (see also the discussion in the 'Markets and operations' article in this *Bulletin*). This would suggest that the pickup in longer-horizon inflation forward rates since 2003 was not primarily driven by higher expected inflation. This seems to accord with other market intelligence, which provides little indication that market participants' expectations of long-term inflation have picked up in recent years.

Conclusions and directions for future research

This article has reviewed recent advances in extracting and interpreting policy-relevant information from the term structures of forward rates and how they can be used to shed light on the downward trend in long-term real forward rates and the upward trend in long-term RPI inflation forward rates since 2003, both developments that have attracted the attention of policymakers.

The theoretical results suggest that the fall in long-term real forward rates over this period is not easy to reconcile with the prolonged macroeconomic stability prior to the onset of the more recent financial market turbulence. Moreover, over this period, the empirical affine model finds it difficult to account fully for the fall in long-term real forward rates. This is also mirrored in the model's inability to account fully for the rise in long-term inflation forward rates over broadly the same period, although estimated long-term inflation expectations have increased somewhat since the start of 2008. The affine model results are broadly consistent with market intelligence, which suggests that to date long-term inflation expectations appear to remain well anchored. However, the apparent stability of market-based measures of long-term inflation expectations contrasts with the increase in survey-based measures of short-term inflation expectations discussed in another article in this *Bulletin*.(2)

The difficulties accounting for and explaining some of the more recent movements in the term structure of forward interest rates and inflation rates pose challenges for future research. One aspect of this is to try to bring the theoretical and empirical approaches to modelling the term structure closer together. This could in principle be done by introducing more structure into empirical models, or by making the general equilibrium models match the data more closely. One reason perhaps that this has yet to be achieved convincingly is that most models of the term structure, like those described here, abstract from what can be broadly described as financial market imperfections. These may include the so-called 'search for yield' that is said to have increased investors' demand for risky assets in order to meet nominal return aspirations, or strong demand for index-linked bonds from particular investor groups such as pension funds. The affine term structure models described here may be picking up some of these effects indirectly. But an understanding of how to characterise such behaviour by different groups of investors more formally and how it impacts on financial prices is not yet well developed.

(1) These issues are discussed further in McGrath and Windle (2006).
(2) For a discussion of short-term inflation expectations see Benford and Driver (2008).

Interest rate concepts and measurement

This box explains some of the terminology used in this article in more detail and how the forward interest rates shown in **Charts 1–4** are estimated.

Concepts

A bond is a promise by the issuer (borrower) to pay interest (coupons) to the investor (lender) until the bond's maturity date, at which point the investor receives the final coupon payment and the principal (the bond's face value). Bonds issued by the UK government are known as gilt-edged securities or gilts.

The *yield to maturity* of a bond is a measure of a bond's implied average interest rate, if it is held to maturity. Though commonly used, this measure has the disadvantage that it is calculated assuming that all coupon payments are reinvested at this same average interest rate. This will not usually be the case unless future interest rates are constant.

The *spot yield* or *spot interest rate* is the same as the yield to maturity in the special case of a bond that does not pay coupons, a so-called 'zero-coupon' bond. It is therefore not affected by any assumptions about the size or timing of coupon payments or the rate at which they can be reinvested.

Forward yields or *forward interest rates* are the interest rates between different horizons in the future implied by current spot rates. Spot rates are averages of forward rates. So any particular forward rate can be derived from the two spot interest rates whose maturities span the period of the forward rate. For example, if the two-year spot rate is 10% and the one-year spot interest rate is 5% then the one-year forward rate relating to the second year is roughly 15%.(1)

The terms *yield curve* and *term structure of interest rates* are often used interchangeably. The term structure of forward rates or the forward yield curve shows implied interest rates at different future horizons on a given date.

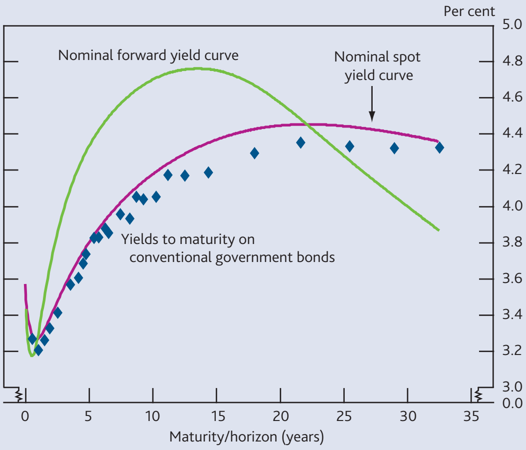
Measurement

The UK government issues nominal bonds (*conventional gilts*) and real bonds (*index-linked gilts*). Most UK government bonds pay coupons semi-annually. Real bonds are indexed to the retail prices index (RPI) and differ from nominal bonds in that both their coupon payments and principal are adjusted in line with inflation. Conventional and index-linked gilts are issued for a limited set of maturities, and the range of outstanding maturities changes through time, reflecting the pattern of new bond issuance and outstanding bonds approaching their maturity date.

In order to produce a yield curve, a method is needed that disentangles the interest payment on coupon-paying bonds to form hypothetical zero-coupon bonds and at the same time 'fills in the gaps' to give a continuous curve at any point in time. The Bank of England achieves this by estimating nominal and real yield curves using a smoothed 'cubic spline' method. This method results in yield curves that show greater flexibility at shorter maturities and less at longer maturities.(2)

As an example, **Chart A** shows the estimated UK government nominal spot and forward yield curves on 30 May 2003 together with the yields to maturity on the outstanding (coupon-paying) government bonds used to construct them.(3) The yields to maturity on most bonds lie slightly below the spot curve, reflecting the fact that the spot curve was upward sloping at this time. Given the way yields to maturity are constructed on coupon-paying bonds, yields in this case tend to be lower than spot rates because they are effectively weighted averages of the lower spot rates at intervening maturities (the opposite relationship would hold if the yield curve was downward sloping). This brings out the point that a curve based on yields to maturity can sometimes give a different and potentially misleading picture. The differences between the forward and spot curves also emphasises the importance of being clear about which measure of long-term interest rates is being used.

Chart A UK government nominal spot and forward yield curves on 30 May 2003



Source: Bank calculations.

A similar method is used to construct real spot and forward curves, though this is a slightly more complicated procedure, as the method needs to allow for the fact that index-linked bonds are not perfectly indexed for inflation because of so-called indexation lags. Inflation spot and forward curves are then derived as the difference between the equivalent nominal and real curves.

(1) More precisely, assuming discrete annual compounding, this can be calculated as 15.24% from the expression $[(1 + 0.1)^2/(1 + 0.05)] - 1$.
(2) This is referred to as a variable roughness penalty method. See Anderson and Sleath (2001). More details on the Bank of England's yield curve estimates are contained on the Bank's website at www.bankofengland.co.uk/statistics/yieldcurve/index.htm.
(3) The curve is also estimated from general collateral repo rates, which are not shown, as these instruments are closer to zero-coupon bonds.

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