Bank of England

Decompositions, forecasts and scenarios from an estimated DSGE model for the UK economy

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Daniel Albuquerque, Jenny Chan, Derrick Kanngiesser, David Latto, Simon Lloyd, Sumer Singh and Jan Žáček

A series designed to document models, analysis and conceptual frameworks for monetary policy preparation – they are written by Bank staff to encourage feedback and foster continued model development.

Macro Technical Paper Series

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This included improving our model maintenance and development. Macroeconomic models and frameworks play an important role in the monetary policy process. To maximise the value of macroeconomic models, they must be well documented and continuously improved as time goes on.

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Importantly, while each MTP will provide insights about a particular model or modelling framework that is an 'input' to policy, no single model can possibly capture all the relevant features to perform even just one of those roles adequately. Models will inevitably have to be updated and will improve over time, including as they are adapted to different constellations of macroeconomic conditions. This is a natural part of real-time monetary policy making. So, inevitably, no MTP will provide definitive answers.

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Clare Lombardelli
Deputy Governor for Monetary Policy

Bank of England

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Decompositions, forecasts and scenarios from an estimated DSGE model for the UK economy

Daniel Albuquerque,[1] Jenny Chan,[2] Derrick Kanngiesser,[3] David Latto,[4] Simon Lloyd,[5] Sumer Singh[6] and Jan Žáček[7]

Abstract

We describe a medium-scale, open-economy dynamic stochastic general equilibrium model of the UK economy, which has its foundations in 'COMPASS', the Bank of England's 'Central Organising Model for Projection Analysis and Scenario Simulation' described in Burgess et al (2013). The model we describe is augmented to include imported energy goods in production and consumption, time-varying trends, an expanded set of economic shocks and real adjustment costs. We parametrise the model via a mix of calibration and full-information Bayesian estimation. The model can match key moments in the UK data and aligns well with salient empirical impulse response functions. The model is part of a broader suite used to inform the monetary policy process at the Bank of England, and it can be used in a range of ways. In this paper, we explain how the model can be applied to produce structural decompositions, forecasts and counterfactual scenarios.

Key words: DSGE, forecasting, macro-modelling, scenario analysis.

JEL classification: E17, E20, E30, E40, E50.

(1) Bank of England. Email: daniel.albuquerque@bankofengland.co.uk
(2) Bank of England. Email: jenny.chan@bankofengland.co.uk
(3) Derrick Kanngiesser worked at the Bank of England until March 2025.
(4) Bank of England. Email: david.latto@bankofengland.co.uk
(5) Bank of England. Email: simon.lloyd@bankofengland.co.uk
(6) Bank of England. Email: sumer.singh@bankofengland.co.uk
(7) Bank of England. Email: jan.zacek@bankofengland.co.uk

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1 Introduction

Estimated medium-scale dynamic stochastic general equilibrium (DSGE) models are important components of central bank toolkits.[1] Grounded in the tradition of Christiano et al. (2005) and Smets and Wouters (2007), these models can provide interpretations of past data and economic forecasts that can be broken down into structural drivers, as well as scenarios with structural underpinnings. However, the usefulness of DSGE models for policy is dependent on their timely estimation and the sources of uncertainty they entertain. In that vein, this *Macro Technical Paper* describes an updated medium-scale DSGE model for the UK economy. The model has its foundations in 'COMPASS', the Bank of England's *Central Organising Model for Projection Analysis and Scenario Simulation* described in Burgess et al. (2013), with both sharing an open-economy structure and a two-agent framework.

A key enhancement of this model is the introduction of an energy sector in the style of Chan et al. (2024), where energy is consumed by households and used as an input to production. This enables an assessment of energy-price shocks of the like seen in 2022 following Russia's invasion of Ukraine. In addition to being consumed by households, energy features in a constant elasticity of substitution (CES) production technology with the realistic feature that imported energy is complementary to domestic labour as a factor of production. This helps the model capture various channels through which energy prices can affect demand and supply in general equilibrium. Alongside this, an updated open-economy setup supports more detailed analyses of global shocks, which have been shown to play an important role for UK macroeconomic dynamics (Cesa-Bianchi et al., 2021).

We also highlight new convex adjustment costs for production inputs, to improve the model's ability to match the observed volatility in macroeconomic data. These real rigidities introduce more realistic lags into the transmission of shocks. In tandem, we describe how our updated model estimation, using quarterly data from 1987 to 2023, accounts for the macroeconomic consequences of the Covid-19 pandemic.

Using the model in practice. Our model serves three main purposes:

- First, the model can be used to analyse and explain different forecasts, as well

[1] Examples include, among others, the European Central Bank's New Area-Wide Model (NAWM) (Coenen et al., 2018), the Norges Bank's open-economy NEMO model (Brubakk et al., 2006), the Sveriges Riksbank's RAMSES (Adolfson et al., 2013) and MAJA (Corbo and Strid, 2020) models, the Czech National Bank's g3+ model (Brázdik et al., 2020, 2025), the Reserve Bank of New Zealand's KITT model (Lees, 2009), and various Federal Reserve system models (e.g., Campbell et al., 2023; Dotsey et al., 2011).

as past data, and cross-check other macroeconomic forecasts. As we describe in this paper, structural model-based decompositions provide a lens through which to interrogate the sources of macroeconomic fluctuations implicit in a forecast to draw out key narratives for policymakers.

- Second, it can be used an organising framework for the construction of macroeconomic forecasts. Although macroeconomic models do not provide definitive answers, with forecasts often requiring judgemental overlays, their general equilibrium structure can help to ensure that projections are internally consistent.

- Third, the model can be used to assess the sensitivity of forecasts to alternative assumptions, offering an opportunity to build model-based scenarios. The Bank of England's May 2025 *Monetary Policy Report* contained two examples of such scenarios. However, given the scale of the model, there is an extensive range of potential scenarios that could be constructed.

While a useful tool, this updated medium-scale DSGE model is just one input to monetary policymaking. As is the case at other central banks (see, e.g., Ciccarelli et al., 2024), staff at the Bank of England draw on a range of different types of models to inform forecasting and scenario analysis. The general principle that "*all models are wrong, but some are useful*" (Box, 1976) is a long-standing feature of the approach staff take when interpreting economic developments. Some models supplement this, relatively tractable, DSGE model to provide a more detailed assessment of key economic channels; others expand the relatively narrow set of forecast-consistent variables to help interpret the outlook through alternative lenses. Some models are used to help reconcile this DSGE model's impulse responses with broader empirical evidence; others provide cross-checks on the central forecast (see those set out in Pill, 2024). Future *Macro Technical Papers* will shed light on some of these tools, highlighting how Bank staff combine insights from these myriad approaches to support their assessment of economic developments.

Outline. This paper has the following structure. Section 2 describes the model, before Section 3 presents its calibration and Bayesian estimation. Section 4 documents the model's properties, including historical shock decompositions that inform our understanding of the cyclical dynamics of the economy. Section 5 sets out the model's forecast performance for key macroeconomic variables, before Section 6 describes how the model can be used for scenario analysis. Section 7 concludes and looks ahead to model development priorities as the Bank continues the process of transforming its approach to monetary policy.

2 The Model

In this section, we outline the main ingredients of the model. A more detailed description of the model, along with derivations, can be found in Appendix A. Figure 1 depicts its overall structure.

The model has a Two-Agent New Keynesian (TANK) structure, featuring fully optimising (or Ricardian) and rule-of-thumb (or hand-to-mouth) households. Household heterogeneity is important for analysing the transmission of shocks, including monetary policy shocks, and for matching households' marginal propensities to consume (Bilbiie, 2008, 2020; Kaplan et al., 2018). A TANK model allows us to capture much of the effect of heterogeneity while remaining fairly tractable (Debortoli and Galí, 2024). In the model, optimising households have access to domestic and foreign risk-free assets, allowing them to smooth consumption over time, while rule-of-thumb households consume all of their disposable income in each period.

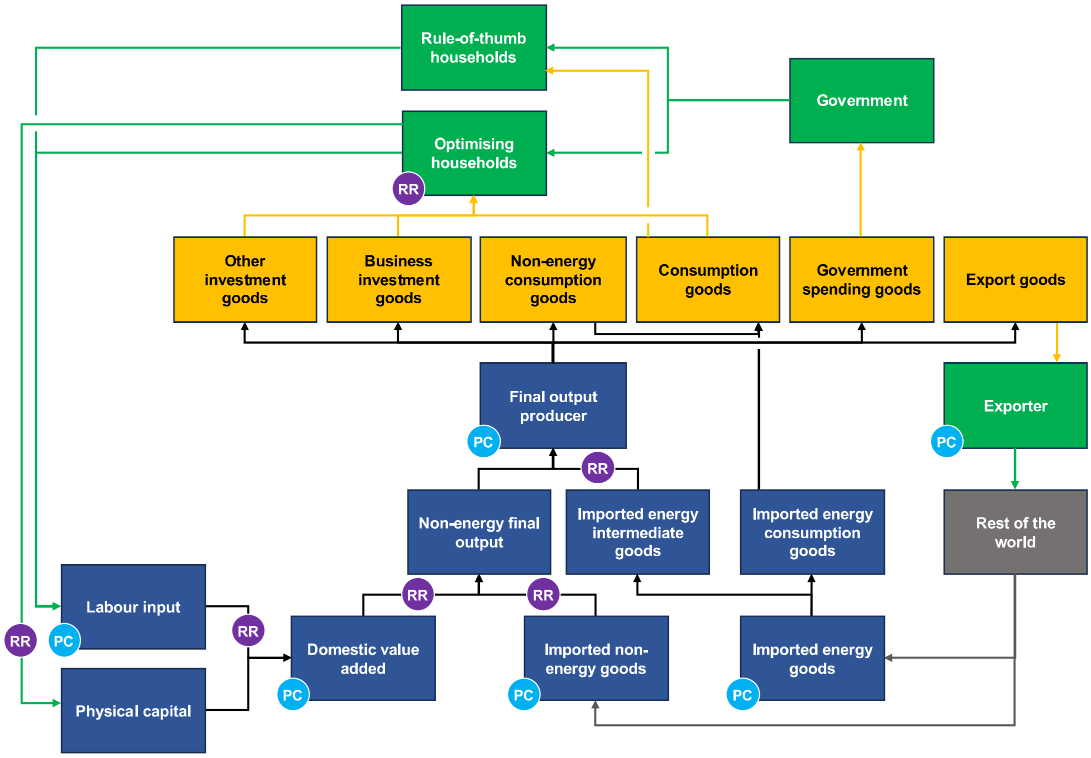
The production structure is multi-layered. First, capital and labour are combined to create domestic value-added. Non-energy imports are then added before imported energy is finally introduced to produce the final good. This final good can be used for investment, government consumption, exports and non-energy consumption. The latter is combined with imported energy consumption into a final household consumption basket. Thus, energy features both as an intermediate input into production and in households' consumption.

The model features exogenous trends (e.g., to population and labour productivity), which drive the long-run behaviour of the economy but imply that many variables are non-stationary. To handle this, we detrend all variables to solve and estimate the model in log-linearised form relative to its balanced growth path. These detrended variables then admit a 'steady-state'. We denote non-stationary variables with a tilde $\widetilde{C}_t$. Their trends and the growth in their trends are given by $\widetilde{\chi}_t^C$ and $\Gamma_t^C = \widetilde{\chi}_t^C / \widetilde{\chi}_{t-1}^C$, respectively. Stationary variables are denoted in upper case $C_t = \widetilde{C}_t / \widetilde{\chi}_t^C$, and their steady state without a time subscript C. Log variables are in lower case $c_t = \log(C_t)$, and log-deviations from the trend with a hat $\widehat{c}_t = \log(C_t) - \log(C) = c_t - c$.

2.1 Households

The economy is populated by a continuum $i \in [0,1]$ of households, a share $\omega^o \in (0,1)$ of which are fully optimising and a share $1-\omega^o$ that are rule-of-thumb. Households derive utility from consumption and disutility from labour. Optimising households also derive utility from real money balances and own the capital stock in the economy, which

Figure 1: OVERALL STRUCTURE OF THE MODEL



Notes: "RR" and "PC" denote real rigidities and Phillips Curves that capture nominal rigidities, respectively. Real rigidities refer to various adjustment costs that slow down the response of real variables to shocks. The source of nominal rigidities is inertia in nominal wages and prices, modelled à la Calvo. The "Labour input" box has a "PC" sign because there are labour unions in the model which set the nominal wage for households under nominal Calvo pricing. The "Optimising households" box has a "RR" sign because of the presence of habit in consumption.

they rent out to firms. They also own the firms, from which they receive profits. They can save in domestic or foreign, risk-free, one period, nominal bonds.

The consumption of optimising and rule of thumb households, in log deviations from the balanced growth path ($\widehat{c}_t^{opt}$ and $\widehat{c}_t^{rot}$, respectively), are given by:

$$\widehat{c}_t^{opt} = \bar{\beta}_h \mathbf{E}_t\left[\widehat{c}_{t+1}^{opt}\right] + \xi_h \widehat{c}_{t-1}^{opt} - \frac{1}{\sigma_h}\left(\widehat{r}_t - \mathbf{E}_t\left[\widehat{\pi}_{t+1}^{CPI}\right] - \mathbf{E}_t\left[\widehat{\gamma}_{t+1}^{z}\right] + \widehat{\varepsilon}_t^{b} - \widehat{\varepsilon}_t^{C} + \mathbf{E}_t[\widehat{\varepsilon}_{t+1}^{C}]\right) \tag{1}$$

$$\widehat{c}_t^{rot} = (w_{ss}L_{ss}/(p_{ss}^{C}C_{ss}))(\widehat{w}_t + \widehat{l}_t - \widehat{p}_t^{c}) \tag{2}$$

with total household consumption given by $\widehat{c}_t = \omega^o \widehat{c}_t^{opt} + (1-\omega^o)\widehat{c}_t^{rot}$.

Equation (1) is the usual dynamic IS curve derived from the Euler equation of optimising households in a New Keynesian model. The term $\xi_h \widehat{c}_{t-1}^{opt}$ appears because of external habit formation, $(\widehat{r}_t - \mathbf{E}_t\left[\widehat{\pi}_{t+1}^{CPI}\right])$ denotes expected changes to the real interest rate, $\mathbf{E}_t[\widehat{\gamma}_{t+1}^Z]$ is the expected trend growth rate of GDP, and $\varepsilon_t^C, \varepsilon_t^b$, are consumption-preference and risk-premium shocks. In contrast, rule-of-thumb households consume all their disposable income by assumption, so changes in consumption follow changes in real labour income, as shown in equation (2).

Relative to Burgess et al. (2013), we augment this model with an energy sector. To account for the role of energy in household consumption, we define aggregate consumption for both optimising and rule-of-thumb households to be a CES bundle of non-energy consumption $\widetilde{C}_t^Z$ and imported energy consumption $\widetilde{E}_t^C$,

$$\widetilde{C}_t = \left((1-\alpha_{ec})^{\frac{1}{\varepsilon_{ec}}} \left(\widetilde{C}_t^z\right)^{\frac{\varepsilon_{ec}-1}{\varepsilon_{ec}}} + \alpha_{ec}^{\frac{1}{\varepsilon_{ec}}} \left(\widetilde{E}_t^c\right)^{\frac{\varepsilon_{ec}-1}{\varepsilon_{ec}}} \right)^{\frac{\varepsilon_{ec}}{\varepsilon_{ec}-1}}.$$

This implies that the cost of energy directly affects the price level of consumption P_t^{CPI}, and thus consumer price index (CPI) inflation. Households' ability to substitute between non-energy consumption and imported energy is captured by the elasticity ε_{ec}.

Optimising households also own the physical capital stock $\widetilde{K}_{t-1}^o$ in the economy, rent it out to value-added firms at the nominal gross interest rate $\widetilde{R}_t^K$, and choose business investment $\widetilde{I}_t^o$ (subject to adjustment costs) to accumulate capital and offset depreciation δ_K. The presence of adjustment costs to investment introduces the notion of a shadow value of capital, or Tobin's Q, $\widehat{tq}_t$, that can differ (in logs) from zero and is given by

$$\widehat{tq}_t = \frac{1-\delta^K}{1-\delta^K+r_{ss}^K}\mathbf{E}_t\widehat{tq}_{t+1} - \left(\widehat{r}_t - \mathbf{E}_t\widehat{\pi}_{t+1}^Z + \widehat{\varepsilon}_t^B\right) + \frac{r_{ss}^K}{1-\delta^K+r_{ss}^K}\mathbf{E}_t\widehat{r}_{t+1}^K$$

where the current value of capital $\widehat{tq}_t$ depends on its future value and on the difference between its expected net return $\left(\mathbf{E}_t\widehat{r}_{t+1}^K - \delta_K\right)$ and its opportunity cost, $\left(\widehat{r}_t - \mathbf{E}_t\widehat{\pi}_{t+1}^Z\right)$.[2] Finally, as a result of adjustment costs, investment depends both on its past and expected future values, and also on the value of capital,

$$\begin{aligned}\widehat{i}_t = {} & \frac{1}{1+\beta\Gamma_{ss}^H}\left(\widehat{i}_{t-1} - \widehat{\gamma}_t^Z\right) + \frac{\beta\Gamma_{ss}^H}{1+\beta\Gamma_{ss}^H}\mathbf{E}_t\left(\widehat{\gamma}_{t+1}^Z + \widehat{i}_{t+1}\right) \\ & + \frac{1}{(1+\beta\Gamma_{ss}^H)\left(\Gamma_{ss}^H\Gamma_{ss}^Z\Gamma_{ss}^I\right)^2}\left(\frac{1}{\psi_I}\widehat{tq}_t + \widehat{\varepsilon}_t^I\right).\end{aligned}$$

2.2 Fiscal Policy

This model is focused on the analysis of monetary policy and monetary policy interactions, so its fiscal side is simplified. Fiscal policy is limited to charging optimising

[2]Notice that this real interest rate $\left(\widehat{r}_t - \mathbf{E}_t\widehat{\pi}_{t+1}^Z\right)$ is relative to the inflation of final domestic output $\widehat{\pi}_t^Z$, which is a measure of domestic producer price inflation, not of CPI inflation.

households lump-sum taxes to finance government consumption, and to transfer resources between household types to ensure that the consumption for both types is equal in steady-state. Since only Ricardian households see their tax bill potentially change over time to finance government consumption, the mix of changes in taxes/debt to finance higher or lower government consumption has no effect on the real economy, i.e., Ricardian equivalence holds. We further assume that domestic government bonds are in zero net supply, thus lump-sum taxes on optimising households adjust over time such that the government's budget constraint holds. Finally, government consumption follows an AR(1) process:

$$\widehat{g}_t = \rho^g \widehat{g}_{t-1} - \widehat{\gamma}_t^z + \widehat{\varepsilon}_t^g \tag{3}$$

where $\widehat{\varepsilon}_t^g$ is a shock to government expenditure.

2.3 Open Economy

The domestic economy imports energy consumption $\widetilde{E}_t^c$ and non-energy goods $\widetilde{M}_t$ from the rest of the world and exports final goods $\widetilde{X}_t$ abroad. We assume a paradigm of Local Currency Pricing (LCP) (Betts and Devereux, 2000) whereby prices of non-energy imports P_t^M and of energy imports P_t^E face nominal rigidities in the domestic currency, while the price of exports P_t^{EXP} is sticky in the foreign currency. International prices abroad are also sticky, giving rise to a Phillips Curve for foreign prices.

We assume that the domestic economy is small relative to the world economy. Therefore, domestic variables do not affect the decisions of foreign households and firms. Accordingly, while domestic exports $\widetilde{X}_t$ depend on total world trade $\widetilde{Z}_t^F$, they do not affect it, and evolve according to:

$$\widetilde{X}_t = \left(\frac{P_t^{EXP}}{P_t^{X^F}} \right)^{-\varepsilon_F} \widetilde{Z}_t^F \overline{\kappa}_t^F$$

where $P_t^{X^F}$ is the price of foreign exports and $\overline{\kappa}_t^F$ depends on adjustment factors.

World trade evolves in line with foreign output $\widetilde{V}_t^F$, except for when a world trade shock $\varepsilon_t^{Z^F}$ disturbs this relationship:

$$\widehat{z}_t^F = \widehat{v}_t^F + \phi^{Z^F} \widehat{\varepsilon}_t^{Z^F},$$

where ϕ^{Z^F} scales the shock.

While the price of foreign exports depends on the developments of the world block, we assume that the foreign price of energy is exogenous:

$$\widehat{p}_t^{E,F} = \rho_E \widehat{p}_{t-1}^{E,F} + \widehat{\varepsilon}_t^E.$$

Finally, monetary policy in the rest of the world is modelled according to a Taylor rule setting the foreign nominal gross interest rate R_t^F. Because households can invest in both domestic and foreign bonds, an Uncovered Interest Parity (UIP) condition links domestic and foreign interest rates with the nominal exchange rate $\mathscr{E}_t$ (defined as the foreign price of domestic currency, such that a decline corresponds to a domestic depreciation):

$$\mathbf{E}_t \left[\frac{\Theta_{t+1}}{\Theta_t} \frac{\widetilde{\Lambda}_{i,t+1}^{C,o}}{\widetilde{\Lambda}_{i,t}^{C,o}} \left[R_t - R_t^F \varepsilon_t^{B^F} \frac{\mathscr{E}_t}{\mathscr{E}_{t+1}} \right] \right] = 0$$

where $\left(\Theta_{t+1}/\Theta_t\right)\left(\widetilde{\Lambda}_{i,t+1}^{C,o}/\widetilde{\Lambda}_{i,t}^{C,o}\right)$ can be interpreted as an endogenous discount factor of the optimising households.

2.4 Monetary Policy

Domestic monetary policy in the model follows a Taylor-type rule (Taylor, 1993) in log deviations:

$$\widehat{r}_t = (1-\theta_r) \left[\frac{\theta_\pi}{4} \left(\widehat{\pi}_t^{CPI,ann} - (1-\theta_E)\widehat{econt}_t - \widehat{\mu}_t^{z,temp,ann} \right) + \theta_y \widehat{y}_t^{gap} \right] + \theta_r \widehat{r}_{t-1} + \widehat{\varepsilon}_t^r$$

where $\Pi_t^{CPI,ann} = P_t^{CPI}/P_{t-4}^{CPI}$ is annual CPI inflation, ε_t^r is a monetary policy shock and $\widehat{y}_t^{gap}$ is the output gap, defined as the deviation of output from its flexible price and wage equilibrium counterpart. The shock $\widehat{\mu}_t^{z,temp,ann}$ is the sum over four quarters of one-period, fully transitory, final-output markup shocks. The idea is that while the policymaker should respond to inflation caused by persistent shocks μ_t^z, it should not respond to inflation driven by purely temporary shocks $\mu_t^{z,temp}$.

Instead of imposing a policy rule that targets either core or headline inflation, we adopt a more agnostic approach. Specifically, we introduce a parameter θ_E, where $\theta_E = 1$ corresponds to pure headline inflation, while $\theta_E = 0$ represents an adjusted measure closer to core inflation, where the energy contribution, $econt_t$, is excluded from headline inflation (although food prices remain in the index).[3] This allows us to estimate θ_E as well, and we find $\theta_E = 0.25$, which suggests that monetary policy reacts primarily to a measure of inflation that is closer to core. This formulation also

[3]The measure $econt_t$ represents the contribution of the items "04.5 Electricity, gas and other fuels" and "07.2.2 Fuels and lubricants" to the CPI.

captures the limited ability of a small-open economy to influence energy prices, which are exogenous and largely determined outside the UK market.

2.5 Aggregate Supply

The production structure of the domestic economy is multi-layered. First, labour and capital are combined to generate domestic value added. Then, this value added is combined with imported non-energy goods, and finally with imported energy goods, to generate final domestic output. We denote this nested production structure as (((KL)M)E), where K, L, M, and E, represent capital, labour, non-energy imports, and energy, respectively. While capital and labour are combined under a Cobb-Douglas production function, the other layers use CES aggregators.

Nominal and real rigidities are included in the model to match better fluctuations in the data, as well as empirical impulse response functions. Many of the real rigidities are introduced with adjustment costs, while nominal-price stickiness, modelled à la Calvo (1983), gives rise to Phillips curves in the model.

Figure 1 highlights where nominal and real rigidities are present in the model. Real rigidities include, for example, investment-adjustment costs, which generate frictions in households' investment decision. Relative to Burgess et al. (2013), the model includes non-energy final-output firms. We introduce real rigidities for these firms in the form of adjustment costs with respect to their inputs. Moreover, even for firms that were present in Burgess et al. (2013), we have introduced additional real rigidities to improve the fit relative to the data. In particular, now final-output producers pay adjustment costs when adjusting their imported-energy intermediate goods, and domestic value-added producers pay costs to adjust their labour input.

Moving to nominal rigidities, as Figure 1 demonstrates, there are several Phillips curves (PCs) in the model. For example, amongst final-output producers, a fraction ϕ_Z of firms adjusts their price according to an average of the steady-state inflation and previous-period inflation, with weights $1-\xi_Z$ and ξ_Z, respectively. For the other $1-\phi_Z$ share of firms that do not follow the above rule of thumb, they can only choose their nominal prices with some exogenous probably $1-\omega_Z$, otherwise they follow the same indexation as above. This gives rise to the following PC:

$$\widehat{\pi}_t^z = \beta_z \mathbf{E}_t\left[\widehat{\pi}_{t+1}^z\right] + \zeta_z \widehat{\pi}_{t-1}^z + \kappa_z \widehat{mc}_t^z + \widehat{\mu}_t^z + \widehat{\mu}_t^{z,temp} \tag{4}$$

where inflation $\widehat{\pi}_t^z$ is a function of markup gaps $\widehat{mc}_t^z$, expected future inflation $\mathbf{E}_t\left[\widehat{\pi}_{t+1}^z\right]$, previous period inflation $\widehat{\pi}_{t-1}^z$, and markup shocks $\widehat{\mu}_t^z, \widehat{\mu}_t^{z,temp}$. The parameters $\beta_z, \zeta_z, \kappa_z$

depend on the primitives of the model, such as ϕ_Z, ω_Z, ξ_Z. Other PCs in the model have a similar setup, including for wages, since nominal wages are also sticky due to the presence of labour unions who set wages for households.

2.6 Aggregate Demand

Domestic output in the model is demand-determined in the short-run. The demand for final output $\widehat{z}_t$ (aggregate demand) is a function of demand for (non-energy) consumption goods $\widehat{c}_t^z$, business investment demand $\widehat{i}_t$, government expenditure $\widehat{g}_t$, and export goods $\widehat{x}_t$:

$$\widehat{z}_t = (\alpha_{cz} C_{ss}/Z_{ss})\widehat{c}_t^z + (I_{ss}/Z_{ss})\widehat{i}_t + (G_{ss}/Z_{ss})\widehat{g}_t + (X_{ss}/Z_{ss})\widehat{x}_t + (I_{ss}^O/Z_{ss})\widehat{i}_t^O. \quad (5)$$

We also include other investment, denoted by $\widehat{i}_t^O$, which is paid by optimising households but follows an exogenous process that returns it to its long-trend:

$$\widehat{i}_t^O = \rho_{io}\widehat{i}_{t-1}^O - \widehat{\gamma}_t^z + \widehat{\varepsilon}_t^{I^O}.$$

This extension helps align aggregate demand more closely with elements in national accounts data. Specifically, other investment I_t^O is intended to capture components such as housing investment and stockbuilding.

3 Parametrisation

This section describes our model parametrisation, defining calibrated parameters before turning to Bayesian estimation of others. We group the parameters, a non-exhaustive set of which are listed in Table 1, into six categories: (i) energy, (ii) households, (iii) firms, (iv) monetary policy, (v) the world and (vi) steady-state ratios/growth. Within these groupings, we distinguish calibrated and estimated parameters by calibrating parameters that do not materially impact the dynamic properties of the model (e.g., expenditure shares) and estimating those that do (e.g., Calvo-pricing and real-adjustment costs). Estimation is carried out over a quarterly sample spanning 1987Q1 to 2023Q4. This period largely covers the UK's transition to an inflation-targeting regime, with data prior to 1992 serving as a training sample to initialise the Kalman filter.[4] When parameters cannot be estimated with macroeconomic time series, we draw on micro-level evidence from internal staff analysis or reference standard values from the academic literature.

[4]For a longer sample period estimated on UK data, see Harrison and Oomen (2010).

3.1 Calibration

Energy. We set the elasticity of substitution between energy and non-energy goods to 0.4 for firm production and 0.2 for household consumption. The elasticity of substitution between energy and non-energy goods in firm production is at the higher end of estimates in the literature (Adjemian and Pariès, 2008; Backus and Crucini, 2000; Bodenstein et al., 2012; Moll et al., 2023; Montoro, 2012; Natal, 2012; Plante, 2014; Stevens, 2015), as internal staff analysis using industry-level data for the UK suggests a slightly higher estimate of 0.46. The elasticity of substitution between non-energy imports and domestic value-added in production is set to 0.75, close to the value estimated by Huo et al. (2025) for the G7. Since a large share of non-energy imports for the UK cannot easily be produced domestically, it seems plausible that this parameter should be less than 1. For household consumption, our calibration for the elasticity of substitution between energy and non-energy goods lies within the range of estimates in the literature (Auclert et al., 2023; Bachmann et al., 2024; Harrison et al., 2011). The energy shares in household consumption and firm production are calibrated to 5% and 3.5%, respectively (see, e.g., Bachmann et al., 2024; Harrison et al., 2011; Natal, 2012; Stevens, 2015).

Households. The household discount factor is adjusted to ensure a steady-state nominal annual interest rate of 2.25%.[5] As in Burgess et al. (2013), the parameter that governs the endogenous discount factor is set at 0.01. The inverse Frisch elasticity is set to 2, in line with Chetty et al. (2011). We also maintain the rule-of-thumb household share at 17%, which lies at the lower end of the range in the literature (see, e.g., Benito and Mumtaz, 2006; Cloyne and Surico, 2017).[6]

Other calibrated household parameters are set based on the literature or using micro-data targets. For instance, the labour share is set to 0.62, which aligns with the post-GFC average ratio of total labour compensation relative to GDP.

The habit formation parameter is not well-identified by the data, in part due to the presence of rule-of-thumb households.[7] The prior is therefore based on the follow-

[5]This 'natural rate of interest', R^*, has varied over the sample period. The calibration here reflects an assessment adopted following the August 2018 Inflation Report, marking a decline from its pre-Global Financial Crisis (GFC) level of 4.5%. Nevertheless, the model can be adapted to explore different assessments of R^*.

[6]The marginal propensity to consume (MPC) is approximately given by the weighted average $\omega^o \times (1-\beta) + (1-\omega^o)$, where β is the discount factor and ω^o represents the fraction of households who consume out of permanent income (with MPC $1-\beta$) and $1-\omega^o$ represents hand-to-mouth consumers who spend all their disposable income each period (MPC = 1). With a hand-to-mouth share of 17%, the aggregate MPC is approximately 17.35%.

[7]There is a trade-off between the share of rule-of-thumb households and the degree of habit for-

ing evidence. First, a meta-analysis of parameter estimates for habit formation using EU countries suggests values between 0.5 and 0.6 (Elminejad et al., 2022). However, the UK consumption series is relatively volatile, which is one justification for a lower degree of consumption habit formation.

The degree of risk aversion typically takes values close to one (Elminejad et al., 2022). This parameter is usually calibrated, since it is poorly identified from macro time series data. However, in this model we estimate a value of 1.34.

Firms. Many of the price-setting parameters are estimated, but we calibrate some key parameters, including the value-added output share, the capital depreciation rate, and the degree of substitutability between non-energy imports and domestic value-added.

Relative to Burgess et al. (2013), the value-added share in final output is slightly lower, at 0.74, to reflect a larger import share. The capital depreciation rate has been increased to 2.25% (annual depreciation rate of 9%), based on internal staff analysis following an updated ONS methodology for measuring capital.

The steady-state value-added markup is calibrated to 1.4. This follows from a ratio of the labour share in value-added production ($\alpha_L = 0.85$) to the share of income received by labour (average share of labour compensation over nominal GDP at basic prices over 1993 - 2023 $= 0.62$).[8] Finally, we leave the final output steady-state markup at 1.005, as in Burgess et al. (2013).

Monetary Policy. The monetary policy reaction function parameters are estimated, except for the inflation target value of 2% and the degree of interest rate smoothing, which is set to discipline impulse responses functions to monetary policy shocks.

Steady-State Ratios and Growth. These parameters were mechanically updated by calculating the growth rates for the respective variables in the post-GFC sample period, and by calculating the post-GFC average ratios of government spending and business investment to final output (measured as total factor expenditures at basic prices).

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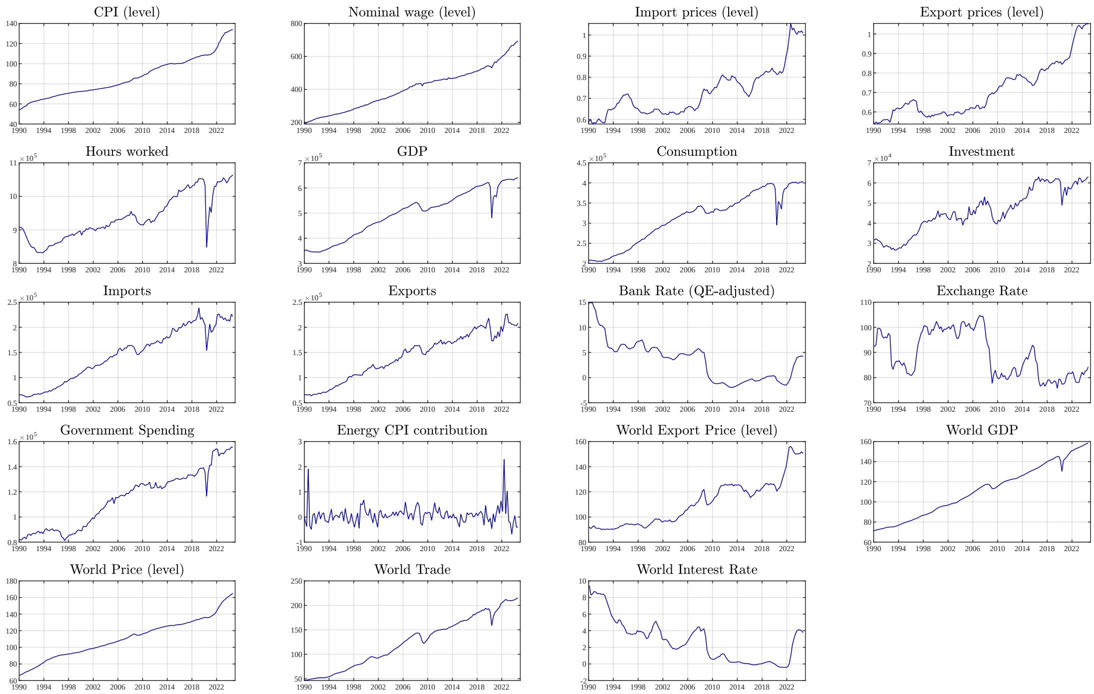
mation. Since the consumption of rule-of-thumb households inherits the stickiness in wages, a higher share of such households can impart sluggishness to consumption, similar to higher habit formation (see Burgess et al., 2013).

[8] The high labour share in production reflects the fact that capital, as measured in the model, is linked specifically to business investment, which constitutes a relatively small share of final output. Other types of investment do not contribute to capital in production. A measure of α_L at this magnitude can also be motivated by a measure of pure profit share and a market sector concept of GDP. To reconcile a low labour share of income in the model, one can also appeal to a high import intensity of capital goods, so that the role of capital essentially comes via imports.

Table 1: MODEL PARAMETERS

Parameter	Definition	Value	Source/Target
Energy			
ε_e	Elasticity of substitution for energy (Firms)	0.4	Staff estimate, micro evidence
ε_{ce}	Elasticity of substitution for energy (Households)	0.2	Staff estimate, micro evidence
α_{cz}	Share of non-energy in consumption	0.95	Household energy share (5%)
α_{zz}	Share of non-energy in final output	0.965	Firm energy share (3.5%)
Households			
β	Household discount factor	0.996	Match nominal rate ($R^* = 2.25\%$)
$betaFactor$	Endogenous discount factor	0.01	Schmitt-Grohé and Uribe (2003a)
ε_C	Coefficient of relative risk aversion	1.3336	*Estimated*
ε_L	Labour supply elasticity	2	Chetty et al. (2011) and Peterman (2012)
ψ_C	Habit formation parameter	0.3696	*Estimated*
$labshare$	Steady state labour share	0.6173	Avg. total labour compensation divide by NGDP (post-GFC)
ω^o	Share of optimising households	0.8324	Benito and Mumtaz (2006) and Cloyne and Surico (2017)
ϕ_W	Nominal wage Calvo parameter	0.8637	*Estimated*
ξ_W	Indexation of nominal wages	0.0827	*Estimated*
ω_W	Share of rule-of-thumb wage setters	0.4081	*Estimated*
Firms			
$valaddshare$	Steady state value-added share	0.7405	Avg. nom. import share in total factor exp.
ε_v	Value-added elasticity	0.75	< 1, complementarity (Huo et al. (2025))
δ_k	Capital depreciation rate	0.0225	Staff estimate, 9% annual depreciation
ψ_I	Investment adjustment cost	6.5172	*Estimated*
ψ_M	Non-energy import adjustment cost	0.8474	*Estimated*
ψ_E	Energy import adjustment cost	99.9486	*Estimated*
ψ_V	Value-added adjustment cost	10.5388	*Estimated*
ψ_L	Labour adjustment cost	7.6636	*Estimated*
ϕ_M	Non-energy import price Calvo parameter	0.4569	*Estimated*
ϕ_E	Energy import price Calvo parameter	0.6244	*Estimated*
ϕ_X	Export price Calvo parameter	0.3293	*Estimated*
ϕ_Z	Final output price Calvo parameter	0.8554	*Estimated*
ϕ_V	Value-added price Calvo parameter	0.6077	*Estimated*
Monetary Policy			
Π^*	Inflation target	1.0050	2%
θ_π	Monetary policy rule, inflation response	1.4990	*Estimated*
θ_y	Monetary policy rule, output gap response	0.2556	*Estimated*
θ_r	Monetary policy rule, interest rate smoothing	0.91	Target monetary policy shock IRF
θ_E	Monetary policy rule, non-core inflation weight	0.2464	*Estimated*
World/Trade			
ε_f	Price elasticity of world demand for UK exports	0.2619	*Estimated*
$\varepsilon_{L,f}$	World labour supply elasticity	2	Chetty et al. (2011) and Peterman (2012)
$\varepsilon_{C,f}$	World CRRA	1.0755	*Estimated*
x$labsharef$	World labour share	0.65	Staff estimate
$\psi_{C,f}$	World habit parameter	0.8826	*Estimated*
Steady State Ratios/Growth			
Γ^G_{ss}	Trend govt. spending growth rel. to final-output growth	0.99825	Match post-GFC mean growth rate
Γ^H_{ss}	Trend population/hours worked growth	1.00244	Match post-GFC mean growth rate
Γ^I_{ss}	Trend investment growth relative to final-output growth	1.00336	Match post-GFC mean growth rate
Γ^X_{ss}	Trend export growth relative to to final-output growth	1.00453	Match post-GFC mean growth rate
Γ^Z_{ss}	Trend productivity growth	1.00140	Match post-GFC mean growth rate

Figure 2: RAW OBSERVABLE DATA



Notes: This figure presents the untransformed data series. The energy contribution to CPI inflation refers to its contribution to the quarterly change in CPI inflation.

3.2 Estimation

Data. Figure 2 presents the 19 time series used to estimate the model. These include seasonally adjusted CPI, nominal wages (level), import price (level), and export price (level), along with total hours worked, GDP, consumption, investment, imports and exports. We use a 'shadow-rate' series for nominal interest rates, which adjusts Bank Rate to account for the estimated effects of quantitative easing policies (summarised in Busetto et al., 2022). The nominal effective exchange rate is computed as a weighted average of bilateral exchange rates based on UK trade shares. The dataset also includes real government spending and energy CPI contribution. For the rest of the world, we have series for export prices (level), GDP, prices (level), trade, and a nominal interest rate, constructed using trade-weighted measures based on UK trade shares.

Detrending. The model dynamics are estimated as a local approximation around a detrended steady state. We therefore need to transform raw (and non-stationary) data into stationary and detrended 'model observables'. This detrending step is crucial to ensure that the data correspond to the model counterparts. Consider GDP growth: if the model has a steady-state growth rate of 2.5% annually for hours worked and productivity, but actual GDP growth averaged only around 1.5% over the sample period, then the model would constantly have to invoke contractionary shocks to explain the discrepancy. This mismatch between (low) historical sample means and (high) theoretical model steady-state growth rates can lead to over-estimated shock persistence (potentially tending towards unit root behaviour) and erroneous historical shock decompositions and forecasts.

We take two additional steps when detrending the data. First, motivated by evidence of a structural break in UK productivity growth in the aftermath of the GFC (e.g. Haldane, 2018), we introduce variable-specific time-varying trends, which differ pre- and post-2008. The estimated trends (green line, lower left panel of Figure 3) ostensibly 'pull down/up' on pre-/post-GFC series to align cyclical fluctuations across the two periods.

Second, we smooth through the extreme volatility seen during the Covid-19 period. To do this, we use a Kalman filter in Dynare to interpolate the data for 2020-21, which we in effect set as 'missing'. The Kalman-smoothed series (cyan line, lower right panel of Figure 3) is the model's best guess of how the unobserved series would have evolved.[9] The difference between the detrended growth rates and the Kalman-smoothed series allows us to construct a variable-specific 'Covid trend', as shown in the lower left panel of Figure 3. This trend is always zero except for the eight quarters in 2020 and 2021.

To summarise, we apply a three-step transformation to the data:

(i) The non-stationary data (e.g., GDP, its components, and price levels) is converted into growth rates by taking log differences.

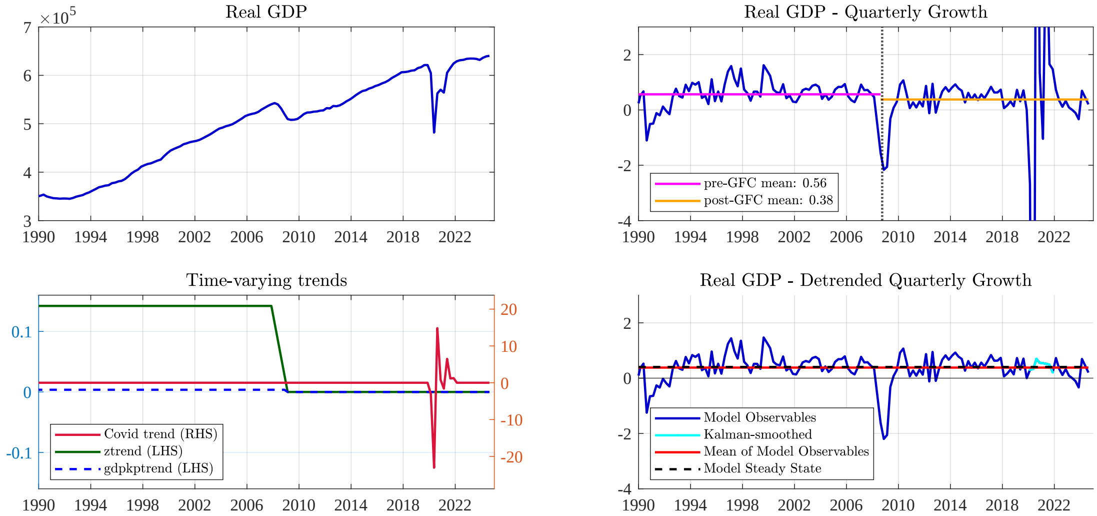
(ii) The growth rates are further de-trended using variable-specific time-varying trends to account for a structural break around the GFC.

(iii) Finally, we introduce a correction for anomalies in the Covid-19 period, interpolating missing data using a Kalman filter.

In Figure 3, we illustrate these steps for the case of GDP. In the upper right panel, we plot the growth rate of GDP, the pre-GFC sample mean (in magenta, at 0.56%) and

[9] We only apply this step for quantities (e.g., GDP and its expenditure components, domestic and foreign, including world trade), not for prices and interest rates. The Kalman filter uses the observed time series and estimated co-movements to infer missing values. A hybrid approach could entail constructing the Kalman-smoothed variable using information from a subset of observables (i.e., by not setting prices and interest rates to missing). More formal approaches include extending the model to feature stochastic volatility, or introducing a transitory shock process that is activated only during the 2020–21 period to capture the effects of the COVID-19 episode (Ferroni et al., 2024).

Figure 3: TRANSFORMATION OF GDP



Notes: This figure shows the raw data for real GDP in the upper left panel, its quarterly growth rate in the upper right panel, the detrended quarterly growth rate ('the model observable') in the lower right panel and time-varying trends in the lower left panel: Covid-trend (red), productivity trend (green), and world GDP growth trend (dashed blue).

the post-GFC sample mean (in yellow, at 0.38%). If we were to align the full sample mean with the model steady state it would imply that an unconditional forecast from the model would revert to a mean that is presumably too high, given the assumption that we are in a lower 'growth regime' since the GFC in 2008-09. We therefore detrend the growth rate with time-varying trends, as shown in the lower left panel of Figure 3. In Figure 4, we illustrate the transformed series for all the time series.

Priors and Posteriors. Using the transformed data, the next step is to estimate the remaining parameters in Table 1 using Bayesian methods (Smets and Wouters, 2003). Letting θ denote the vector of parameters to be estimated, we proceed in two steps: first, we specify prior distributions for the parameters with the standard from the literature on DSGE estimation (see Table B.1). Next, we combine these priors with the model likelihood to characterise the posterior distribution.[10] Using Bayes' rule, the posterior is given by

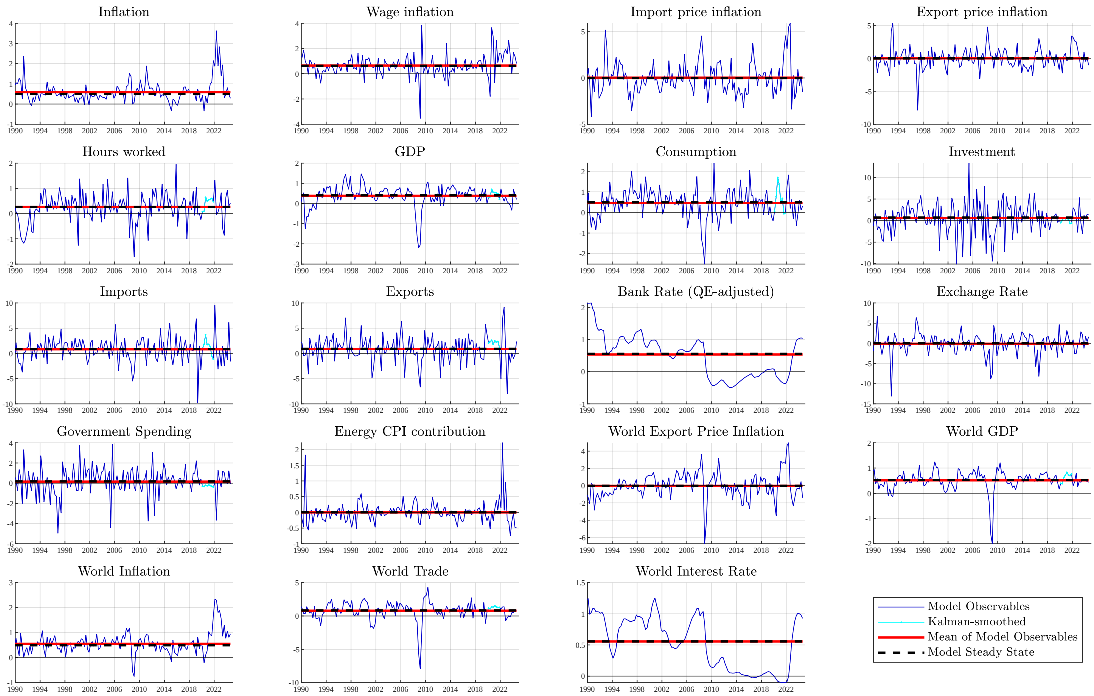
$$p(\theta|Y_T) \propto p(\theta)\,p(Y_T|\theta),$$

where Y_T represents the observed time series.

To characterise the full posterior distribution, we use Markov-Chain Monte Carlo

[10]The posterior mode is obtained by using the `csminwel` optimization algorithm (Sims, 1999).

Figure 4: MODEL OBSERVABLES



Notes: This figure shows the transformed data used to estimate model parameters. The energy contribution to CPI inflation refers to its contribution to the quarterly change in CPI inflation.

(MCMC) methods, specifically a Random-Walk Metropolis-Hastings (RWMH) sampler. We use four chains, each generating 50,000 draws, using a burn-in period of 1,000 draws. Thinning is applied to reduce autocorrelation between draws, leaving 10,000 draws in each of the four chains.

Moments. Finally, we assess how well this updated model captures properties of the data. Table 2 shows the volatility, in terms of standard deviations, of the observed empirical data and the model-generated data. The comparison indicates that the estimated model is capable of replicating the volatility observed in the actual data with a reasonable degree of accuracy. This alignment suggests that the model captures the underlying stochastic processes driving the data. Although some deviations are evident, particularly in variables such as total hours worked and export-price inflation, the model exhibits a strong alignment with the data across several core macroeconomic indicators. This overall consistency reinforces the model's credibility in reproducing second-moment properties and supports its use for policy analysis and forecasting.

Table 2: VOLATILITY IN THE DATA AND IN THE MODEL

Variable	Data Standard Deviation (1987–2023)	Simulated Data StDev
CPI inflation	0.54	0.59 (+9.26%)
Wage inflation	0.88	0.95 (+7.95%)
Import price inflation	1.81	2.20 (+21.55%)
Export price inflation	1.65	2.74 (+66.06%)
Total hours worked growth	0.64	0.99 (+54.69%)
GDP growth	0.55	0.74 (+34.55%)
Consumption growth	0.72	1.14 (+58.33%)
Investment growth	3.63	4.14 (+14.05%)
Imports growth	2.43	3.56 (+46.50%)
Exports growth	2.67	2.77 (+3.75%)
Nominal interest rate	0.70	0.38 (-45.71%)
Nominal exchange rate	2.62	3.50 (+33.59%)
Real government spending growth	1.49	1.46 (-2.01%)
Energy contribution to quarterly CPI inflation	0.30	0.37 (+23.33%)
World export price inflation	1.51	1.38 (-8.61%)
World GDP growth	0.41	0.42 (+2.44%)
World CPI inflation	0.42	0.43 (+2.38%)
World demand growth	1.43	1.24 (-13.29%)
World interest rate	0.45	0.33 (-26.67%)
Average absolute deviation		24.99%

4 Model Properties

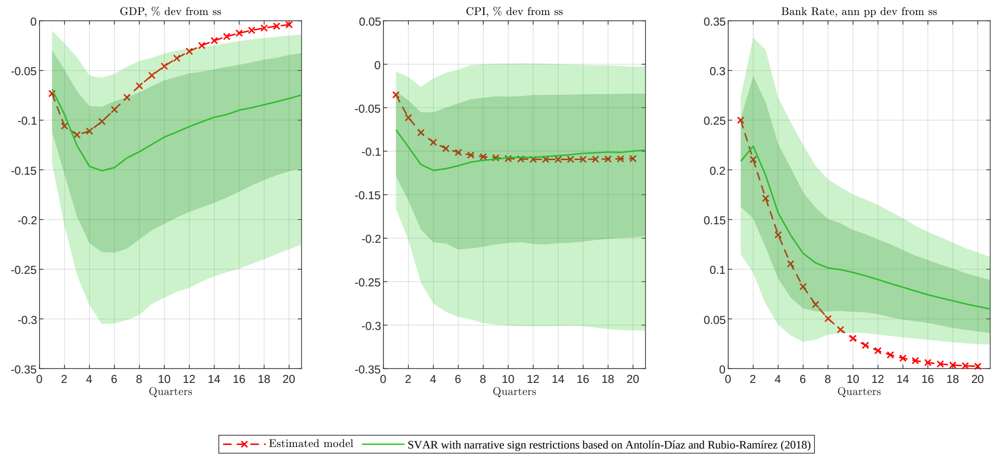
To demonstrate model properties, we present impulse response functions (IRFs) to two important shocks in the model: monetary policy and global energy prices. We use these to show how the responses from our theoretical model align with empirical estimates. The IRFs for the full set of shocks are presented in Appendix C. Additionally, we evaluate historical shock decompositions for two core macroeconomic variables, CPI inflation and GDP growth, to showcase the model's ability to offer narrative and structural interpretations of cyclical economic fluctuations. Beyond the standard decompositions, we introduce a specialised analysis that isolates the effects of monetary policy on inflation deviations from the 2% target, thereby deepening our understanding of the historical role of monetary policy.

4.1 Impulse Response Functions

Monetary Policy Shock. In Figure 5, we show the effects of a monetary policy shock, where red lines show the responses from the estimated model. We cross-check the model results with those from an estimated structural vector autoregression (SVAR) model with narrative sign restrictions based on Antolín-Díaz and Rubio-Ramírez (2018).[11] Green lines represent the median SVAR responses with corresponding swathes for the

[11] This model estimates the impact of monetary policy by combining the usual sign restrictions for monetary shocks with high-frequency surprises from financial markets.

Figure 5: IMPULSE RESPONSES TO A MONETARY POLICY SHOCK



Notes: This figure shows the effects of a monetary policy shock. Red lines show the responses from the estimated model. Green lines represent the median responses from an estimated SVAR model with narrative sign restrictions based on Antolín-Díaz and Rubio-Ramírez (2018), with corresponding swathes for the 68% and 90% confidence intervals.

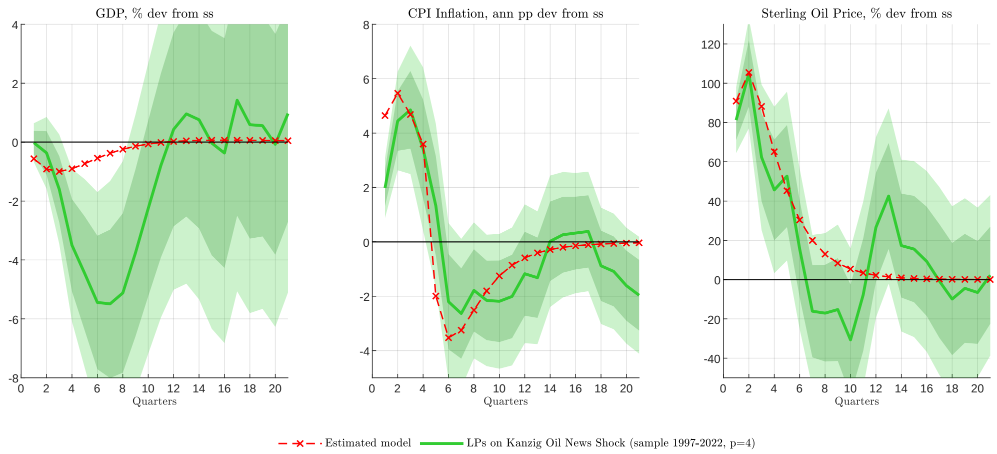
68% and 90% confidence intervals. In both cases, we scale the responses such that there is a contemporaneous shock to Bank Rate in the initial quarter of 25bps.

Despite not being a target of the estimation, the monetary transmission mechanism in the DSGE model is broadly in line with the SVAR: a monetary tightening shock leads to falls in GDP and CPI. The average impact of Bank Rate on CPI over the duration of its response is similar in the DSGE and SVAR models, although the impact on GDP is somewhat more persistent in the latter.

Figure 5 explores only *unanticipated* contemporaneous shocks because our model exhibits a 'Forward Guidance puzzle' like other New Keynesian DSGE models. In particular, variables respond too strongly when monetary policy shocks are *anticipated* by economic agents, and so the model can deliver unintuitive IRFs for very distant anticipated shocks (Del Negro et al., 2023; McKay et al., 2016). To alleviate this puzzle, we are exploring an alternative model structure where agents are myopic following Gabaix (2020), which can attenuate responses to anticipated shocks and bring them closer to our empirical cross-check from more persistent monetary-policy shocks in the SVAR model.

Global Energy-Price Shock Figure 6 plots responses of GDP, CPI inflation and the sterling oil price to a global energy-price shock. Red lines denote responses from the estimated model. Green lines are median responses based on a local-projection (LP) regression of each variable on an oil-news shock series from Känzig (2021), with cor-

Figure 6: IMPULSE RESPONSES TO A GLOBAL ENERGY-PRICE SHOCK



Notes: This figure shows the effects of an unanticipated global energy price shock. Red lines are responses from the estimated DSGE model. Green lines are Local Projections (LPs) on an oil shock series from Känzig (2021), with corresponding swathes for the 68% and 90% confidence intervals. The LP is scaled up such that the energy shock leads to an increase in inflation of around 4pp, consistent with the energy shock the UK experienced in 2022. The impulse responses from the estimated DSGE model are also scaled such that the energy shock leads to an increase in the energy price in line with the LPs.

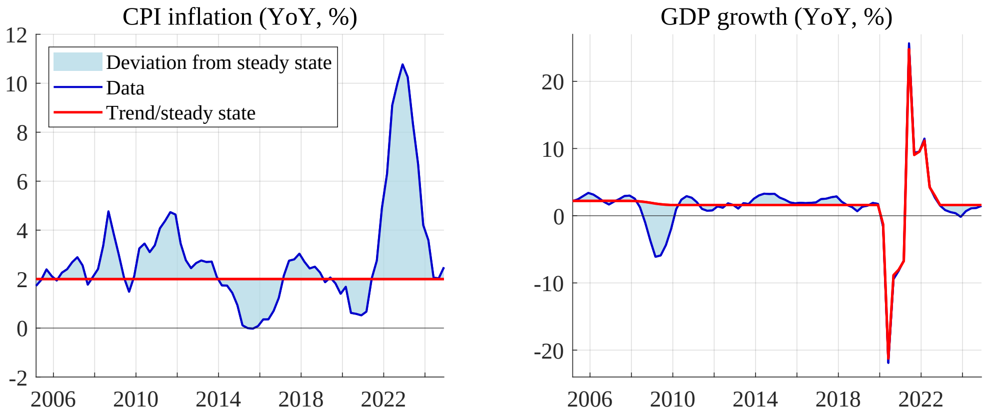
responding swathes for the 68% and 90% confidence intervals from Newey and West (1987) standard errors. We use the LP results to cross-check the dynamic responses in the estimated DSGE model. The LP results are scaled such that the energy shock leads to an increase in inflation of around 4pp, consistent with the energy shock the UK experienced in 2022. The IRFs from the estimated DSGE model are also scaled such that the energy shock leads to an increase in the energy price in line with the LP results.

Although these LP regressions are only one potential empirical cross-check for the IRFs in the estimated theoretical model, the model performs well in matching the dynamics and magnitude of the response of CPI inflation. However, notice that the LP responses imply a substantial fall in GDP (greater than 5% after six quarters). The GDP response in the estimated DSGE model is weaker, albeit in line with the LPs in the initial quarters, such that there is a 1% fall in GDP in response to an energy shock that gives rise to around a 4pp inflation overshoot on impact.

4.2 Historical Shock Decompositions

Model-based historical shock decompositions are helpful for interrogating the drivers of macroeconomic dynamics. To construct them within our model for key variables, we first calculate deviations of CPI inflation from target and deviations of year-on-

Figure 7: DATA TRANSFORMATION INTO DEVIATIONS FROM STEADY STATE



Notes: This figure shows the transformation of observed CPI inflation and GDP growth into deviations from its target and steady state, respectively. To retrieve deviations from the steady states, data inputs are adjusted for (1) off-model trends, which capture structural breaks, transition periods and extraordinary events; and (2) the steady states, which account for long-term dynamics, as explained in Section 3.

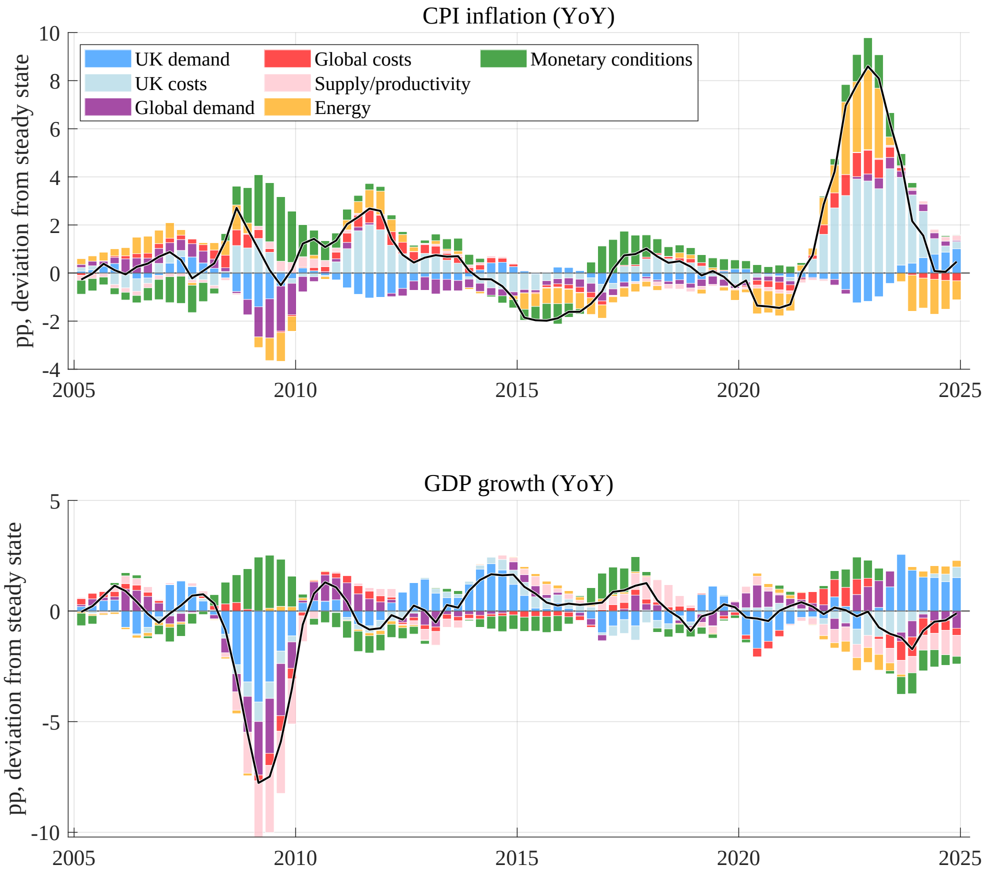
year real GDP growth from trend (both in percentage points), as Figure 7 documents. Figure 8 then shows the historical shock decompositions for CPI inflation deviations from target and detrended year-on-year real GDP growth (i.e., Covid-19-adjusted model observables). To ease readability and interpretation, we group the shocks into seven categories, including UK demand, monetary conditions, and energy.[12]

CPI Inflation. The historical shock decomposition for deviations of CPI inflation from target reveals that the model attributes most cyclical inflation fluctuations to cost-push and demand shocks. During the GFC, the UK economy entered a deep recession, driven by both domestic and global demand, putting downward pressure on inflation. At the same time, the Bank of England cut interest rates to (at the time) a historic low and initiated quantitative easing to support demand and stabilise financial markets, as demonstrated by the positive contributions of monetary conditions at the time.

Between 2010 and 2015, UK inflation remained persistently above target for almost the entire period. Through the lens of the model, this elevated inflation was largely

[12] Shocks are grouped as follows: UK demand – risk premium shock, consumption preference shock, import demand shock, investment shock, other investment shock, government spending shock; UK costs – final output markup shock, temporary final output markup shock, export price markup shock, value-added price markup shock, import price markup shock, wage markup shock; global demand – world consumption preference shock, world monetary policy shock, world demand shock, world preference shock; global costs – world price markup shock, world export price shock; supply/productivity – TFP shock, labour-augmenting productivity shock, hours growth shock, relative productivity shock, labour supply shock; energy – energy price shock; monetary conditions – monetary policy shock, UIP shock.

Figure 8: HISTORICAL SHOCK DECOMPOSITION OF MODEL VARIABLES



Notes: This figure shows the historical shock decompositions of year-on-year CPI inflation and real GDP growth, as model observables, in percentage point deviations from their steady states. The bars represent the contributions of model shocks grouped into aggregate categories.

driven by cost-push factors (such as rising global commodity prices, and a VAT increase in 2011) and increases in domestic energy prices. From 2014 onwards, falling oil prices and a stronger currency (captured by 'energy' and 'monetary conditions' contributions) helped to push down on inflation, with the UK briefly experiencing deflation in 2015.

As the economy reopened after the Covid-19 pandemic in 2021, inflationary pressures emerged mainly from global supply-chain disruptions and labour shortages. These were further intensified by Russia's invasion of Ukraine in 2022, which drove up energy prices and other costs, pushing inflation to its peak, just above 10%. In 2024, inflation had gradually returned to near-target levels due to negative contributions from energy, although domestic demand pushed up on inflation.

GDP Growth. The historical decomposition for year-on-year GDP growth shows that, through the lens of our model, the main driving forces behind GDP growth departing from trend are domestic and global demand factors, accompanied by contributions from supply, productivity and monetary conditions. The severe contraction of the UK economy during the GFC was driven by several factors: falling global demand, weak domestic demand (especially investment), and weak supply. The decomposition shows that, at that time, monetary policy expansionary, and the only opposing factor pushing economic activity up substantively.

In the aftermath of the GFC, the UK economy entered a period of sluggish and variable growth. Growth began to strengthen more consistently from 2013 onwards, driven mainly by consumer spending, other investment, and a more stable global environment, although productivity growth remained weak, especially in 2013 and 2014.

Our model cannot decompose the unprecedented drop in economic activity due to the Covid-19 pandemic in 2020 and the 2021 rebound as lockdowns eased and stimulus measures took effect, as we applied 'Kalman-smoothing' to handle this unprecedented economic event.[13] However, in the period that follows (2022-2023), our model can decompose the slowdown in economic growth due to high inflation and energy-price shocks from the war in Ukraine. Since 2022, we can see the negative impacts of weak productivity on UK GDP growth and strong contributions from domestic demand since the second half of 2023.

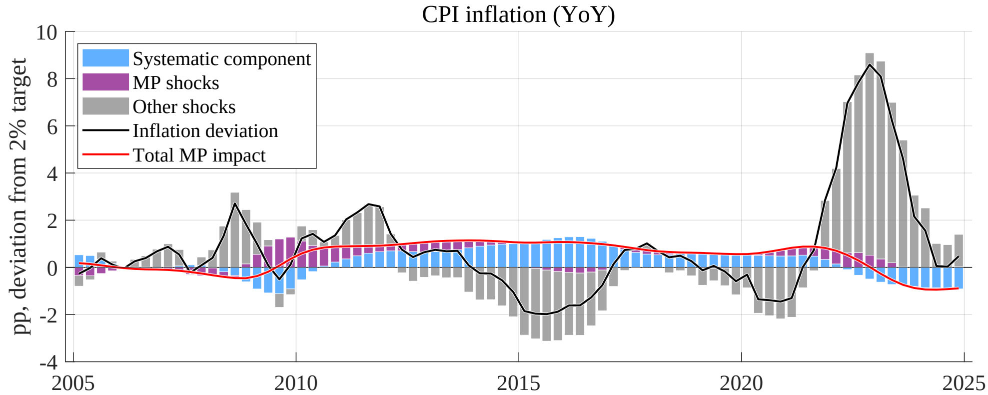
Role of Monetary Policy. Our model can also be used to further examine the influence of monetary policy on CPI inflation. To do this, we decompose the effects of 'systematic' and 'non-systematic' monetary policy, replicating an experiment carried out for the euro area by the ECB in Ciccarelli et al. (2024).

There are multiple ways to operationalise this. We do so by using the Taylor-type rule that governs the evolution of Bank Rate, as detailed in Section 2. The 'systematic' component accounts for interest-rate movements that are consistent with the rule, i.e., fundamental factors (like deviations of inflation from the target). The 'non-systematic' component captures deviations from the rule, i.e., model-implied monetary-policy shocks.[14] With this, we disentangle past inflation deviations from the 2% target into non-systematic fluctuations explained by monetary policy shocks, other shocks

[13]As this period would require expert judgements, we resorted to using the Kalman-filter to input GDP observables while considering other observed variables in the identification. See Section 3 for more details.

[14]Monetary-policy shocks can be interpreted as the wedge between the observed policy rate and a notional rule-implied policy rate. Positive (negative) monetary policy shocks in the monetary policy decomposition indicate a more contractionary (expansionary) monetary policy setting compared to what macroeconomic fundamentals would prescribe. This implicitly assumes inflation expectations are anchored around the policy rule, and does not take into account the role of monetary policy signalling when large shocks hit the economy.

Figure 9: HISTORICAL SHOCK DECOMPOSITION AND THE ROLE OF MONETARY POLICY



Notes: This figure shows the historical shock decomposition of year-on-year CPI inflation as a model observable, in percentage point deviations from its steady states. The decomposition shows contributions of (1) a 'systematic' component of the model Taylor-type rule, accounting for interest rate movements driven by fundamental factors, such as deviations of inflation from the target, (2) a 'non-systematic' component, consisting of monetary policy shocks, and (3) other shocks.

that drive CPI inflation and the systematic response of monetary policy to those shocks, which we assume is sufficiently captured by the Taylor rule. We present this decomposition in Figure 9.

From a historical perspective, the model indicates that the total impact of monetary policy has predominantly been driven by its systematic component, with two notable instances where monetary policy shocks played a more pronounced role. According to the model, monetary policy was looser than the policy rule would prescribe following the GFC, contributing to inflation rates above the 2% target. Persistent contributions from the systematic component between 2010 and 2022 reflect the Bank of England's efforts to stimulate the UK economy during and after the GFC, to return inflation to the target, and to further stimulate the economy during the Covid-19 pandemic through unconventional monetary policy measures.[15] Since late-2022, the systematic component of monetary policy has shown a negative contribution, suggesting that the lagged effects of Bank-Rate hikes since late-2021 have helped to mitigate inflationary pressures and bring inflation back to the target.

[15]Unconventional monetary policy measures, such as quantitative easing and tightening, are represented by shadow rates in the model.

5 Model Forecast Performance

As outlined in the introduction, the model can provide an organising framework for the construction of macroeconomic forecasts and can serve as a cross-check for the MPC's central projections. With that in mind, in this section, we demonstrate the model's forecasting ability.

First, we present recursive conditional model forecasts and compare them with actual data outturns. Second, we evaluate the accuracy of the model's forecasts for CPI inflation and GDP. Third, to help understand some important forecast errors ex-post, we construct a counterfactual simulation that starts prior to the sharp increase in energy prices in 2022. This exercise demonstrates that with perfect foresight of the outturns for key forecast conditioning paths like energy prices and Bank Rate, the model would have produced an inflation forecast closely resembling the observed profile of inflation since 2022. In so doing, it highlights the challenges associated with predicting the rise in inflation in 2022 within a conditional forecast based on information available in 2021.

5.1 Forecasts Relative to Data Outturns

Figure 10 showcases the estimated model's 13-quarter-ahead conditional forecasts (light blue lines) for year-on-year (%) CPI inflation and real GDP growth from 2014 to 2024 relative to data outturns (black lines) over the same period.[16] The forecasts are based on data vintages available at the projection start date (i.e., in real time), and are conditional on multiple domestic and global variables over the forecast horizon.[17]

The domestic and world conditioning paths over the whole forecast horizon match those used for the MPC's UK forecast and include the overnight-indexed swap (OIS) forward curve for Bank Rate, a path for the nominal effective exchange rate, the outlook for energy prices and fiscal policy, world interest rates, world GDP, world trade, world export price deflator, and world CPI outlook. Additionally, we constrain the short-term outlook of the forecasts based on an additional set of UK variables, following usual practice, by including nowcasts and nearcasts based on high-frequency forecasting techniques (Moreira, 2025).[18]

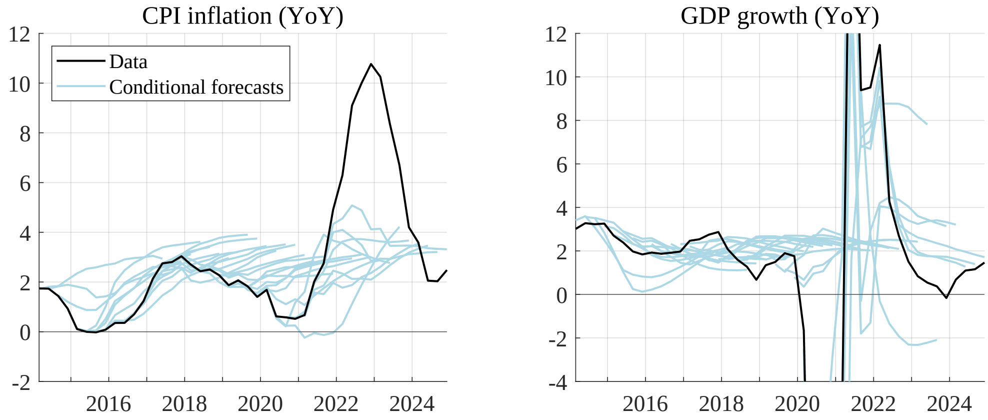
Figure 10 demonstrates that the model performs reasonably well at forecasting inflation and GDP growth during 'standard times'. Over the 2010s, the model's conditional

[16] The 13-quarter forecast horizon matches that of the MPC's forecast.

[17] We focus on direct comparisons between data outturns and conditional forecasts. Kanngiesser and Willems (2024) offer a more detailed set of metrics and techniques for evaluation of conditional forecasts.

[18] We constrain real GDP and its expenditure components, hours worked, average aggregate wage, and export and import price deflators for the first quarter. Furthermore, we constrain the CPI outlook for two-quarters ahead.

Figure 10: OUTTURNS VERSUS MODEL CONDITIONAL FORECASTS



Notes: This figure shows the model's 13 quarters ahead conditional forecasts (light blue lines) for year-on-year (%) inflation and real GDP growth from 2014Q1 to 2024Q4 relative to data outturns (black lines) over this period. The forecasts are based on data vintages available as of the projection start date, and conditional on multiple domestic and global variables over the whole forecast horizon.

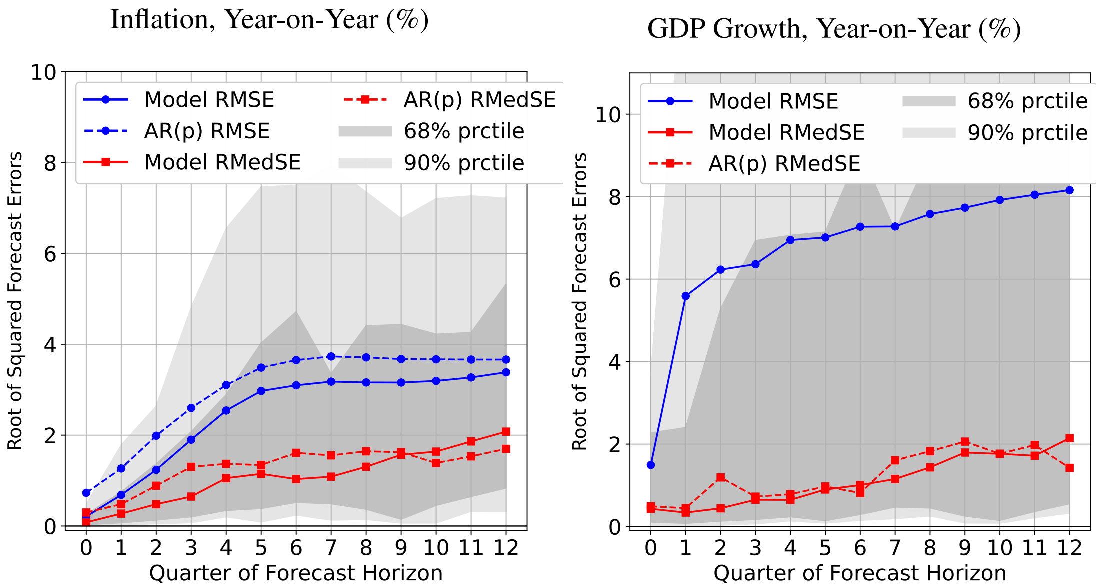
forecasts deviate relatively little from the data outturns and, although some of the conditional forecasts for CPI inflation lie above the data outturns from 2017 to 2019, this is in large part due to the conditioning paths for Bank Rate remaining low relative to actual values over this period. However, the estimated model struggles most to predict abrupt changes in the economic environment. For example, real-time conditional forecasts under-predict the inflation pick-up in 2022 ahead of the significant observed spike in energy prices. It is also unable to capture the steep drop in GDP growth during the Covid-19 pandemic, which is unsurprising given the unusual nature of the period.

To formalise these observations, we present forecast-accuracy metrics. Figure 11 plots the root-mean squared error (RMSE, blue lines) with corresponding 68% and 90% confidence intervals (grey swathes) and the root-median squared error (RMedSE, red lines) from real-time forecasts that we attain from the estimated DSGE models, as well as an AR(p) model, at forecast horizons $h = 0, \ldots, 13$. We present these statistics for two headline forecast variables: year-on-year CPI inflation and real GDP growth.[19] All metrics are calculated over the period 2014Q1-2024Q4.

The results show that the estimated DSGE model is relatively accurate at forecasting inflation and GDP growth over a 3-year horizon, outperforming the AR(p) model at all horizons on a RMSE basis and most horizons when comparing RMedSE. Both RMSE and RMedSE generally increase over the forecast horizon, but level-off after about six

[19] In the right-hand figure, we omit the AR(p) RMSE results for GDP growth as these are large, due to volatility in UK GDP data during the Covid-19 period, and go beyond the y-axis scale.

Figure 11: CONDITIONAL FORECAST ACCURACY



Notes: This figure contains the root-mean squared error (RMSE, blue lines) with corresponding 68% and 90% confidence intervals (grey swathes), and the root median square error (RMedSE, red lines) at forecast horizons $h = 0, \ldots, 13$ for year-on-year (%) inflation and GDP growth from the estimated DSGE model and an AR(p) model. RMSE from AR(p) omitted from right-hand figure as it goes beyond the y-axis scale.

quarters. The latter metric is less sensitive to outliers, and lower than RMSE throughout the 3-year evaluation period. This suggests that the model is less accurate at predicting large outliers in outturns, in line with our discussion of the 2022-23 energy-price spike, albeit more so than the AR(p) benchmark.

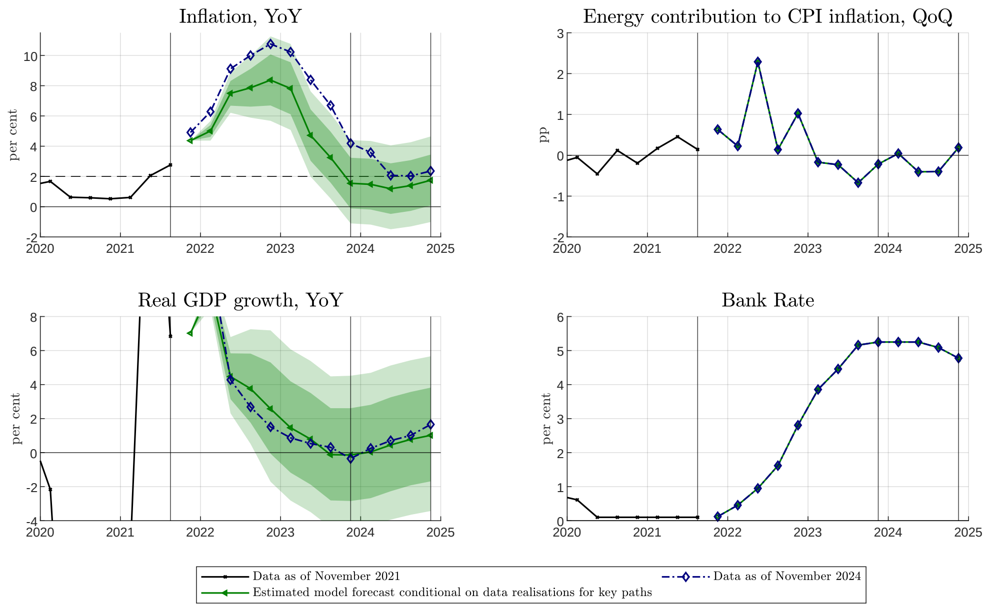
5.2 Counterfactual Model Forecasts

To further demonstrate the model's forecasting ability, as well as some of the challenges associated with real-time conditional forecasting, we produce counterfactual forecasts for CPI inflation and GDP growth based on data available as of 2021, but conditional on realised values of key forecast conditioning paths as of November 2024. This exercise aims to answer the question: '*Had we known ex-ante how key conditioning paths actually turned out, what would the model have predicted for CPI inflation and GDP growth ahead of the energy crisis in 2022?*'[20] To do this, we use realised values for Bank Rate, the energy contribution to inflation, the exchange rate path, government spending growth and global variables (inflation, export prices, GDP, trade and interest

[20] See Ciccarelli et al. (2024) and Lane (2024) for a similar analysis of euro-area forecasts.

rates). We refrain from imposing any constraints on the forecasts using the quarterly nowcasts and nearcasts.

Figure 12: COUNTERFACTUAL MODEL FORECASTS



Notes: This figure shows the counterfactual projection for year-on-year inflation and GDP growth based on data available as of 2021, but conditional on realised values of key paths as of November 2024. Black lines show the data as of November 2021, blue lines represent the data as of November 2024, and green lines are the model's counterfactual forecasts with 68% and 90% confidence intervals (green swathes)

Figure 12 presents the results of this counterfactual exercise. Black lines show the data as of November 2021, blue lines represent the data as of November 2024, and green lines are the model's conditional forecasts with 68% and 90% confidence intervals (green swathes) as discussed above. Conditional on the realised values of key paths, the model would have produced a CPI inflation forecast that closely resembles the observed profile. Forecasted inflation would have peaked around 8% in 2022Q4, before falling back to target quickly. The discrepancy between the conditional forecast for inflation and the actual realisation could be interpreted as a manifestation of second-round effects and non-linearities, which are not well captured in our linearised model. However, the outturns for inflation are close to the upper bound of the 90% confidence interval, reflecting the parameter uncertainty inherent in the Bayesian estimation approach. The forecasts for real GDP growth would also have largely been in line with the observed outturns.

6 Scenario Analysis

As noted by Lombardelli (2025), scenarios and various policy simulations are a useful tool to help policymakers understand risks and uncertainties around their central projection, and infer policy implications. Due to their flexibility, structural models, such as our estimated DSGE model, can be used to generate these simulations. In this section, we describe and provide examples for two classes of scenarios that can be constructed using the model: those that rely on alternative conditioning paths or shocks, and those that alter the structure of the economy. We also describe how the model was used in practice to construct scenarios for the Bank's *Monetary Policy Report* in May 2025. These scenarios included a combination of alternative shocks, capturing different economic events and structural changes, which led to varying macroeconomic outcomes.

6.1 Types of Scenarios

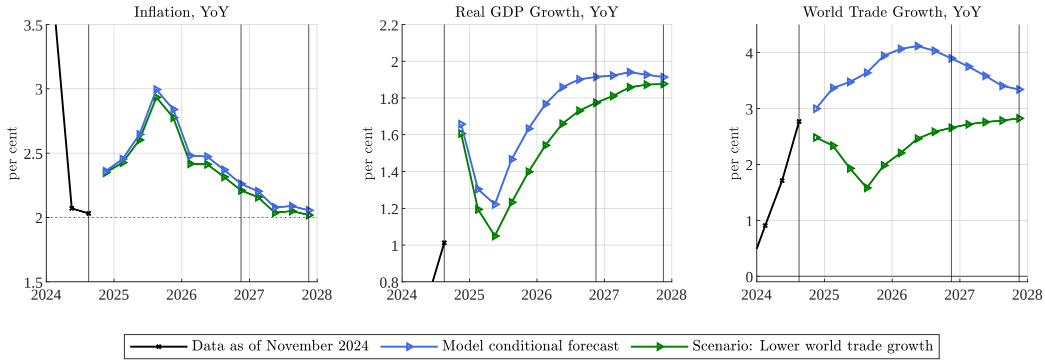
We first describe the *types* of scenarios that one can construct using our estimated DSGE model—subject, of course, to the caveat that they are simplifications of a complex reality and can abstract from realistic channels of transmission.

Alternate Conditioning Paths/Shocks Scenarios. These are scenarios that can be based on different sets of underlying conditioning paths or constraints (such as the world outlook), or can reflect specific economic events, represented by a single shock or a combination of shocks.

Figure 13 presents a hypothetical example of this type of scenario, from the perspective of a forecaster in November 2024, where a world trade shock is assumed to reduce world-trade growth by approximately 1pp throughout 2025. The black lines represent data available as of November 2024, the blue lines show the model's conditional forecasts from that period, and the green lines illustrate the alternative projections under the scenario. Within the model, lower world-trade growth reduces external demand for UK exports. At the same time, it dampens firms' willingness to invest and weakens household consumption. Together, these effects drag on UK GDP growth. Although a negative trade shock leads to a depreciation of sterling, the overall impact on inflation is negative, as weaker demand reduces inflationary pressures.

Structural Scenarios. In these scenarios, key structural features of the economy may differ. Such scenarios can be built by using alternative assumptions about economic processes, parameters, or even the model's structure. This allows for, among other things,

Figure 13: ALTERNATE SHOCKS AND CONDITIONING PATHS SCENARIO - LOWER WORLD TRADE GROWTH

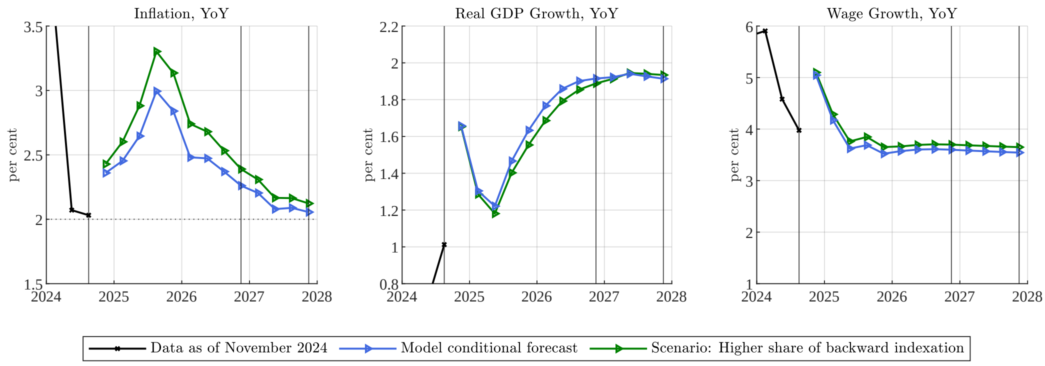


Notes: This figure contains an example of an alternate shocks scenario where a world trade shock is assumed to lead to a decline in world trade growth by around 1 percentage point throughout 2025. The black lines represent data available as of November 2024, the blue lines show the model's conditional forecasts from that period, and the green lines illustrate the alternative projections under the scenario.

an analysis of how the economy could evolve if it were subjected to the same constellation of shocks as in the central conditional projection, but under different structural conditions.

Figure 14 provides an example of this type of scenario, again from the perspective of a forecaster in November 2024, in which we assume that price- and wage-setting behaviour is structurally different. Specifically, the degree of backward indexation for prices and wages is doubled and tripled, respectively, compared to the baseline model setting. This implies that a larger share of firms and households in the model become backward-looking, indexing their prices and wages to previously observed values. The black lines represent data as of November 2024, the blue lines show the model's conditional forecasts from that period, and the green lines illustrate the alternative projections under the scenario. The structural change in price- and wage-setting behaviour leads to higher inflation and wage growth compared to the model's conditional forecast. Since the impact on prices is more pronounced than on wages, real wages decline. As labour becomes less rewarding, households reduce their hours worked. Consequently, output grows more slowly due to both reduced labour input and higher prices. However, the real effects remain relatively modest in scale.

Figure 14: STRUCTURAL SCENARIO - HIGHER BACKWARDS INDEXATION IN PRICE AND WAGE SETTING



Notes: This figure provides an example of a structural scenario, in which we assume that price- and wage-setting behaviour is structurally different such that the degree of backward indexation for price setting doubles (from 0.1 to 0.2), and that it triples for wage setting (from 0.075 to 0.225). The black lines represent data as of November 2024, the blue lines show the model's conditional forecasts from that period, and the green lines illustrate the alternative projections under the scenario.

6.2 Scenarios in the May 2025 Monetary Policy Report

We now describe the details of two specific scenarios described by the MPC in their May 2025 *Monetary Policy Report*: a weaker-demand scenario and a higher-persistence scenario.

Weaker-Demand Scenario. The weaker-demand scenario explored the risk that a heightened degree of uncertainty in the UK, driven by both domestic and global developments, might have dragged on UK activity more than already incorporated into the baseline forecast. The weakness in demand could have added to previous pressures on firms, precipitating a rapid loosening in the labour market, which contributed to a standard business-cycle feedback loop of higher saving and lower income, consumption, and investment. The much larger and faster opening of slack in this scenario could also have triggered a more rapid unwind of excess inflation persistence.

In stage one of this scenario, Bank staff delivered a weaker-demand outlook by imposing risk-premium shocks over the forecast horizon, in combination with investment-cost shocks. To calibrate these shocks, the staff utilised their work on uncertainty by updating a UK uncertainty index (see May 2025 *Monetary Policy Report*) and estimating its impact on UK GDP via an SVAR model.

In stage two of this scenario, staff assumed a quicker than usual unwind of inflation persistence relative to the baseline. The baseline forecast was consistent with excess inflation persistence fading gradually. Given the much more rapid opening of slack in the scenario, this should have acted to unwind these pressures more quickly. The faster unwind of inflation persistence was calibrated to match the second-round inflation impacts identified by the Bernanke and Blanchard (2025) model, while accounting for weaker domestic activity from stage one. Staff achieved this by increasing the frequency of price and wage setting for final output producers and wage setters (lowering the Calvo stickiness parameters ϕ_W and ϕ_Z) so that slack fed through to inflation more quickly – akin to steepening the wage and price Phillips curves.

Higher-Persistence Scenario. The higher persistence scenario explored the risk that households' and businesses' inflation expectations were more sensitive to recent price rises than normal. This could generate more persistence in price and wage setting, and so the impact of the renewed cost push shock in mid-2025 could have more lasting effects on inflation than in the baseline. A further step explored how this might have interacted with a more supply-constrained state of the economy.

In stage one of this scenario, staff assumed that firms gave greater weight to past inflation developments in their price setting compared to the baseline forecast. At the same time, staff assumed that wages were indexed to past wage growth to a greater degree than in the baseline projection. Both changes were implemented by adjusting parameters in the price and wage New Keynesian Phillips curves (for final output producers in the case of prices), ξ_W and ξ_Z, resulting in greater backward-looking behaviour in prices and wages, and therefore, higher inflation persistence. The exact change in the parameters was informed by internal empirical analysis and survey data.

In stage two, staff explored how this increased degree of persistence in price and wage setting processes might have interacted with a more supply-constrained state of the economy. Productivity had been weak for quite some time, and this had not been reflected in wages, which have been stronger than expected. In this stage of the scenario, staff assumed that this trend continued, and that potential productivity quarter-over-quarter growth was around 0.1pp weaker each quarter compared to the baseline projection by imposing negative labour augmenting productivity shocks. To capture continued wage strength despite weakness in potential productivity, staff imposed additional wage markup shocks to offset the effect stemming from weaker productivity.

7 Conclusion

In this paper, we have presented an updated and re-estimated medium-scale DSGE model for the UK economy. The model has its foundations in Burgess et al. (2013), including an open-economy structure and a two-agent framework. Relative to this, our model includes imported energy goods in production and consumption, time-varying trends, an expanded set of economic shocks and real adjustment costs.

As we have explained, although the model is just one of many inputs into the monetary-policy process, it serves multiple purposes, including (but not limited to) providing a tool for interrogating historical data, offering a framework for constructing forecasts, and a means by which to assess the sensitivity of forecasts to alternative assumptions. Nevertheless, as the economic landscape changes and new data arrives, we will need to continually update and develop the model in the future. In this vein, Bank staff are currently working on extensions to the model to account for a wider array of fiscal tools, a non-linear version to better capture extreme deviations from steady state, and a refined estimation of bounded-rationality parameters. Alongside this, further work could include the development of a richer labour market setup within the model, drawing on (for example) search-and-matching-style mechanisms to incorporate flows between employment and unemployment in the model.

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Appendix

Contents

A Derivation of Model Equations

A.1 Households

There is a continuum of identical households defined on the unit interval $i \in [0,1]$. We assume that a share, ω^o, of households are 'optimising' or 'unconstrained' or 'Ricardian'. Those households have access to financial markets and are able to save and borrow in domestic and foreign bonds. The remaining share, $1-\omega^o$, are 'rule of thumb' or 'constrained'. Those households have no access to financial markets, and therefore consume all their labour income in each period. We also assume that they supply any labour demanded given the wage. All individual households (regardless of type) are denoted with subscript i. Individual optimising households are denoted with superscript o so that the (detrended) consumption of an optimising household is referred to as $C^o_{i,t}$, while individual 'rule of thumb' households are denoted with superscript rot so that consumption of these households is given by $C^{rot}_{i,t}$.

The size of each household, $\widetilde{\chi}^H_t$, is assumed to grow at a constant rate $\Gamma^H = \widetilde{\chi}^H_t / \widetilde{\chi}^H_{t-1}$. Given that households are defined as a continuum on the unit interval, this means that the total population is also given by $\widetilde{\chi}^H_t$.

Members of optimising households consume, hold money, save, invest, work and pay taxes. Each household derives utility from the sum of the utilities of the individual household members. Since the members of individual households are identical and the size of each household is equal to the total population, $\widetilde{\chi}^H_t$, we can express the utility function for the representative optimising household in per capita terms. In any arbitrary period $t=s$ an optimising household i maximises their lifetime utility $\mathscr{U}_{is}$

$$\mathscr{U}_{is} = \mathbf{E}_s \left[\sum_{t=s}^{\infty} \Theta_t \widetilde{\chi}^H_t \left\{ U_{i,t} \left(\widetilde{C}^o_{i,t}, L^o_{i,t}, \frac{\widetilde{Mon}^o_{i,t}}{P^Z_t}; \widetilde{C}^o_{t-1}, \varepsilon^L_t, \varepsilon^C_t \right) \right\} \right]$$

where $\mathbf{E}_s[\cdot]$ is the conditional expectations operator of the household, Θ_t is a discount factor (defined below), $U_{i,t}(\cdot)$ is the period utility function (defined below), $\widetilde{C}^o_{t-1}$ is lagged *aggregate* per capita consumption of optimising households, $L^o_{i,t}$ is the optimising household's labour supply, $\widetilde{Mon}^o_{i,t}/P^Z_t$ denotes real money holdings and P^Z_t is the final output price level. The period utility function is

$$U_{i,t} = \left[\frac{\left(\frac{\widetilde{C}^o_{i,t}}{\widetilde{\chi}^Z_t} - \psi_C \frac{\widetilde{C}^o_{t-1}}{\widetilde{\chi}^Z_{t-1}} \right)^{1-\varepsilon_C} - 1}{1-\varepsilon_C} - \frac{\nu_L \varepsilon^L_t \left(L^o_{i,t} \right)^{1+\varepsilon_L}}{1+\varepsilon_L} + \frac{\nu_M \left(\frac{\widetilde{Mon}^o_{i,t}}{\widetilde{\chi}^Z_t P^Z_t} \right)^{1-\varepsilon_C} - 1}{1-\varepsilon_C} \right] \varepsilon^C_t$$

where the marginal utility of consumption and real money balances are defined relative to the trend in overall productivity growth, $\widetilde{\chi}^Z_t$ (defined below), ε_C is the inverse of the intertemporal elasticity of substitution, ψ_C is the parameter governing external habit formation, ε_L is the elasticity of labour supply, ν_L is the relative weight on the disutility of working and ν_M is the relative weight on real money balances. ε^L_t is a disturbance that raises the disutility of supplying labour and ε_t^C is a consumption preference shock. The shocks follow standard AR(1) processes

$$\log \varepsilon_t^j = \rho_j \log \varepsilon_{t-1}^j + \left(1-\rho_j^2\right)^{1/2} \sigma_j \eta_t^j, \;\; \eta_t^j \sim N(0,1), \quad j \in \{L, C\}.$$

Utility is maximised with respect to the per capita budget constraint

$$\begin{aligned} & \widetilde{W}_t^H L_{i,t}^o + \frac{\widetilde{Mon}_{i,t-1}^o}{\Gamma^H} + \widetilde{R}_t^K \widetilde{K}_{i,t-1}^o + \frac{R_{t-1}\widetilde{B}_{i,t-1}^o}{\Gamma^H} + \frac{R_{t-1}^F \widetilde{B}_{i,t-1}^{F,o}}{\mathscr{E}_t \Gamma^H} + \widetilde{D}_{i,t}^o \\ & = P_t^{CPI} \widetilde{C}_{i,t}^o + \widetilde{Mon}_{i,t}^o + \frac{\widetilde{B}_{i,t}^o}{\varepsilon_t^B} + \frac{\widetilde{B}_{i,t}^{F,o}}{\varepsilon_t^B \varepsilon_t^{B^F} \mathscr{E}_t} + P_t^I \widetilde{I}_{i,t}^o + P_t^{I^O} \widetilde{I}_{i,t}^{O,o} + \widetilde{T}_{i,t}^{D,o} + \widetilde{T}_{i,t}^{G,o} + P_t^Z \widetilde{\chi}_t^Z \mathscr{T}_i^o \end{aligned}$$

where $\widetilde{W}_t^H$ denotes the nominal wage received by households, $\widetilde{K}_{t-1}^o$ is physical capital inherited from the previous period, $\widetilde{B}_{i,t}^o$ and $\widetilde{B}_{i,t}^{F,o}$ denote domestic and foreign nominal risk-less bonds, which provide a nominal gross returns of R_t and R_t^F to the household, $\mathscr{E}_t$ denotes the nominal exchange rate (foreign currency relative to domestic currency), $\widetilde{D}_{i,t}^o$ are the nominal profits/dividends made by monopolistic firms that are re-distributed lump-sum to optimising households. Since there are five monopolistically-competitive domestic entities[21] in this model, total profits of firm ownership will be

$$\widetilde{D}_{i,t}^o \;\; = \;\; \widetilde{D}_{i,t}^{U,o} + \widetilde{D}_{i,t}^{Z,o} + \widetilde{D}_{i,t}^{V,o} + \widetilde{D}_{i,t}^{M,o} + \widetilde{D}_{i,t}^{X,o}.$$

$P_t^I \widetilde{I}_{i,t}^o$ and $P_t^I \widetilde{I}_{i,t}^{O,o}$ represent nominal investment (described below) and $\widetilde{T}_{i,t}^{G,o}$ and $\mathscr{T}_i^o$ are nominal lump-sum taxes paid to the government and transfers distributed among household to equalise steady state consumption. ε_t^B and $\varepsilon_t^{B^F}$ denote a domestic and foreign risk-premium shock

$$\log \varepsilon_t^j = \rho_j \log \varepsilon_{t-1}^j + \left(1-\rho_j^2\right)^{1/2} \sigma_j \eta_t^j, \;\; \eta_t^j \sim N(0,1), \quad j \in \{B, B^F\}.$$

Capital Accumulation and Investment. Optimising households own the capital stock and rent it out to firms at the rental rate R_t^K. Capital accumulates according to the following law of motion[22]

$$\Gamma^H \widetilde{K}_{i,t}^o = \left(1-\delta^K\right) \widetilde{K}_{i,t-1}^o + \Psi_I\left(\zeta_{i,t}^{I,o}, \varepsilon_t^I\right) \widetilde{I}_{i,t}^o$$

where δ_K is the depreciation rate of capital and $\Psi_I(\cdot)$ is a function that determines the cost of adjusting investment away from it's long-run growth rate, specified as

$$\Psi_I\left(\zeta_{i,t}^{I,o}, \varepsilon_t^I\right) = \left(1 - \psi_I \left(\zeta_{i,t}^{I,o} - \Gamma^H \Gamma^Z \Gamma^I\right)^2 / 2\right) \varepsilon_t^I, \qquad \zeta_{i,t}^{I,o} \equiv \Gamma^H \widetilde{I}_{i,t}^o / \widetilde{I}_{i,t-1}^o$$

[21]The five entities are unions (U), final output good firms (Z), value-added good firms (V), importers (M) and exporters (X).

[22]The capital accumulation identity is expressed in terms of the per capita capital stock, so Γ^H enters this equation to account for population growth between periods t and $t+1$.

where ψ_I is a parameter that governs the size of the adjustment costs, $\Gamma^H \Gamma^Z \Gamma^I$ denotes investment growth along the balanced growth path (discussed below). Optimising households also finance 'other investment', $\widetilde{I}^{O,o}_{i,t}$, at the price $P^{I^O}_t$. This variable is included in the model so that the aggregate resource constraint comes as close as possible to matching the national accounting identity for GDP into expenditure components. The mapping of the model to the data implies that this variable captures expenditure components of GDP not explicitly included in the model, like housing investment and stockbuilding.[23] For simplicity, growth in other investment is assumed to exhibit 'error correction' to its long-run trend

$$\widetilde{I}^{O,o}_{i,t} / \widetilde{I}^{O,o}_{i,t-1} = \left(\widetilde{I}^{O,o}_{i,t-1} / \widetilde{\chi}^Z_{t-1} \right)^{\rho_{I^O} - 1} \varepsilon^{I^O}_t ,$$

ε^I_t is a shock that affects the rate at which investment is converted into capital and $\varepsilon^{I^O}_t$ denotes a disturbance to other investment where

$$\log \varepsilon^j_t = \rho_j \log \varepsilon^j_{t-1} + \left(1 - \rho_j^2\right)^{1/2} \sigma_j \eta^j_t , \;\; \eta^j_t \sim N(0,1) , \quad j \in \{I, I^O\}.$$

A.1.1 Household Optimality Conditions

Lagrangian. Each optimising household, i, solves the following Lagrangian in any arbitrary period t

$$\begin{aligned} \mathscr{L}^o_{i,t} \quad &= \sum_{\mathscr{S}^t} \pi_{\mathscr{S}^t} \sum_{t=1}^{\infty} \Theta_t \widetilde{\chi}^H_t \Bigg\{ U_{i,t}(.) + \widetilde{\Lambda}^{C,o}_{i,t} \Bigg[\widetilde{W}^H_t L^o_{i,t} + \frac{\widetilde{Mon}^o_{i,t-1}}{\Gamma^H} + \widetilde{R}^K_t \widetilde{K}^o_{i,t-1} + \frac{R_{t-1} \widetilde{B}^o_{i,t-1}}{\Gamma^H} + \frac{R^F_{t-1} \widetilde{B}^{F,o}_{i,t-1}}{\mathscr{E}_t \Gamma^H} + \widetilde{D}^o_{i,t} - P^{CPI}_t \widetilde{C}^o_{i,t} \\ &- \widetilde{Mon}^o_{i,t} - \frac{\widetilde{B}^o_{i,t}}{\varepsilon^B_t} - \frac{\widetilde{B}^{F,o}_{i,t}}{\varepsilon^B_t \varepsilon^{B^F}_t \mathscr{E}_t} - P^I_t \widetilde{I}^o_{i,t} - P^{I^O}_t \widetilde{I}^{O,o}_{i,t} - \widetilde{T}^o_{i,t} - P^Z_t \widetilde{\chi}^Z_t \mathscr{T}^o_i \Bigg] - \widetilde{\Lambda}^{K,o}_{i,t} \left[\widetilde{K}^o_{i,t} - \left(1 - \delta^K\right) \frac{\widetilde{K}^o_{i,t-1}}{\Gamma^H} - \Psi_I \left(\zeta^{I,o}_{i,t}, \varepsilon^I_t \right) \widetilde{I}^o_{i,t} \right] \Bigg\} \end{aligned}$$

where $\widetilde{\Lambda}^{C,o}_{i,t}$ and $\widetilde{\Lambda}^{K,o}_{i,t}$ are the Lagrange multipliers associated with the optimising households' resource constraint and capital accumulation equation respectively. The term, Θ_t, is an endogenous discount factor

$$\Theta_t = \Theta_{t-1} \beta \exp \left\{ \varepsilon_\beta \left(\frac{C^o_{t-1}}{C^o_{ss}} - 1 \right) \right\} , \qquad \mathbf{B}_t \equiv \frac{\Theta_t}{\Theta_{t-1}} = \beta \exp \left\{ \varepsilon_\beta \left(\frac{C^o_{t-1}}{C^o_{ss}} - 1 \right) \right\}$$

where $\mathbf{B}_t$ is a function of the ratio of *aggregate* per capita consumption of optimising households, relative to its steady state level. We assume that the function is specified so that $\mathbf{B}(1) = \beta$ and has elasticity ε_β with respect to its argument when evaluated at steady state. Since Θ_t depends on *aggregate* per capita consumption of optimising households, each individual household treats it parametrically. The 'endogenous discount factor' is included in the model to ensure that the model returns to a unique steady-state net foreign asset position following temporary shocks.[24]

[23] The model explicitly includes consumption, business investment, total government spending, exports and imports. The components of GDP (measured at market prices) not explicitly modelled are: dwellings investment, so-called 'other investment' (which includes stamp duty and so is correlated with dwellings investment), stockbuilding and the alignment adjustment.

[24] There are a range of technical assumptions of this type that can be made to deliver this results. Schmitt-Grohé and Uribe (2003b) argue that these approaches deliver similar quantitative properties if

Optimal Choice of $\widetilde{C}^o_{i,t}$. The first-order condition for consumption is

$$\widetilde{\Lambda}^{C,o}_{i,t} = \widetilde{U}^{C,o}_{i,t}/P^{CPI}_t, \qquad \widetilde{U}^{C,o}_{i,t} = \left(\widetilde{C}^o_{i,t}/\widetilde{\chi}^Z_t - \psi_C \widetilde{C}^o_{t-1}/\widetilde{\chi}^Z_{t-1}\right)^{-\varepsilon_C} \varepsilon^C_t/\widetilde{\chi}^Z_t.$$

MRS of Optimising Households. The marginal rate of substitution (MRS) of optimisers is

$$\widetilde{MRS}^o_{i,t} \equiv -U^L_{i,t}/\widetilde{\Lambda}^{C,o}_{i,t} = -U^L_{i,t}/(\widetilde{U}^{C,o}_{i,t}/P^{CPI}_t), \qquad U^L_{i,t} = -\nu_L \varepsilon^C_t \varepsilon^L_t \left(L^o_{i,t}\right)^{\varepsilon_L}.$$

Optimal Choice of $\widetilde{Mon}^o_{i,t}$. Taking the derivative with respect to $\widetilde{Mon}^o_{i,t}$ gives the intertemporal optimality condition associated with money holdings

$$\widetilde{U}^{M,o}_{i,t} = \widetilde{\Lambda}^{C,o}_{i,t} - \mathbf{E}_t\left[\frac{\Theta_{t+1}}{\Theta_t}\left\{\widetilde{\Lambda}^{C,o}_{i,t+1}\right\}\right], \ \frac{\widetilde{U}^{M,o}_{i,t}}{\widetilde{\Lambda}^{C,o}_{i,t}} = 1 - \mathbf{E}_t\left[\frac{\Theta_{t+1}}{\Theta_t}\left\{\frac{\widetilde{\Lambda}^{C,o}_{i,t+1}}{\widetilde{\Lambda}^{C,o}_{i,t}}\right\}\right], \ \widetilde{U}^{M,o}_{i,t} = \nu_M\left(\frac{\widetilde{Mon}^o_{i,t}}{\widetilde{\chi}^Z_t P^Z_t}\right)^{-\varepsilon_C}\frac{\varepsilon^C_t}{\widetilde{\chi}^Z_t P^Z_t}.$$

Optimal Choice of $\widetilde{B}^o_{i,t}$ and $\widetilde{B}^{F,o}_{i,t}$. The first-order conditions for domestic and foreign bonds are

$$\begin{aligned}
\widetilde{\Lambda}^{C,o}_{i,t} &= \mathbf{E}_t\left[\frac{\widetilde{\chi}^H_{t+1}}{\widetilde{\chi}^H_t}\frac{\Theta_{t+1}}{\Theta_t}\widetilde{\Lambda}^{C,o}_{i,t+1}\frac{R_t}{\Gamma^H}\varepsilon^B_t\right], \quad \widetilde{\Lambda}^{C,o}_{i,t} = \mathbf{E}_t\left[\frac{\Theta_{t+1}}{\Theta_t}\widetilde{\Lambda}^{C,o}_{i,t+1}R_t\varepsilon^B_t\right], \\
\widetilde{\Lambda}^{C,o}_{i,t} &= \mathbf{E}_t\left[\frac{\Theta_{t+1}}{\Theta_t}\widetilde{\Lambda}^{C,o}_{i,t+1}R^F_t\varepsilon^B_t\varepsilon^{B^F}_t\frac{\mathscr{E}_t}{\mathscr{E}_{t+1}}\right],
\end{aligned}$$

so that no-arbitrage implies the uncovered interest parity (UIP) condition

$$\mathbf{E}_t\left[\frac{\Theta_{t+1}}{\Theta_t}\frac{\widetilde{\Lambda}^{C,o}_{i,t+1}}{\widetilde{\Lambda}^{C,o}_{i,t}}\left[R_t - R^F_t\varepsilon^{B^F}_t\frac{\mathscr{E}_t}{\mathscr{E}_{t+1}}\right]\right] = 0.$$

Optimal Choice of $\widetilde{I}^o_{i,t}$. The first-order condition for investment is

$$P^I_{i,t} = \frac{\widetilde{\Lambda}^{K,o}_{i,t}}{\widetilde{\Lambda}^{C,o}_{i,t}}\left[\Psi_I\left(\zeta^{I,o}_{i,t},\varepsilon^I_t\right) + \Psi'_I\left(\zeta^{I,o}_{i,t},\varepsilon^I_t\right)\widetilde{I}^o_{i,t}\right] + \mathbf{E}_t\left[\frac{\Theta_{t+1}}{\Theta_t}\frac{\widetilde{\chi}^H_{t+1}}{\widetilde{\chi}^H_t}\frac{\widetilde{\Lambda}^{C,o}_{i,t+1}}{\widetilde{\Lambda}^{C,o}_{i,t}}\frac{\widetilde{\Lambda}^{K,o}_{i,t+1}}{\widetilde{\Lambda}^{C,o}_{i,t+1}}\Psi'_I\left(\zeta^{I,o}_{i,t+1},\varepsilon^I_{t+1}\right)\widetilde{I}^o_{i,t+1}\right]$$

where

$$\begin{aligned}
\Psi'_I\left(\zeta^{I,o}_{i,t},\varepsilon^I_t\right) &= -\psi_I\left(\zeta^{I,o}_{i,t} - \Gamma^H\Gamma^Z\Gamma^I\right)\frac{\zeta^{I,o}_{i,t}\varepsilon^I_t}{\widetilde{I}^o_{i,t}} \\
\Psi'_I\left(\zeta^{I,o}_{i,t+1},\varepsilon^I_{t+1}\right) &= \psi_I\left(\zeta^{I,o}_{i,t+1} - \Gamma^H\Gamma^Z\Gamma^I\right)\frac{\zeta^{I,o}_{i,t+1}\varepsilon^I_{t+1}}{\widetilde{I}^o_{i,t}}.
\end{aligned}$$

suitably parametrised. We calibrate ε_β to be small, so that it does not play an important role in determining the quantitative properties of the model.

Optimal Choice of $\widetilde{K}^{o}_{i,t}$. The first-order condition for capital introduces Tobin's Q, $\widetilde{TQ}_{i,t}$

$$\widetilde{TQ}_{i,t} = \mathbf{E}_t \left[\frac{\Theta_{t+1}}{\Theta_t} \frac{\widetilde{\Lambda}^{C,o}_{i,t+1}}{\widetilde{\Lambda}^{C,o}_{i,t}} \left\{ \widetilde{R}^{K}_{t+1} + \widetilde{TQ}_{i,t+1} \left(1-\delta^{K}\right) \right\} \right], \qquad \widetilde{TQ}_{i,t} = \frac{\widetilde{\Lambda}^{K,o}_{i,t}}{\widetilde{\Lambda}^{C,o}_{i,t}}.$$

Rule-of-Thumb Households. Finally, rule-of-thumb or non-optimising households are assumed to consume their labour income plus union profits and a transfer that equalises consumption between the two types of households in steady state

$$P^{CPI}_t \widetilde{C}^{rot}_{i,t} = \widetilde{W}^{H}_t L^{rot}_{i,t} + \widetilde{D}^{U,rot}_{i,t} + P^{Z}_t \widetilde{\chi}^{Z}_t \mathscr{T}^{rot}_i .$$

MRS of Rule-of-Thumb Households. The MRS of rule-of-thumb households is

$$\widetilde{MRS}^{rot}_{i,t} \equiv -U^{L,rot}_{i,t} / \left(\frac{\widetilde{U}^{C,rot}_{i,t}}{P^{CPI}_t} \right), \; U^{L,rot}_{i,t} = -\nu_L \varepsilon^{C}_t \varepsilon^{L}_t \left(L^{rot}_{i,t}\right)^{\varepsilon_L}, \; \widetilde{U}^{C,rot}_{i,t} = \left(\frac{\widetilde{C}^{rot}_{i,t}}{\widetilde{\chi}^{Z}_t} - \psi_C \frac{\widetilde{C}^{rot}_{t-1}}{\widetilde{\chi}^{Z}_{t-1}} \right)^{-\varepsilon_C} \frac{\varepsilon^{C}_t}{\widetilde{\chi}^{Z}_t}.$$

A.1.2 Detrending

Detrending the Optimising Household Budget Constraint. We divide the optimiser's budget by the final-output price level P^{Z}_t and by the productivity trend $\widetilde{\chi}^{Z}_t$. Note that lower cases denote stationary relative prices and wages, i.e., $w^{H}_t \equiv \widetilde{W}^{H}_t / (\widetilde{\chi}^{Z}_t P^{Z}_t)$, $p^{CPI}_t = P^{CPI}_t / P^{Z}_t$, $p^{I}_t = P^{I}_t / P^{Z}_t$, $p^{I^O}_t = P^{I^O}_t / P^{Z}_t$, $r^{K}_t \equiv \widetilde{R}^{K}_t \widetilde{\chi}^{I}_t / P^{Z}_t$ and quantity variables without a tilde denote stationary real versions, i.e. $K_t = \widetilde{K}_t / (\widetilde{\chi}^{Z}_t \widetilde{\chi}^{I}_t)$, $B_t \equiv \widetilde{B}_t / (\widetilde{\chi}^{Z}_t P^{Z}_t)$, $B^{F}_t \equiv \widetilde{B}_t / (\widetilde{\chi}^{V^F}_t P^{V^F}_t)$, $Mon_t \equiv \widetilde{Mon}_t / (\widetilde{\chi}^{Z}_t P^{Z}_t)$, $C_t = \widetilde{C}_t / \widetilde{\chi}^{Z}_t$, $T_t \equiv \widetilde{T}_t / (\widetilde{\chi}^{Z}_t P^{Z}_t)$ so that

$$w^{H}_t L^{o}_{i,t} + \frac{r^{K}_t K^{o}_{i,t-1}}{\Gamma^{Z}_t \Gamma^{I}} + \frac{R_{t-1} B^{o}_{i,t-1} + M^{o}_{i,t-1}}{\Gamma^{H} \Gamma^{Z}_t \Pi^{Z}_t} + D^{o}_{i,t} - M^{o}_{i,t} + \frac{R^{F}_{t-1} \widetilde{B}^{F,o}_{i,t-1}}{\widetilde{\chi}^{Z}_t P^{Z}_t \mathscr{E}_t \Gamma^{H}} = p^{CPI}_t C^{o}_{i,t} + \frac{B^{o}_{i,t}}{\varepsilon^{B}_t} + p^{I}_t I^{o}_{i,t} + p^{I^O}_t I^{O,o}_{i,t} + T^{o}_{i,t} + \mathscr{T}^{o}_i + \frac{\widetilde{\chi}^{V^F}_t P^{V^F}_t B^{F,o}_{i,t}}{\widetilde{\chi}^{Z}_t P^{Z}_t \varepsilon^{B}_t \varepsilon^{B^F}_t \mathscr{E}_t}.$$

Next, we introduce the definition of the real exchange rate $Q_t \equiv \mathscr{E}_t P^{Z}_t / P^{V^F}_t$ and the definition of a wedge between the domestic and foreign[25] productivity trend, $\Omega^{F}_t \equiv \widetilde{\chi}^{V^F}_t / \widetilde{\chi}^{Z}_t$, so that

$$w^{H}_t L^{o}_{i,t} + \frac{r^{K}_t K^{o}_{i,t-1}}{\Gamma^{Z}_t \Gamma^{I}} + \frac{R_{t-1} B^{o}_{i,t-1} + M^{o}_{i,t-1}}{\Gamma^{H} \Gamma^{Z}_t \Pi^{Z}_t} + D^{o}_{i,t} - M^{o}_{i,t} + \frac{\Omega^{F}_{t-1} R^{F}_{t-1} B^{F,o}_{i,t-1}}{Q_t \Gamma^{H} \Gamma^{Z}_t \Pi^{V^F}_t} = p^{CPI}_t C^{o}_{i,t} + \frac{B^{o}_{i,t}}{\varepsilon^{B}_t} + p^{I}_t I^{o}_{i,t} + p^{I^O}_t I^{O,o}_{i,t} + T^{o}_{i,t} + \mathscr{T}^{o}_i + \frac{\Omega^{F}_t}{Q_t} \frac{B^{F,o}_{i,t}}{\varepsilon^{B}_t \varepsilon^{B^F}_t}.$$

Detrending Total Profits from Firm/Union Ownership.

$$\frac{\widetilde{D}^{o}_{i,t}}{P^{Z}_t \widetilde{\chi}^{Z}_t} = \frac{1}{P^{Z}_t \widetilde{\chi}^{Z}_t} \left(\widetilde{D}^{U,o}_{i,t} + \widetilde{D}^{Z,o}_{i,t} + \widetilde{D}^{V,o}_{i,t} + \widetilde{D}^{M,o}_{i,t} + \widetilde{D}^{X,o}_{i,t} \right), \; D^{o}_{i,t} = D^{U,o}_{i,t} + D^{Z,o}_{i,t} + D^{V,o}_{i,t} + D^{M,o}_{i,t} + D^{X,o}_{i,t}.$$

[25] Note that in the world block there is no distinction into final output Z and value-added output V since production on the world level is assumed to abstract from imports. Therefore, world GDP and the associated price level will be denoted using V^F. Thus, world inflation will be $\Pi^{V^Z}_t$.

Detrending $\widetilde{C}_{i,t}$, $\widetilde{U}_{i,t}^{C}$, $\widetilde{\Lambda}_{i,t}^{C,o}$ and $\widetilde{MRS}_t$. Note that $U_t^C = \widetilde{U}_t^C \widetilde{\chi}_t^Z$, $\Lambda_t^{C,o} \equiv \widetilde{\Lambda}_t^{C,o} \widetilde{\chi}_t^Z P_t^Z$ so that

$$\left(C_{i,t}^o - \psi_C C_{t-1}\right)^{-\varepsilon_C} \varepsilon_t^C = \widetilde{U}_{i,t}^{C,o} \widetilde{\chi}_t^Z, \quad \left(C_{i,t}^o - \psi_C C_{t-1}^o\right)^{-\varepsilon_C} \varepsilon_t^C = U_{i,t}^{C,o}$$

and for the real MRS we define, $mrs_{i,t}^o = \widetilde{MRS}_{i,t}^o / (P_t^Z \widetilde{\chi}_t^Z)$ so that

$$\begin{aligned} mrs_{i,t}^o &= -\frac{U_{i,t}^L}{U_{i,t}^{C,o}/p_t^{CPI}}, \quad \widetilde{\Lambda}_{i,t}^{C,o} = \widetilde{U}_{i,t}^{C,o}/P_t^{CPI} \Leftrightarrow \Lambda_{i,t}^{C,o} = \widetilde{\chi}_t^Z P_t^Z \widetilde{\Lambda}_{i,t}^{C,o} = \widetilde{\chi}_t^Z \widetilde{U}_{i,t}^{C,o}/(P_t^{CPI}/P_t^Z) \\ U_{i,t}^L &= -\nu_L \left(L_{i,t}^o\right)^{\varepsilon_L} \varepsilon_t^L \varepsilon_t^C. \end{aligned}$$

For the domestic saving and consumption Euler equation this implies

$$1 = \mathbf{E}_t \left[\Lambda_{t,t+1}^o \frac{1}{\Pi_{t+1}^{CPI}} R_t \varepsilon_t^B\right], \qquad \Pi_t^{CPI} = \frac{P_t^{CPI}}{P_{t-1}^{CPI}} = \frac{p_t^{CPI}}{p_{t-1}^{CPI}} \Pi_t^Z$$

and for the domestic vs. foreign saving no-arbitrage Euler equation this implies a UIP condition

$$0 = \mathbf{E}_t \left[\Lambda_{t,t+1}^o \frac{1}{\Pi_{t+1}^{CPI}} \left(R_t - R_t^F \frac{\Pi_{t+1}^Z}{\Pi_{t+1}^{V^F}} \frac{Q_t}{Q_{t+1}} \varepsilon_t^{B^F}\right)\right],$$

where we introduce the definition of optimising household's stochastic discount factor

$$\Lambda_{t,t+1}^o \equiv \frac{\Theta_{t+1}}{\Theta_t} \frac{1}{\Gamma_{t+1}^Z} \frac{U_{i,t+1}^{C,o}}{U_{i,t}^{C,o}}.$$

Detrending $\widetilde{M}_{i,t}^o$. Detrending the first-order condition for money demand delivers

$$1 - \frac{U_{i,t}^{M,o}}{U_{i,t}^{C,o}/p_t^{CPI}} = \mathbf{E}_t \left[\Lambda_{t,t+1}^o \frac{1}{\Pi_{t+1}^{CPI}}\right], \quad U_{i,t}^{M,o} = \nu_M \left(M_{i,t}^o\right)^{-\varepsilon_C} \varepsilon_t^C.$$

Detrending $\widetilde{C}_{i,t}^{rot}$. Assuming that rule-of-thumb household simply consume their labour income plus transfers and that they work the same number of hours as their optimising counterparts, we have

$$P_t^{CPI} \widetilde{C}_{i,t}^{rot} = \widetilde{W}_t^H L_{i,t}^{rot} + \widetilde{D}_t^{U,rot} + P_t^Z \widetilde{\chi}_t^Z \mathscr{T}_i^{rot}, \quad p_t^{CPI} C_{i,t}^{rot} = w_t^H L_{i,t}^{rot} + D_t^{U,rot} + \mathscr{T}_i^{rot}$$

Detrending $\widetilde{I}_{i,t}^{O,o}$. We detrend other investment, so that

$$\frac{\widetilde{I}_{i,t}^{O,o}}{\widetilde{I}_{i,t-1}^{O,o}} = \left(\frac{\widetilde{I}_{i,t-1}^{O,o}}{\widetilde{\chi}_{t-1}^Z}\right)^{\rho_{I^O}-1} \varepsilon_t^{I^O} \quad \Leftrightarrow \quad \frac{I_{i,t}^{O,o}}{I_{i,t-1}^{O,o}} \Gamma_t^Z = \left(I_{i,t-1}^{O,o}\right)^{\rho_{I^O}-1} \varepsilon_t^{I^O}.$$

Detrending $\zeta_{i,t}^{I,o}$. Using $I_t \equiv \widetilde{I}_t / (\widetilde{\chi}_t^Z \widetilde{\chi}_t^I)$, we can write the investment growth rate ζ^I as

$$\zeta_{i,t}^{I,o} \equiv \frac{\Gamma^H \widetilde{I}_{i,t}^o}{\widetilde{I}_{i,t-1}^o}, \quad \zeta_{i,t}^{I,o} = \Gamma^H \frac{I_{i,t}^o}{I_{i,t-1}^o} \frac{\widetilde{\chi}_t^Z \widetilde{\chi}_t^I}{\widetilde{\chi}_{t-1}^Z \widetilde{\chi}_{t-1}^I}, \quad \zeta_{i,t}^{I,o} = \Gamma^H \Gamma^I \Gamma_t^Z \frac{I_{i,t}^o}{I_{i,t-1}^o},$$

where we introduced a deterministic trend for investment $\Gamma^I \equiv \widetilde{\chi}_t^I / \widetilde{\chi}_{t-1}^I$.

Detrending $\widetilde{I}_{i,t}^o$. Note that $TQ_{i,t} \equiv \widetilde{\Lambda}_{i,t}^{K,o} / \widetilde{\Lambda}_{i,t}^{C,o}$ and $tq_{i,t} \equiv \widetilde{TQ}_{i,t} \widetilde{\chi}_t^I / P_t^Z$

$$\begin{aligned}
P_t^I &= \frac{\widetilde{\Lambda}_{i,t}^{K,o}}{\widetilde{\Lambda}_{i,t}^{C,o}} \left[\Psi_I\left(\zeta_{i,t}^{I,o}, \varepsilon_t^I\right) + \Psi_I'\left(\zeta_{i,t}^{I,o}, \varepsilon_t^I\right) \widetilde{I}_{i,t}^o\right] + \mathbf{E}_t \left[\frac{\Theta_{t+1}}{\Theta_t} \frac{\widetilde{\chi}_{t+1}^H}{\widetilde{\chi}_t^H} \frac{\widetilde{\Lambda}_{i,t+1}^{C,o}}{\widetilde{\Lambda}_{i,t}^{C,o}} \frac{\widetilde{\Lambda}_{i,t+1}^{K,o}}{\widetilde{\Lambda}_{i,t+1}^{C,o}} \Psi_I'\left(\zeta_{i,t+1}^{I,o}, \varepsilon_{t+1}^I\right) \widetilde{I}_{i,t+1}^o\right] \\
p_t^I &= tq_{i,t} \left[\Psi_I\left(\zeta_{i,t}^{I,o}, \varepsilon_t^I\right) + \Psi_I'\left(\zeta_{i,t}^{I,o}, \varepsilon_t^I\right) \widetilde{I}_{i,t}^o\right] + \Gamma^H \mathbf{E}_t \left[\Lambda_{t,t+1}^o \frac{\Pi_{t+1}^Z}{\Pi_{t+1}^{CPI}} \frac{1}{\Gamma^I} tq_{i,t+1} \Psi_I'\left(\zeta_{i,t+1}^{I,o}, \varepsilon_{t+1}^I\right) \widetilde{I}_{i,t+1}^o\right]
\end{aligned}$$

where

$$\begin{aligned}
\Psi_I\left(\zeta_{i,t}^{I,o}, \varepsilon_t^I\right) &= \left(1 - \frac{\psi_I \left(\zeta_{i,t}^{I,o} - \Gamma^H \Gamma^Z \Gamma^I\right)^2}{2}\right) \varepsilon_t^I, \Psi_I'\left(\zeta_{i,t}^{I,o}, \varepsilon_t^I\right) \\
&= -\psi_I \left(\zeta_{i,t}^{I,o} - \Gamma^H \Gamma^Z \Gamma^I\right) \frac{\zeta_{i,t}^{I,o} \varepsilon_t^I}{\widetilde{I}_{i,t}^o}, \\
\Psi_I'\left(\zeta_{i,t+1}^{I,o}, \varepsilon_{t+1}^I\right) &= \psi_I \left(\zeta_{i,t+1}^{I,o} - \Gamma^H \Gamma^Z \Gamma^I\right) \frac{\left(\zeta_{i,t+1}^{I,o}\right)^2 \varepsilon_{t+1}^I}{\widetilde{I}_{i,t+1}^o \Gamma^H}
\end{aligned}$$

so that

$$\begin{aligned}
p_t^I = {} & tq_{i,t} \varepsilon_t^I \left[\left(1 - \frac{\psi_I \left(\zeta_{i,t}^{I,o} - \Gamma^H \Gamma^Z \Gamma^I\right)^2}{2}\right) - \psi_I \left(\zeta_{i,t}^{I,o} - \Gamma^H \Gamma^Z \Gamma^I\right) \zeta_{i,t}^{I,o}\right] \\
& + \mathbf{E}_t \left[\Lambda_{t,t+1}^o \frac{\Pi_{t+1}^Z}{\Pi_{t+1}^{CPI}} \frac{1}{\Gamma^I} tq_{i,t+1} \psi_I \left(\zeta_{i,t+1}^{I,o} - \Gamma^H \Gamma^Z \Gamma^I\right) \left(\zeta_{i,t+1}^{I,o}\right)^2 \varepsilon_{t+1}^I\right].
\end{aligned}$$

Detrending $\widetilde{K}_{i,t}^o$. The detrended first-order condition for the optimal choice of capital is given by

$$tq_{i,t} = \mathbf{E}_t \left[\Lambda_{t,t+1}^o \frac{1}{\Gamma^I} \frac{\Pi_{t+1}^Z}{\Pi_{t+1}^{CPI}} \left\{r_{t+1}^K + tq_{i,t+1}\left(1 - \delta^K\right)\right\}\right]$$

and the law of motion for capital which is given by

$$\Gamma^H K_{i,t} = (1 - \delta^K) \frac{K_{i,t-1}}{\Gamma_t^Z \Gamma^I} + \varepsilon_t^I \left(1 - \frac{\psi_I}{2} \left(\frac{\Gamma^I \Gamma^H \Gamma_t^Z I_{i,t}}{I_{i,t-1}} - \Gamma^H \Gamma^Z \Gamma^I\right)^2\right) I_{i,t}.$$

A.1.3 Aggregation

Aggregate consumption, employment, real balances, investment, other investment, capital, union profits, firm profits, domestic and foreign bonds are given by

$$
\begin{aligned}
C_t &= \omega^o C^o_{i,t} + (1-\omega^o) C^{rot}_{i,t}, \quad L^s_t = \omega^o L^o_{i,t} + (1-\omega^o) L^{rot}_{i,t}, \quad mrs_t = \omega^o mrs^o_{i,t} + (1-\omega^o) mrs^{rot}_{i,t} \\
M_t &= \omega^o M^o_{i,t}, \quad I_t = \omega^o I^o_{i,t}, \quad I^O_t = \omega^o I^{O,o}_{i,t}, \quad K_t = \omega^o K^o_{i,t}, \quad D^U_t = \omega^o D^{U,o}_{i,t} + (1-\omega^o) D^{U,rot}_{i,t} \\
D^Z_t &= \omega^o D^{Z,o}_{i,t},\ D^V_t = \omega^o D^{V,o}_{i,t},\ D^X_t = \omega^o D^{X,o}_{i,t},\ D^M_t = \omega^o D^{M,o}_{i,t}, \quad B_t = \omega^o B^o_{i,t}, B^F_t = \omega^o B^{F,o}_{i,t}
\end{aligned}
$$

A.1.4 Log-linearisation

Euler Equation for Optimising Households. Log-linearising the Euler equation for optimising households,

$$
1 = \mathbf{E}_t \left[\Lambda^o_{t,t+1} \frac{1}{\Pi^{CPI}_{t+1}} R_t \varepsilon^B_t \right], \quad \Leftrightarrow \quad 0 = \left[\mathbf{E}_t \widehat{\lambda}^o_{t,t+1} + \widehat{r}_t - \mathbf{E}_t \widehat{\pi}^{CPI}_{t+1} + \widehat{\varepsilon}^B_t \right]
$$

and using the definition of the SDF,

$$
\Lambda^o_{t,t+1} \equiv \mathbf{B}_{t+1} \frac{1}{\Gamma^Z_{t+1}} \frac{U^{C,o}_{i,t+1}}{U^{C,o}_{i,t}} \quad \Leftrightarrow \quad \widehat{\lambda}^o_{t,t+1} = \widehat{\mathbf{b}}_t - \widehat{\gamma}^Z_{t+1} + \widehat{u}^{C,o}_{i,t+1} - \widehat{u}^{C,o}_{i,t}
$$

also recall

$$
\mathbf{B}_{t+1} \equiv \frac{\Theta_{t+1}}{\Theta_t} = \beta \exp\left\{ \varepsilon_\beta \left(\frac{C^o_t}{C^o_{ss}} - 1 \right) \right\}, \mathbf{B}_{ss} \left(1 + \widehat{\mathbf{b}}_t \right) = \beta \left(1 + \left\{ \varepsilon_\beta \left(\frac{C^o_t - 1}{C^o_{ss}} \right) \right\} \right), \widehat{\mathbf{b}}_t = \varepsilon_\beta \widehat{c}^o_t
$$

and

$$
\begin{aligned}
U^{C,o}_{i,t} &= \left(C^o_{i,t} - \psi_C C^o_{t-1} \right)^{-\varepsilon_C} \varepsilon^C_t, \quad \left(\frac{\varepsilon^C_t}{U^{C,o}_{i,t}} \right)^{\frac{1}{\varepsilon_C}} = C^o_{i,t} - \psi_C C^o_{t-1} \\
C^o_{ss}(1-\psi_C) \left(1 + \frac{1}{\varepsilon_C} (\widehat{\varepsilon}^C_t - \widehat{u}^{C,o}_t) \right) &= C^o_{ss}(1+\widehat{c}^o_t) - \psi_C C^o_{ss}(1+\widehat{c}^o_{t-1}) \\
\widehat{u}^{C,o}_t &= -\varepsilon_C \frac{1}{1-\psi_C} \left(\widehat{c}^o_t - \psi_C \widehat{c}^o_{t-1} \right) + \widehat{\varepsilon}^C_t
\end{aligned}
$$

which implies

$$
\widehat{\mathbf{b}}_t + \widehat{u}^{C,o}_{t+1} - \widehat{u}^{C,o}_t = \left(\frac{\varepsilon_C(1+\psi_C) + (1-\psi_C)\varepsilon_\beta}{1-\psi_C} \right) \widehat{c}^o_t - \varepsilon_C \frac{\psi_C}{1-\psi_C} \widehat{c}^o_{t-1} - \varepsilon_C \frac{1}{1-\psi_C} \widehat{c}^o_{t+1} + \widehat{\varepsilon}^C_{t+1} - \widehat{\varepsilon}^C_t .
$$

So that we arrive at

$$
\widehat{c}^o_t = \frac{\mathbf{E}_t \widehat{c}^o_{t+1}}{1+\psi_C+\frac{\varepsilon_\beta(1-\psi_C)}{\varepsilon_C}} + \frac{\psi_C \widehat{c}^o_{t-1}}{1+\psi_C+\frac{\varepsilon_\beta(1-\psi_C)}{\varepsilon_C}} - \frac{1-\psi_C}{\varepsilon_\beta(1-\psi_C)+(1+\psi_C)\varepsilon_C} \left(\widehat{r}_t - \mathbf{E}_t \widehat{\pi}^{CPI}_{t+1} - \mathbf{E}_t \widehat{\gamma}^Z_{t+1} + \widehat{\varepsilon}^B_t - \widehat{\varepsilon}^C_t + \mathbf{E}_t \widehat{\varepsilon}^C_{t+1} \right). \tag{A.1}
$$

Consumption of Rule-of-Thumb Households. Note that union profits are rebated proportionately to the rule-of-thumb households. Thus, we can substitute the wage received by households W^H by the wage received by labour unions W and omit the profits of labour unions

$$
\begin{aligned}
p_t^{CPI} C_t^{rot} &= w_t L_t^{rot} + \mathscr{T}_i^{rot} \\
p_{ss}^{CPI} C_{ss} = w_{ss} L_{ss}^{rot} + \mathscr{T}_i^{rot}, \quad C_{ss}\left(1+\widehat{p}_t^{CPI}+\widehat{c}_t^{rot}\right) &= W_{ss}\left(1+\widehat{w}_t\right) + L_{ss}^{rot}\left(1+\widehat{l}_t^{rot}\right) + \mathscr{T}_i^{rot} \\
C_{ss}\widehat{p}_t^{CPI} + C_{ss}\widehat{c}_t^{rot} &= W_{ss}\widehat{w}_t + L_{ss}^{rot}\widehat{l}_t^{rot} \qquad \text{(A.2)}
\end{aligned}
$$

Money Demand. Combine the money demand equations

$$
\begin{aligned}
1-\frac{U_t^{M,o}}{U_t^{C,o}/p_t^{CPI}} &= \mathbf{E}_t\left[\Lambda_{t,t+1}^o \frac{1}{\Pi_{t+1}^{CPI}}\right] \simeq (R_t \varepsilon_t^B)^{-1}, \; 1-\frac{U_{ss}^{M,o}}{U_{ss}^{C,o}} = \frac{1}{R_{ss}} \Leftrightarrow \frac{U_{ss}^{M,o}}{U_{ss}^{C,o}} = \frac{R_{ss}-1}{R_{ss}} \\
1-\frac{U_{ss}^{M,o}}{U_{ss}^{C,o}}\left(1+\widehat{u}_t^{M,o}-\widehat{u}_t^{C,o}+\widehat{p}_t^{CPI}\right) &= \frac{1}{R_{ss}}(1+(-1)(\widehat{r}_t+\widehat{\varepsilon}_t^B)), \quad \frac{U_{ss}^{M,o}}{U_{ss}^{C,o}}(\widehat{u}_t^{M,o}-\widehat{u}_t^{C,o}+\widehat{p}_t^{CPI}) = -\frac{1}{R_{ss}}(\widehat{r}_t+\widehat{\varepsilon}_t^B) \\
(R_{ss}-1)(\widehat{u}_t^{M,o}-\widehat{u}_t^{C,o}+\widehat{p}_t^{CPI}) &= -(\widehat{r}_t+\widehat{\varepsilon}_t^B)
\end{aligned}
$$

and the marginal utility of consumption log-linear expression

$$
\begin{aligned}
\widehat{u}_t^{C,o} &= \varepsilon_C \frac{\psi_C}{1-\psi_C}\widehat{c}_{t-1}^o - \varepsilon_C \frac{1}{1-\psi_C}\widehat{c}_t^o + \widehat{\varepsilon}_t^C, \; U_t^{M,o} = \nu_M \left(M_t^o\right)^{-\varepsilon_C}\varepsilon_t^C, \quad \widehat{u}_t^{M,o} = -\varepsilon_C \widehat{mon}_t + \widehat{\varepsilon}_t^C \\
-(\widehat{r}_t+\widehat{\varepsilon}_t^B) &= (R_{ss}-1)(-\varepsilon_C \widehat{mon}_t + \widehat{\varepsilon}_t^C - (\varepsilon_C \frac{\psi_C}{1-\psi_C}\widehat{c}_{t-1}^o - \varepsilon_C \frac{1}{1-\psi_C}\widehat{c}_t^o + \widehat{\varepsilon}_t^C) + \widehat{p}_t^{CPI})
\end{aligned}
$$

to arrive at

$$
\widehat{mon}_t = \tfrac{1}{1-\psi_C}\left(\widehat{c}_t^o - \psi_C \widehat{c}_{t-1}^o\right) - \left(\widehat{r}_t + \widehat{\varepsilon}_t^B\right)\tfrac{1}{\varepsilon_C R_{ss}} + \tfrac{\widehat{p}_t^{CPI}}{\varepsilon_C}. \qquad \text{(A.3)}
$$

UIP Condition. Recall

$$
0 = \mathbf{E}_t\left[\Lambda_{t,t+1}^o \frac{1}{\Pi_{t+1}^{CPI}}\left(R_t - R_t^F \frac{\Pi_{t+1}^Z}{\Pi_{t+1}^{V^F}}\frac{Q_t}{Q_{t+1}}\varepsilon_t^{B^F}\right)\right].
$$

which can easily be log-linearised to

$$
\widehat{q}_t = \widehat{r}_t + \mathbf{E}_t \widehat{q}_{t+1} - \widehat{r}_t^F - \mathbf{E}_t \widehat{\pi}_{t+1}^Z + \mathbf{E}_t \widehat{\pi}_{t+1}^{V^F} - \widehat{\varepsilon}_t^{B^F} \qquad \text{(A.4)}
$$

Total Consumption.

$$
C_t = \omega^o C_{i,t}^o + (1-\omega^o) C_{i,t}^{rot}, \quad \widehat{c}_t = \omega_o \widehat{c}_t^o + (1-\omega_o)\widehat{c}_t^{rot} \qquad \text{(A.5)}
$$

Combine Household and Firm Budgets into Aggregate Resource Constraint

Households

$$
w_t^H L_t^o + \frac{r_t^K K_{t-1}^o}{\Gamma_t^Z \Gamma^I} + \frac{R_{t-1}B_{t-1}^o + M_{t-1}^o}{\Gamma^H \Gamma_t^Z \Pi_t^Z} + D_t^o - M_t^o + \frac{\Omega_{t-1}^F R_{t-1}^F B_{t-1}^{F,o}}{Q_t \Gamma^H \Gamma_t^Z \Pi_t^{V^F}} = p_t^{CPI} C_t^o + \frac{B_t^o}{\varepsilon_t^B} + p_t^I I_t^o + p_t^{I^O} I_t^{O,o} + T_t^{G,o} + T_t^{D,o} + \mathscr{T}^o + \frac{\Omega_t^F B_t^{F,o}}{Q_t \varepsilon_t^B \varepsilon_t^{B^F}}.
$$

$$
p_t^{CPI} C_t^{rot} = w_t^H L_t^{rot} + D_t^{U,rot} + \mathscr{T}^{rot}
$$

Energy Retailer

$$p_t^{CPI} C_t = p_t^C C_t^z + p_t^E E_t^c$$

Government

$$p_t^G G_t + (B_{t-1} R_{t-1} + Mon_{t-1}) / (\Gamma^H \Gamma_t^Z \Pi_t^Z) = T_t^G + B_t + Mon_t$$

Firm Profits

$$\begin{aligned}
D_t^o &= D_t^{U,o} + D_t^{Z,o} + D_t^{V,o} + D_t^{M,o} + D_t^{X,o}, \quad T_t^{D,o} = T_t^{U,o} + T_t^{Z,o} + T_t^{V,o} + T_t^{M,o} + T_t^{X,o} \\
D_t^Z &= Z_t - \tau_t^{\mathscr{M}_Z} \left(p_t^V V_t (1 + \Psi_t^V(.)) + p_t^M M_t^z (1 + \Psi_t^M(.)) + p_t^E E_t^z (1 + \Psi_t^E(.)) \right) \\
D_t^U &= w_t L_t - \tau_t^{\mathscr{M}_W} w_t^H L_t, \qquad D_t^V = p_t^V V_t - \tau_t^{\mathscr{M}_V} \left(w_t L_t + r_t^K \frac{K_{t-1}}{\Gamma^I \Gamma_t^Z} \right) \\
D_t^M &= p_t^M M_t^z - \tau_t^{\mathscr{M}_M} \left(\frac{p_t^{X^F}}{Q_t} M_t^z \right), \quad D_t^X = \frac{p_t^{EXP}}{Q_t} X_t - \tau_t^{\mathscr{M}_X} \left(p_t^X X_t \right)
\end{aligned}$$

Total Firm Profits net of Lump-Sum Subsidy

$$\begin{aligned}
D_t^o - T_t^{o,D} &= -w_t^H L_t + Z_t - \underbrace{\left(p_t^V V_t (\Psi_t^V(.)) + p_t^M M_t^z (\Psi_t^M(.)) + p_t^E E_t^z (\Psi_t^E(.)) \right)}_{\equiv \Psi_t^{\text{Firm}}} - p_t^E E_t^z \\
&\quad - \left(r_t^K \frac{K_{t-1}}{\Gamma^I \Gamma_t^Z} \right) - \left(\frac{p_t^{X^F}}{Q_t} M_t^z \right) + \frac{p_t^{EXP}}{Q_t} X_t - \left(p_t^X X_t \right)
\end{aligned}$$

Trade Balance and Net Foreign Asset Position

$$\begin{aligned}
TB_t &\equiv \frac{p_t^{EXP}}{Q_t} X_t - p_t^E (E_t^z + E_t^c) - \frac{p_t^{X^F}}{Q_t} M_t^z \\
NFA_t &= \frac{\Omega_t^F}{Q_t} \frac{B_t^F}{\varepsilon_t^B \varepsilon_t^{B^F}} - \frac{\Omega_{t-1}^F}{Q_t} \frac{R_{t-1}^F B_{t-1}^F}{\Gamma^H \Gamma_t^Z \Pi_t^{V^F}}, \qquad TB_t = NFA_t
\end{aligned}$$

Aggregate Resource Constraint

$$\begin{aligned}
Z_t &= C_t^z + X_t + G_t + I_t + I_t^O + X_t \\
Z_{ss} \widehat{z}_t &= C_{ss}^z \widehat{c}_t^z + X_{ss} \widehat{x}_t + G_{ss} \widehat{g}_t + I_{ss} \widehat{i}_t + I_{ss}^O \widehat{i}_t^O + X_{ss} \widehat{x}_t
\end{aligned} \tag{A.6}$$

CPI Inflation Definition

$$\widehat{\pi}_t^{CPI} = \widehat{p}_t^{CPI} - \widehat{p}_{t-1}^{CPI} + \widehat{\pi}_t^Z \tag{A.7}$$

Residual component of TFE rule

$$\frac{I_{i,t}^{O}}{I_{i,t-1}^{O}}\Gamma_t^Z = \left(I_{i,t-1}^{O}/\omega^o\right)^{\rho_{I^O}-1}\varepsilon_t^{I^O}, \quad \Leftrightarrow \quad \widehat{\gamma}_t^Z + \widehat{i}_t^O - \widehat{i}_{t-1}^O \quad = \widehat{i}_{t-1}^O\left(\rho_{I^o}-1\right) + \widehat{\varepsilon}_t^{I^o} \quad \text{(A.8)}$$

Investment Euler Equation

$$\begin{aligned}
p_t^I = 1 \quad &= \quad TQ_t\varepsilon_t^I\left[\left(1-\frac{\psi_I}{2}\left(\zeta_t^{I,o}-\Gamma^H\Gamma^Z\Gamma^I\right)^2\right)-\left(\psi_I\left(\zeta_t^{I,o}-\Gamma^H\Gamma^Z\Gamma^I\right)\zeta_t^{I,o}\right)\right]+\mathbf{E}_t\left[\mathbf{B}_{t+1}\frac{\Lambda_{t+1}^C}{\Lambda_t^C}\frac{TQ_{t+1}}{\Gamma^I\Gamma_{t+1}^Z}\left(\psi_I\left(\zeta_{t+1}^{I,o}-\Gamma^H\Gamma^Z\Gamma^I\right)\left(\zeta_{t+1}^{I,o}\right)^2\varepsilon_{t+1}^I\right)\right], \quad 1 = TQ_{ss}\left[1-0\right]+0 \\
1 \quad &= \quad TQ_{ss}\left(1+\widehat{tq}_t+\widehat{\varepsilon}_t^I\right)-\frac{\psi_I}{2}TQ_{ss}(\zeta_{ss}^I)^2\left(1+\widehat{tq}_t+\widehat{\varepsilon}_t^I+2(\widehat{\zeta}_t^{I,o})\right)+\psi_I\Gamma^H\Gamma^Z\Gamma^I TQ_{ss}\zeta_{ss}^I\left(1+\widehat{\zeta}_t^{I,o}+\widehat{tq}_t+\widehat{\varepsilon}_t^I\right) \\
&\quad -\frac{\psi_I}{2}\left((\Gamma^H\Gamma^Z\Gamma^I)^2\right)TQ_{ss}\left(1+\widehat{tq}_t+\widehat{\varepsilon}_t^I\right)-TQ_t\varepsilon_t^I\psi_I(1+\zeta_t^{I,o})^2+\psi_I\Gamma^H\Gamma^Z\Gamma^I\zeta_t^{I,o}TQ_t\varepsilon_t^I \\
&\quad +\mathbf{E}_t\left[\mathbf{B}_{t+1}\frac{\Lambda_{t+1}^C}{\Lambda_t^C}\frac{TQ_{t+1}}{\Gamma^I\Gamma_{t+1}^Z}\left(\left(\psi_I\zeta_{t+1}^{I,o}\left(\zeta_{t+1}^{I,o}\right)^2\varepsilon_{t+1}^I\right)\right)\right]-\mathbf{E}_t\left[\mathbf{B}_{t+1}\frac{\Lambda_{t+1}^C}{\Lambda_t^C}\frac{TQ_{t+1}}{\Gamma^I\Gamma_{t+1}^Z}\left(\left(\psi_I\Gamma^H\Gamma^Z\Gamma^I\left(\zeta_{t+1}^{I,o}\right)^2\varepsilon_{t+1}^I\right)\right)\right] \\
0 \quad &= \quad TQ_{ss}\left(\widehat{tq}_t+\widehat{\varepsilon}_t^I\right)-\frac{\psi_I}{2}TQ_{ss}(\zeta_{ss}^I)^2\left(\widehat{tq}_t+\widehat{\varepsilon}_t^I+2(\widehat{\zeta}_t^{I,o})\right)+\psi_I\Gamma^H\Gamma^Z\Gamma^I TQ_{ss}\zeta_{ss}^I\left(\widehat{\zeta}_t^{I,o}+\widehat{tq}_t+\widehat{\varepsilon}_t^I\right)-\frac{\psi_I}{2}\left((\Gamma^H\Gamma^Z\Gamma^I)^2\right)TQ_{ss}\left(\widehat{tq}_t+\widehat{\varepsilon}_t^I\right) \\
&\quad -\psi_I TQ_{ss}(\zeta_{ss}^{I,o})^2\left(\widehat{tq}_t+\widehat{\varepsilon}_t^I+2(\widehat{\zeta}_t^{I,o})\right)+\psi_I\Gamma^H\Gamma^Z\Gamma^I\zeta_{ss}^{I,o}TQ_{ss}\left(\widehat{\zeta}_t^{I,o}+\widehat{tq}_t+\widehat{\varepsilon}_t^I\right)+\beta\frac{TQ_{ss}}{\Gamma^I\Gamma_{ss}^Z}\psi_I\left(\zeta_{ss}^{I,o}\right)^3\mathbf{E}_t\left[\widehat{b}_{t+1}+\widehat{\lambda}_{t+1}^C-\widehat{\lambda}_t^C+\widehat{tq}_{t+1}-\widehat{\gamma}_{t+1}^Z+3\widehat{\zeta}_{t+1}^I+\widehat{\varepsilon}_{t+1}^I\right] \\
&\quad -\beta\frac{TQ_{ss}}{\Gamma^I\Gamma_{ss}^Z}\psi_I\Gamma^H\Gamma^Z\Gamma^I\left(\zeta_{ss}^{I,o}\right)^2\mathbf{E}_t\left[\widehat{b}_{t+1}+\widehat{\lambda}_{t+1}^C-\widehat{\lambda}_t^C+\widehat{tq}_{t+1}-\widehat{\gamma}_{t+1}^Z+2\widehat{\zeta}_{t+1}^{I,o}+\widehat{\varepsilon}_{t+1}^I\right]
\end{aligned}$$

Recall:

$$\zeta_t^{I,o} = \Gamma^H\Gamma^I\Gamma_t^Z\frac{I_t^o}{I_{t-1}^o}, \qquad \zeta_{ss}^{I,o} = \Gamma^H\Gamma^I\Gamma_{ss}^Z, \quad \zeta_{ss}^{I,o}\widehat{\zeta}_t^{I,o} = \Gamma^H\Gamma^I\Gamma_{ss}^Z\left(\widehat{i}_t-\widehat{i}_{t-1}+\widehat{\gamma}_t^Z\right), \qquad \widehat{\zeta}_t^{I,o} = \left(\widehat{i}_t-\widehat{i}_{t-1}+\widehat{\gamma}_t^Z\right)$$

So

$$\begin{aligned}
0 \quad &= \quad TQ_{ss}\left(\widehat{tq}_t+\widehat{\varepsilon}_t^I\right)-\frac{\psi_I}{2}TQ_{ss}(\zeta_{ss}^I)^2\left(\widehat{tq}_t+\widehat{\varepsilon}_t^I+2(\widehat{\zeta}_t^{I,o})\right)+\psi_I\Gamma^H\Gamma^Z\Gamma^I TQ_{ss}\zeta_{ss}^I\left(\widehat{\zeta}_t^{I,o}+\widehat{tq}_t+\widehat{\varepsilon}_t^I\right)-\frac{\psi_I}{2}\left((\Gamma^H\Gamma^Z\Gamma^I)^2\right)TQ_{ss}\left(\widehat{tq}_t+\widehat{\varepsilon}_t^I\right) \\
&\quad -\psi_I TQ_{ss}(\zeta_{ss}^{I,o})^2\left(\widehat{tq}_t+\widehat{\varepsilon}_t^I+2(\widehat{\zeta}_t^{I,o})\right)+\psi_I\Gamma^H\Gamma^Z\Gamma^I\zeta_{ss}^{I,o}TQ_{ss}\left(\widehat{\zeta}_t^{I,o}+\widehat{tq}_t+\widehat{\varepsilon}_t^I\right)+\beta\frac{TQ_{ss}}{\Gamma^I\Gamma_{ss}^Z}\psi_I\left(\zeta_{ss}^{I,o}\right)^3\mathbf{E}_t\left[\widehat{b}_{t+1}+\widehat{\lambda}_{t+1}^C-\widehat{\lambda}_t^C+\widehat{tq}_{t+1}-\widehat{\gamma}_{t+1}^Z+3\widehat{\zeta}_{t+1}^I+\widehat{\varepsilon}_{t+1}^I\right] \\
&\quad -\beta\frac{TQ_{ss}}{\Gamma^I\Gamma_{ss}^Z}\psi_I\left(\zeta_{ss}^{I,o}\right)^3\mathbf{E}_t\left[\widehat{b}_{t+1}+\widehat{\lambda}_{t+1}^C-\widehat{\lambda}_t^C+\widehat{tq}_{t+1}-\widehat{\gamma}_{t+1}^Z+2\widehat{\zeta}_{t+1}^{I,o}+\widehat{\varepsilon}_{t+1}^I\right] \\
0 \quad &= \quad \left(\widehat{tq}_t+\widehat{\varepsilon}_t^I\right)-\frac{\psi_I}{2}(\zeta_{ss}^I)^2\left(\widehat{tq}_t+\widehat{\varepsilon}_t^I+2(\widehat{\zeta}_t^{I,o})\right)+\psi_I\Gamma^H\Gamma^Z\Gamma^I\zeta_{ss}^I\left(\widehat{\zeta}_t^{I,o}+tq_t+\widehat{\varepsilon}_t^I\right)-\frac{\psi_I}{2}\left((\Gamma^H\Gamma^Z\Gamma^I)^2\right)\left(\widehat{tq}_t+\widehat{\varepsilon}_t^I\right)-\psi_I(\zeta_{ss}^{I,o})^2\left(\widehat{tq}_t+\widehat{\varepsilon}_t^I+2(\widehat{\zeta}_t^{I,o})\right) \\
&\quad +\psi_I\Gamma^H\Gamma^Z\Gamma^I\zeta_{ss}^{I,o}\left(\widehat{\zeta}_t^{I,o}+\widehat{tq}_t+\widehat{\varepsilon}_t^I\right)+\frac{\beta}{\Gamma^I\Gamma_{ss}^Z}\psi_I\left(\zeta_{ss}^{I,o}\right)^3\mathbf{E}_t\left[\widehat{\zeta}_{t+1}^I\right] \\
0 \quad &= \quad \left(\widehat{tq}_t+\widehat{\varepsilon}_t^I\right)-\psi_I(\zeta_{ss}^I)^2\left((\widehat{\zeta}_t^{I,o})\right)+\beta\Gamma^H\psi_I\left(\zeta_{ss}^{I,o}\right)^2\mathbf{E}_t\left[\widehat{\zeta}_{t+1}^I\right], \quad \widehat{\zeta}_t^{I,o} = \frac{1}{\psi_I(\zeta_{ss}^I)^2}\left(\widehat{tq}_t+\widehat{\varepsilon}_t^I\right)+\beta\Gamma^H\mathbf{E}_t\left[\widehat{\zeta}_{t+1}^I\right] \\
\widehat{i}_t \quad &= \quad \frac{\widehat{i}_{t-1}-\widehat{\gamma}_t^Z}{(1+\beta\Gamma^H)}+\frac{1}{\psi_I(\widehat{\zeta}_{ss}^I)^2(1+\beta\Gamma^H)}\left(\widehat{tq}_t+\widehat{\varepsilon}_t^I\right)+\frac{\beta\Gamma^H}{(1+\beta\Gamma^H)}\mathbf{E}_t\left[\left(\widehat{i}_{t+1}+\widehat{\gamma}_{t+1}^Z\right)\right]
\end{aligned}$$

In the end, we get

$$\widehat{i}_t \quad = \frac{1}{1+\beta\,\Gamma_{ss}^H}\left(\widehat{i}_{t-1}-\widehat{\gamma}_t^Z\right)+\frac{\beta\,\Gamma_{ss}^H}{1+\beta\,\Gamma_{ss}^H}\mathbf{E}_t\left(\widehat{\gamma}_{t+1}^Z+\widehat{i}_{t+1}\right)+\frac{1}{(1+\beta\,\Gamma_{ss}^H)\,(\Gamma_{ss}^H\,\Gamma_{ss}^Z\,\Gamma_{ss}^I)^2}\left(\frac{1}{\psi_I}\widehat{tq}_t+\widehat{\varepsilon}_t^I\right) \quad \text{(A.9)}$$

Note that an amendment was made in the investment Euler equation, (we introduced $1/\psi_I$ in front of $\widehat{tq}_t$), to decouple the investment adjustment cost parameters, ψ_I, from the forcing process for the investment adjustment cost shock. This modification allows us to separately identify the standard deviation of the adjustment cost shock and the structural parameter. This can be interpreted as the implementation of a 'hierarchical prior' on the standard deviation of the investment adjustment cost shock, conditional on the value of ψ_I.

Tobin's Q. Recall the expression for Tobin's Q

$$
\begin{aligned}
tq_{i,t} &= \mathbf{E}_t\left[\Lambda^o_{t,t+1}\frac{1}{\Gamma^I}\frac{\Pi^Z_{t+1}}{\Pi^{CPI}_{t+1}}\left\{r^K_{t+1}+tq_{i,t+1}\left(1-\delta^K\right)\right\}\right],\ tq_{i,t}=\mathbf{E}_t\left[\Lambda^o_{t,t+1}\frac{1}{\Gamma^I}\frac{\Pi^Z_{t+1}}{\Pi^{CPI}_{t+1}}r^K_{t+1}\right]+\mathbf{E}_t\left[\Lambda^o_{t,t+1}\frac{1}{\Gamma^I}\frac{\Pi^Z_{t+1}}{\Pi^{CPI}_{t+1}}tq_{i,t+1}\left(1-\delta^K\right)\right] \\
tq_{i,ss} &= \left[\Lambda^o_{ss}\frac{1}{\Gamma^I}r^K_{ss}\right]+\left[\Lambda^o_{ss}\frac{1}{\Gamma^I}tq_{i,ss}\left(1-\delta^K\right)\right],\quad \frac{\Gamma^I}{\beta}=r^K_{ss}+\left(1-\delta^K\right) \\
\left(\widehat{tq}_t\right) &= \mathbf{E}_t\left[\frac{\beta}{\Gamma^I}r^K_{ss}\right]\left(\widehat{\lambda}^o_{t,t+1}+\widehat{\pi}^Z_{t+1}-\widehat{\pi}^{CPI}_{t+1}+\widehat{r}^K_{t+1}\right)+\mathbf{E}_t\left[\frac{\beta}{\Gamma^I}\left(1-\delta^K\right)\right]\left(\widehat{\lambda}^o_{t,t+1}+\widehat{\pi}^Z_{t+1}-\widehat{\pi}^{CPI}_{t+1}+\widehat{tq}^K_{t+1}\right).
\end{aligned}
$$

Also recall from the optimising household Euler equation $\mathbf{E}_t\widehat{\lambda}^o_{t,t+1}=\mathbf{E}_t\widehat{\pi}^{CPI}_{t+1}-\widehat{r}_t-\widehat{\varepsilon}^B_t$ so that

$$
\frac{\Gamma^I_{ss}}{\beta}\left(\widehat{tq}_t\right) = r^K_{ss}\mathbf{E}_t\left(\widehat{\pi}^Z_{t+1}-\widehat{r}_t-\widehat{\varepsilon}^B_t+\widehat{r}^K_{t+1}\right)+\left(1-\delta^K\right)\mathbf{E}_t\left(\widehat{\pi}^Z_{t+1}-\widehat{r}_t-\widehat{\varepsilon}^B_t+\widehat{tq}^K_{t+1}\right).
$$

We then arrive at

$$
\widehat{tq}_t = \tfrac{1-\delta^K}{1-\delta^K+r^K_{ss}}\mathbf{E}_t\widehat{tq}_{t+1}-\left(\widehat{r}_t-\mathbf{E}_t\widehat{\pi}^Z_{t+1}+\widehat{\varepsilon}^B_t\right)+\tfrac{r^K_{ss}}{1-\delta^K+r^K_{ss}}\mathbf{E}_t\widehat{r}^K_{t+1}. \tag{A.10}
$$

Capital Accumulation. Recall the law of motion for capital and rewrite it

$$
\Gamma^H K_t = (1-\delta^K)\frac{K_{t-1}}{\Gamma^Z_t\Gamma^I}+\varepsilon^I_t I^o_t-\varepsilon^I_t I^o_t\frac{\psi_I}{2}\left(\frac{\Gamma^I\Gamma^H\Gamma^Z_t I^o_t}{I^o_{t-1}}-\Gamma^H\Gamma^Z\Gamma^I\right)^2
$$

to obtain the log-linear expression

$$
\begin{aligned}
\Gamma^H K_t &= (1-\delta^K)\frac{K_{ss}}{\Gamma^Z_{ss}\Gamma^I}\left(k_{t-1}-\gamma^Z_t\right)+I_{ss}\left(\widehat{\varepsilon}^I_t+i_t\right)-\frac{\psi_I}{2}(\zeta^I_{ss})^2 I_{ss}\left(2\widehat{\varepsilon}^I_t+2i_t+2\widehat{\zeta}^I_t\right)+\frac{\psi_I}{2}(\zeta^I_{ss})^2 I_{ss}\left(2\widehat{\varepsilon}^I_t+2i_t+2\widehat{\zeta}^I_t\right) \\
\Gamma^H K_{ss}\widehat{k}_t &= (1-\delta^K)\frac{K_{ss}}{\Gamma^Z_{ss}\Gamma^I}\left(\widehat{k}_{t-1}-\widehat{\gamma}^Z_t\right)+I_{ss}\left(\widehat{\varepsilon}^I_t+\widehat{i}_t\right),\quad \widehat{k}_t=\left(\widehat{k}_{t-1}-\widehat{\gamma}^Z_t\right)\frac{1-\delta^K}{\Gamma^H_{ss}\Gamma^Z_{ss}\Gamma^I_{ss}}+\frac{I_{ss}}{\Gamma^H_{ss}K_{ss}}\left(\widehat{i}_t+\widehat{\varepsilon}^I_t\right).
\end{aligned} \tag{A.11}
$$

A.2 Labour Packers and Unions

We follow Schmitt-Grohé and Uribe (2006) and introduce wage stickiness into the model via two types of agents: (i) perfectly competitive labour packers and (ii) monopolistically competitive unions. After households have chosen how much labour to supply in a given period, $L^k_t(j)$, $k\in\{o,rot\}$, this labor is supplied to a union, in return for a nominal wage $\widetilde{W}^H_t$. The union unpacks the homogenous labour supplied by households and differentiates it into different varieties $L_t(j)$, $j\in[0,1]$ and sells these units of labour varieties at wage $\widetilde{W}_t(j)$. Due to imperfect substitutability the union can act as a monopolist.

Labour Packers. Varieties $L_t(j)$ are assembled by labour packers according to a CES production function. $L_t(j)$ denotes the demand for a specific labour variety j and L_t denotes aggregate labour demand. ε_w is the elasticity of substitution between labour varieties and thus $\mathscr{M}_W=\varepsilon_w/(\varepsilon_w-1)$ is the corresponding gross wage markup of monopolistically competitive unions. After the packers have assembled the labour bundle, they sell it to firms at wage W_t who then use it in the production process. The packers' production function, and the implied demand schedule associated with the cost minimisation are

$$L_t = \left[\int_0^1 (L_t(j))^{\frac{\varepsilon_w-1}{\varepsilon_w}} \mathrm{d}j\right]^{\frac{\varepsilon_w}{\varepsilon_w-1}}, \quad L_t(j) = \left(\widetilde{W}_t(j)/\widetilde{W}_t\right)^{\frac{\mathscr{M}_W}{1-\mathscr{M}_W}} L_t, \quad \widetilde{W}_t \equiv \left(\int_0^1 \left(\widetilde{W}_t(j)\right)^{\frac{1}{1-\mathscr{M}_W}} \mathrm{d}j\right)^{1-\mathscr{M}_W}$$

where $\widetilde{W}_t$ is the aggregate wage index and optimal behaviour by the labour packers implies that $\widetilde{W}_t L_t = \int_0^1 \widetilde{W}_t(j) L_t(j) dj$.

Labour Unions. Each individual labour union who sells its imperfectly substitutable labour variety $L_t(j)$ to the packer is subject to *nominal wage rigidities*. The probability that the union cannot reset its wage is ϕ_w. It is convenient to split the problem of a monopolistically competitive labour union into two steps: (i) the intra-temporal cost minimisation problem and (ii) the inter-temporal wage setting problem.

Cost Minimisation. A union will choose to minimise its costs $\tau_t^{\mathscr{M}_W} \widetilde{W}_t^H L_t(j)$ subject to meeting the packer's labour demand. The Lagrange multiplier $\widetilde{MC}_t^W(j)$ is the union's (nominal) shadow cost of providing one more unit of labour, i.e., the nominal marginal cost and $\tau_t^{\mathscr{M}_W}$ is a subsidy to marginal costs that eliminates the steady-state distortion associated with monopolistic competition. Note that the Lagrange multiplier of an individual union j does not depend on its own quantities of inputs demanded, so that all unions have the same marginal costs $\widetilde{MC}_t^W(j) = \widetilde{MC}_t^W$. The wage paid to households,[26] $\widetilde{W}_t^H$ corresponds to the marginal rate of substitution so that $\widetilde{MC}_t^W = \tau_t^{\mathscr{M}_W} \widetilde{W}_t^H = \tau_t^{\mathscr{M}_W} \widetilde{MRS}_t$. Recall that we use lower cases to denote stationary real (final output price level) terms $w_t^H \equiv \widetilde{W}_t^H/(P_t^Z \widetilde{\chi}_t^Z)$ so that

$$mc_t^W = \tau_t^{\mathscr{M}_W} w_t^H, \quad w_t^H = mrs_t, \quad \text{where} \quad mrs_t = \omega^o mrs_t^o + (1-\omega^o) mrs_t^{rot}.$$

Following Galí et al. (2007), we assume that the union takes into account the fact that firms allocate labour demand uniformly across different workers of type j, independently of their household type $\{o, rot\}$, $L_t^o(j) = L_t^{rot}(j)$.

Rule-of-Thumb Wage Setting. The objective of each union j is to maximise its nominal profits $\widetilde{D}_t^U(j)$

$$\widetilde{D}_t^U(j) = \widetilde{W}_t(j) L_t(j) - \left\{\tau_t^{\mathscr{M}_W}\left(\widetilde{W}_t^H L_t(j)\right)\right\}, \quad D_t^U(j) = \left(w_t - mc_t^W\right) L_t(j).$$

A fraction ω_W of 'rule-of-thumb' wage setters will set their wage based on an index of previous-period and steady-state wage inflation. Only the remaining fraction $(1-\omega_W)$ attempts to implement the optimal wage $w_t^{\#}$. An expression for the aggregate wage

[26] We assume that both unconstrained and constrained households receive the same wage.

index

$$\widetilde{W}_t = \left(\int_F \left(\widehat{\widetilde{W}}^{\#}_t(j)\right)^{\frac{1}{1-\mathscr{M}_W}} dj + \int_{\bar{F}} \left(\left[\left(\Pi^W_{ss}\right)^{(1-\xi_W)} \left(\Pi^W_{t-1}\right)^{\xi_W}\right] \widetilde{W}_{t-1}(j)\right)^{\frac{1}{1-\mathscr{M}_W}} dj\right)^{1-\mathscr{M}_W}$$

where F is the set of those unions who can reoptimise their wage.

$$\widetilde{W}_t = \left((1-\phi_W)\left(\widehat{\widetilde{W}}^{\#}_t\right)^{\frac{1}{1-\mathscr{M}_W}} + (\phi_W)\left(\left[\left(\Pi^W_{ss}\right)^{(1-\xi_W)} \left(\Pi^W_{t-1}\right)^{\xi_W}\right] \widetilde{W}_{t-1}\right)^{\frac{1}{1-\mathscr{M}_W}}\right)^{1-\mathscr{M}_W}$$

$$\left(\widetilde{W}_t\right)^{\frac{1}{1-\mathscr{M}_W}} = \left((1-\phi_W)\left\{\left(\widetilde{W}^{\#}_t\right)^{1-\omega_W} \cdot \left(\widetilde{W}_{t-1}\frac{\Pi^W_t}{\zeta^W_t}\right)^{\omega_W}\right\}^{\frac{1}{1-\mathscr{M}_W}} + (\phi_W)\left(\left[\left(\Pi^W_{ss}\right)^{(1-\xi_W)} \left(\Pi^W_{t-1}\right)^{\xi_W}\right] \widetilde{W}_{t-1}\right)^{\frac{1}{1-\mathscr{M}_W}}\right)$$

$$1 = (1-\phi_W)\left\{\left(w^{\#}_t\right)^{1-\omega_W} \cdot \left(\zeta^W_t\right)^{-\omega_W}\right\}^{\frac{1}{1-\mathscr{M}_W}} + (\phi_W)\left(\left[\left(\Pi^W_{ss}\right)^{(1-\xi_W)} \left(\Pi^W_{t-1}\right)^{\xi_W}\right] \frac{1}{\Pi^W_t}\right)^{\frac{1}{1-\mathscr{M}_W}}.$$

With probability $1-\phi_w$ a non-rule-of-thumb union can re-optimise its wage

$$\widetilde{W}_t(j) = \begin{cases} \widetilde{W}^{\#}_t(j) & \text{with probability: } 1-\phi_w \\ \widetilde{W}_{t-1}(j)\left(\left(\Pi^W_{ss}\right)^{1-\xi_W}\left(\Pi^W_{t-1}\right)^{\xi_W}\right) & \text{with probability: } \phi_w \end{cases}$$

where ξ_w is the weight attached to the previous period wage inflation, Π^W_t. Consider a union who can reset its wage in the current period $\widetilde{W}_t(j) = \widetilde{W}^{\#}_t(j)$ and who is then stuck with its wage until future period $t+s$. The wage in this case would be

$$\widetilde{W}_{t+s}(j) = \widetilde{W}^{\#}_t(j)\left(\Pi^W_{ss}\right)^{s(1-\xi_W)}\left(\prod_{g=0}^{s-1}\left(\left(\Pi^W_{t+g}\right)^{\xi_W}\right)\right) = \widetilde{W}^{\#}_t(j)\left[\left(\Pi^W_{ss}\right)^{s(1-\xi_W)}\left(\frac{\widetilde{W}_{t+s-1}}{\widetilde{W}_{t-1}}\right)^{\xi_W}\right].$$

Subject to the above derived demand constraint and assuming that a union j always meets the demand for its labour at the current wage labour unions solve the following optimisation problem

$$\max_{\widetilde{W}^{\#}_t(j)} \mathbf{E}_t \sum_{s=0}^{\infty} (\phi_W)^s \Lambda^{C,o}_{t,t+s} P^Z_{t+s} \widetilde{\chi}^Z_{t+s}\left[\left(\frac{\widetilde{W}_{t+s}(j)}{\widetilde{\chi}^Z_{t+s} P^Z_{t+s}} - mc^W_{t+s}\right) L_{t+s}(j)\right] \quad \text{s.t.} \quad L_{t+s}(j) = \left(\frac{\widetilde{W}^{\#}_t(j)}{\widetilde{W}_{t+s}}\right)^{-\frac{\mathscr{M}_W}{\mathscr{M}_W-1}} L_{t+s}.$$

$$\max_{\widetilde{W}^{\#}_t(j)} \mathbf{E}_t \sum_{s=0}^{\infty} (\phi_W)^s \Lambda^{C,o}_{t,t+s} P^Z_{t+s} \widetilde{\chi}^Z_{t+s}\left[\left(\frac{\widetilde{W}_{t+s}(j)}{\widetilde{\chi}^Z_{t+s} P^Z_{t+s}} - mc^W_{t+s}\right)\left(\frac{\widetilde{W}^{\#}_t(j)}{\widetilde{W}_{t+s}}\right)^{-\frac{\mathscr{M}_W}{\mathscr{M}_W-1}} L_{t+s}\right].$$

Taking the derivative with respect to $\widetilde{W}^{\#}_t(j)$ delivers the familiar wage-inflation schedule

$$w^{\#}_t = \widetilde{W}^{\#}_t/\widetilde{W}_t, \quad 1 = \left(\frac{1-(\phi_W)\left(\zeta^W_t\right)^{\frac{-1}{1-\mathscr{M}_W}}}{1-\phi_W}\right)^{1-\mathscr{M}_W}\left[\left(\frac{f^{W,1}_t \mathscr{M}_W}{f^{W,2}_t}\right)^{1-\omega_W} \cdot \left(\zeta^W_t\right)^{-\omega_W}\right]^{-1}$$

$$f^{W,1}_t = L_t \frac{mc^W_t}{w_t} + \phi_W \Gamma^H \mathbf{E}_t\left[\left(\frac{\Theta_{t+1}}{\Theta_t}\frac{U^{C,o}_{t+1}}{U^{C,o}_t}\frac{1}{\Gamma^Z_{t+1}}\right)\left(\frac{\Pi^W_{t+1}}{\Pi^{CPI}_{t+1}}\right)\left(\zeta^W_{t+1}\right)^{\frac{\mathscr{M}_W}{\mathscr{M}_W-1}} f^{W,1}_{t+1}\right]$$

$$f^{W,2}_t = L_t + \phi_W \Gamma^H \mathbf{E}_t\left[\left(\frac{\Theta_{t+1}}{\Theta_t}\frac{U^{C,o}_{t+1}}{U^{C,o}_t}\frac{1}{\Gamma^Z_{t+1}}\right)\left(\frac{\Pi^W_{t+1}}{\Pi^{CPI}_{t+1}}\right)\left(\zeta^W_{t+1}\right)^{\frac{1}{\mathscr{M}_W-1}} f^{W,2}_{t+1}\right]$$

$$\zeta^W_t = \Pi^W_t/\left(\left(\Pi^W_{ss}\right)^{1-\xi^W}\left(\Pi^W_{t-1}\right)^{\xi_W}\right), \quad \Pi^W_t = w_t/w_{t-1}\Pi^Z_t/\Gamma^Z_t$$

$$\mathscr{D}_t^W = (1-\phi_W)\left(\frac{1-\phi_W\left(\zeta_t^W\right)^{\frac{1}{\mathscr{M}_W-1}}}{1-\phi_W}\right)^{\mathscr{M}_W} + \phi_W\left(\zeta_t^W\right)^{\frac{\mathscr{M}_W}{\mathscr{M}_W-1}}\mathscr{D}_{t-1}^W$$

Wage dispersion is given by $\mathscr{D}_t^W$. Aggregate hours worked in the economy is given by $L_t^s = L_t\mathscr{D}_t^W$.

Log-linearisation of Wage Inflation Equation. This proceeds in the four steps outlined below. Note that the markup shocks are re-scaled.[27]

$$\begin{aligned} mc_t^W &= \tau_t^{\mathscr{M}_W} mrs_t, \quad \Leftrightarrow \quad \widehat{mc}_t^W = \widehat{\tau}_t^{\mathscr{M}_W} + \widehat{mrs}_t, \quad w_t = \Pi_t^W/(\Pi_t^Z\Gamma_t^Z)w_{t-1} \\ \widehat{w}_t &= \widehat{\pi}_t^W - \widehat{\pi}_t^Z - \left(\widehat{\gamma}_t^Z - \widehat{\gamma}_t^L\right) + \widehat{w}_{t-1} \\ \zeta_t^W &= \Pi_t^W/\left(\left(\Pi_{ss}^W\right)^{1-\xi_W}\left(\Pi_{t-1}^W\right)^{\xi_W}\right), \quad \Leftrightarrow \quad \widehat{\zeta}_t^W = \widehat{\pi}_t^W - \xi_W\widehat{\pi}_{t-1}^W \end{aligned} \tag{A.12}$$

$$\begin{aligned} U_t^L &= -\nu_L\varepsilon_t^C\varepsilon_t^L(L_t)^{\varepsilon_L}, \Leftrightarrow \widehat{u}_t^L = \widehat{\varepsilon}_t^C + \widehat{\varepsilon}_t^L + \varepsilon_L\widehat{l}_t \\ \widehat{mrs}_t - \widehat{p}_t^{CPI} &= \widehat{\varepsilon}_t^L + \varepsilon_L\widehat{l}_t + \omega_o\left(\widehat{c}_t^o\frac{\varepsilon_C}{1-\psi_C} - \widehat{c}_{t-1}^o\frac{\psi_C\varepsilon_C}{1-\psi_C}\right) + (1-\omega_o)\left(\widehat{c}_t^{rot}\frac{\varepsilon_C}{1-\psi_C} - \widehat{c}_{t-1}^{rot}\frac{\psi_C\varepsilon_C}{1-\psi_C}\right) \end{aligned} \tag{A.13}$$

$$f_{1t}^{W,1} = \frac{1}{w_t}mc_t^W L_t + (\Gamma^H\phi_W)\mathbf{E}_t\left[\mathbf{B}_{t+1}\frac{U_{t+1}^{C,o}}{U_t^{C,o}}\frac{1}{\Gamma_{t+1}^Z}\frac{\Pi_{t+1}^W}{\Pi_{t+1}^{CPI}}\left(\zeta_{t+1}^{\Pi^W}\right)^{\frac{\mathscr{M}_W}{\mathscr{M}_W-1}}\right]f_{t+1}^{W,1},\ 1-\beta\phi_W\Gamma^H = \frac{\frac{1}{w_{ss}}mc_{ss}^W L_{ss}}{f_{ss}^{W,1}}$$

$$\begin{aligned} \widehat{f}_t^{W,1} &= \left(1-\Gamma^H\phi_W\beta\right)\left(-\widehat{w}_t + \widehat{mc}_t^W + \widehat{l}_t\right) \\ &+ (\Gamma^H\phi_W)\beta\mathbf{E}_t\left[\widehat{b}_{t+1} + \widehat{u}_{t+1}^{C,o} - \widehat{u}_t^{C,o} + \widehat{\pi}_{t+1}^W - \widehat{\pi}_{t+1}^{CPI} - \widehat{\gamma}_{t+1}^Z + \left(\frac{\mathscr{M}_W}{\mathscr{M}_W-1}\right)\widehat{\zeta}_{t+1}^W + \widehat{f}_{t+1}^{W,1}\right] \end{aligned}$$

$$\widehat{f}_t^{W,2} = \left(1-\Gamma^H\phi_W\beta\right)\widehat{l}_t + \Gamma^H\phi_W\beta\mathbf{E}_t\left[\widehat{b}_{t+1} + \widehat{u}_{t+1}^{C,o} - \widehat{u}_t^{C,o} + \widehat{\pi}_{t+1}^W - \widehat{\pi}_{t+1}^{CPI} - \widehat{\gamma}_{t+1}^Z + \frac{\widehat{\zeta}_{t+1}^W}{\mathscr{M}_W-1} + \widehat{f}_{t+1}^{W,2}\right]$$

$$\left[\frac{1-(\phi_W)\left(\zeta_t^W\right)^{\frac{-1}{1-\mathscr{M}_W}}}{1-\phi_W}\right]^{1-\mathscr{M}_W} = \left(\frac{f_t^{W,1}\mathscr{M}_W}{f_t^{W,2}}\right)^{1-\omega_W}\left(\zeta_t^W\right)^{-\omega_W}, \Leftrightarrow \widehat{\zeta}_t^W\left(\frac{\omega_W - \phi_W\omega_W + \phi_W}{(1-\phi_W)(1-\omega_W)}\right) = \widehat{f}_t^{W,1} - \widehat{f}_t^{W,2}$$

$$\begin{aligned} \widehat{\pi}_t^W &= \widehat{\mu}_t^W + \frac{(1-\phi_W)\left(1-\omega_W-(1-\omega_W)\bar{\Gamma}^H\beta\phi_W\right)}{\phi_W\left(1+\bar{\Gamma}^H\beta\xi_W\right)+(1-\phi_W)\omega_W}\left(\widehat{mrs}_t - \widehat{w}_t\right) \\ &+ \frac{\xi_W\left(\phi_W+(1-\phi_W)\omega_W\right)}{\phi_W\left(1+\bar{\Gamma}^H\beta\xi_W\right)+(1-\phi_W)\omega_W}\widehat{\pi}_{t-1}^W + \frac{\beta\bar{\Gamma}^H\phi_W}{\phi_W\left(1+\bar{\Gamma}^H\beta\xi_W\right)+(1-\phi_W)\omega_W}\mathbf{E}_t\widehat{\pi}_{t+1}^W \end{aligned} \tag{A.14}$$

[27] We apply a similar approach to the remaining inflation equations.

A.3 Final-Output Firms

Final-output goods production involves two types of agents: (i) perfectly competitive final-output packers and (ii) monopolistically competitive final output producers.

Final-Output Packers. Final-output packers demand and aggregate infinitely many varieties of final-output goods $\widetilde{Z}_t(i),\ i \in [0,1]$ into a final-output good $\widetilde{Z}_t$. $\widetilde{Z}_t(i)$ denotes the demand for a specific variety i of the final-output good and $\widetilde{Z}_t$ denotes the aggregate demand of the final-output good. ε_z is the elasticity of substitution and $\mathscr{M}_Z = \varepsilon_z/(\varepsilon_z - 1)$ is the corresponding gross markup of monopolistically competitive final-output good producers. Final-output packers purchase a single variety at given prices $P_t^Z(i)$ and sell the final-output good $\widetilde{Z}_t$ at price P_t^Z to a sectoral retailer. The packers' CES production function, and the implied demand schedule associated with the cost minimisation are

$$\widetilde{Z}_t = \left[\int_0^1 \left(\widetilde{Z}_t(i)\right)^{1-\frac{1}{\varepsilon_z}} \mathrm{d}i\right]^{\frac{\varepsilon_z}{\varepsilon_z-1}}, \quad \widetilde{Z}_t(i) = \left(\frac{P_t^Z(i)}{P_t^Z}\right)^{\frac{\mathscr{M}_Z}{1-\mathscr{M}_Z}} \widetilde{Z}_t, \quad P_t^Z \equiv \left(\int_0^1 \left(P_t^Z(i)\right)^{\frac{1}{1-\mathscr{M}_Z}} \mathrm{d}i\right)^{1-\mathscr{M}_Z}$$

where P_t^Z is the price index and optimal behaviour implies $P_t^Z\widetilde{Z}_t = \int_0^1 P_t^Z(i)\widetilde{Z}_t(i)di$.

Final-Output Producers Each variety $\widetilde{Z}_t(i)$ that the final-output good packer demands and assembles is produced and supplied by a single *monopolistically competitive* final-output producer $i \in [0,1]$ according to the final-output CES production function

$$(\widetilde{Z}_t(i))^{\frac{\varepsilon_{ez}-1}{\varepsilon_{ez}}} = (1-\alpha_{ez})^{\frac{1}{\varepsilon_{ez}}} \left(\widetilde{Z}_t^z(i)\right)^{\frac{\varepsilon_{ez}-1}{\varepsilon_{ez}}} + (\alpha_{ez})^{\frac{1}{\varepsilon_{ez}}} \left(\widetilde{E}_t^z(i)\right)^{\frac{\varepsilon_{ez}-1}{\varepsilon_{ez}}} \quad \Leftrightarrow \quad \widehat{z}_t = (1-\alpha_{ez})\widehat{z}_t^z + \alpha_{ez}\widehat{e}_t^z. \tag{A.15}$$

The production inputs demanded by a specific firm i are non-energy final output $\widetilde{Z}_t^z(i)$ and imported energy goods $E_t^z(i)$. We assume that outputs and inputs have the same growth trend. α_{ez} denotes the share of energy in production and ψ_{ez} denotes the elasticity of substitution between non-energy inputs and the imported energy good. Firm i purchases energy imports E_t^z from the energy importer. Each individual final-output producer is subject to *nominal rigidities*. The probability that they cannot reset their price is ϕ_z. We split the firms problem into two steps: (i) the intra-temporal cost minimisation problem and (ii) the inter-temporal price setting problem.

Non-Energy Final Output. Final-output production combines imported energy with 'non-energy final output', which is a combination of domestic value-added, $\widetilde{V}$, and imported non-energy goods, $\widetilde{M}$

$$(\widetilde{Z}_t^z(i))^{\frac{\varepsilon_v-1}{\varepsilon_v}} = (\alpha_v)^{\frac{1}{\varepsilon_v}} \left(\widetilde{V}_t(i)/c_{vz}\right)^{\frac{\varepsilon_v-1}{\varepsilon_v}} + (1-\alpha_v)^{\frac{1}{\varepsilon_v}} \left(\widetilde{M}_t(i)/\widetilde{\chi}_t^M\right)^{\frac{\varepsilon_v-1}{\varepsilon_v}} \Leftrightarrow \widehat{z}_t^z = \alpha_v\widehat{v}_t + (1-\alpha_v)\widehat{m}_t. \tag{A.16}$$

The constant scaling factor c_{vz} ensures that value added and final output have the same growth trend, $\widetilde{\chi}_t^Z = \widetilde{\chi}_t^V$.[28] It is costly to adjust the quantity of the production inputs (ψ_M, ψ_V). Import demand is subject to a shock (ε_t^M). The cost minimisation problem

[28] See further details in Appendix A.11.

of non-energy final output producers is

$$\mathscr{L}_t^{Z^z} = \mathbf{E}_t \sum_{s=0}^{\infty} \frac{\Theta_{t+s}}{\Theta_t} \frac{\widetilde{\Lambda}_{t+s}^{C,o}}{\widetilde{\Lambda}_t^{C,o}} \frac{\widetilde{\chi}_{t+s}^H}{\widetilde{\chi}_t^H} \left\{ -\left(P_{t+s}^V \widetilde{V}_{t+s}(i) \left(1+\Psi_t^V(.)\right) + P_{t+s}^M \widetilde{M}_{t+s}(i) \left\{ \varepsilon_{t+s}^M + \frac{\psi^M}{2} \left(\frac{\Gamma^H \widetilde{M}_{t+s}(i)}{\widetilde{M}_{t-1+s}(i)} - \Gamma^M \Gamma^Z \Gamma^H \right)^2 \right\} \right) \right.$$
$$\left. + \widetilde{MC}_{t+s}^{Z^z}(i) \left(\widetilde{Z}_{t+s}^z(i) - \left((\alpha_v)^{\frac{1}{\varepsilon_v}} \left(\widetilde{V}_{t+s}(i) \right)^{\frac{\varepsilon_v - 1}{\varepsilon_v}} + (1-\alpha_v)^{\frac{1}{\varepsilon_v}} \left(\widetilde{M}_{t+s}(i) \right)^{\frac{\varepsilon_v - 1}{\varepsilon_v}} \right)^{\frac{\varepsilon_v}{\varepsilon_v - 1}} \right) \right\}.$$

Non-Energy Import Demand. Taking the derivative with respect to $\widetilde{M}_t$ delivers

$$0 = -\varepsilon_t^M \left(1 + \frac{\psi^M}{2} \left(\iota_t^M - \Gamma^H \Gamma^Z \Gamma^M \right)^2 + \psi^M \left(\iota_t^M - \Gamma^H \Gamma^Z \Gamma^M \right) \iota_t^M \right) + \widetilde{MC}_t^{Z^z} \frac{\partial \widetilde{Z}_t^z}{\partial \widetilde{M}_t} \frac{1}{P_t^M}$$
$$+ \mathbf{E}_t \left[\mathbf{B}_{t+1} \frac{\widetilde{\Lambda}_{t+1}^{C,o}}{\widetilde{\Lambda}_t^{C,o}} \Pi_{t+1}^M \psi^M \left(\iota_{t+1}^M - \Gamma^H \Gamma^Z \Gamma^M \right) \left(\iota_{t+1}^M \right)^2 \varepsilon_{t+1}^M \right], \quad \frac{\partial \widetilde{Z}_t^z}{\partial \widetilde{M}_t} = (1-\alpha_v)^{\frac{1}{\varepsilon_v}} \left(\frac{Z_t^z}{M_t} \right)^{\frac{1}{\varepsilon_v}} \frac{1}{\widetilde{\chi}_t^M}$$

Log-linearise the optimality condition

$$-\widehat{\varepsilon}_t^M - \frac{\psi^M}{2} (\iota_{ss}^M)^2 (2\widehat{\iota}_t^M) + \psi^M \iota_{ss}^M \Gamma^H \Gamma^Z \Gamma^M (\widehat{\iota}_t^M) - \psi^M (\iota_{ss}^M)^2 \left(2\widehat{\iota}_t^M\right) + \psi^M \Gamma^H \Gamma^Z \Gamma^M \iota_{ss}^M \left(\widehat{\iota}_t^M\right)$$
$$+ \underbrace{\frac{mc_{ss}^{Z^z}}{p_{ss}^M} \left((1-\alpha_v) \frac{Z_{ss}^z}{M_{ss}} \right)^{\frac{1}{\varepsilon_v}}}_{=1, \text{ in ss}} \left(\widehat{mc}_t^{Z^z} - \widehat{p}_t^M + \frac{\widehat{z}_t^z - \widehat{m}_t}{\varepsilon_v} \right) + \frac{\psi^M \beta \left(\iota_{ss}^M\right)^3}{\Gamma^M \Gamma^Z} \mathbf{E}_t \left(\widehat{b}_{t+1} + \widehat{\lambda}_{t+1}^{C,o} - \widehat{\lambda}_t^{C,o} + \widehat{\pi}_{t+1}^M - \widehat{\pi}_{t+1}^Z - \widehat{\gamma}_{t+1}^Z + 3\widehat{\iota}_{t+1}^M + \widehat{\varepsilon}_{t+1}^M \right)$$
$$- \frac{\psi^M \beta}{\Gamma^M \Gamma^Z} \left(\iota_{ss}^M\right)^2 \left(\underbrace{\Gamma^H \Gamma^Z \Gamma^M}_{\iota_{ss}^M} \right) \mathbf{E}_t \left(\widehat{b}_{t+1} + \widehat{\lambda}_{t+1}^{C,o} - \widehat{\lambda}_t^{C,o} + \widehat{\pi}_{t+1}^M - \widehat{\pi}_{t+1}^Z - \widehat{\gamma}_{t+1}^Z + 2\widehat{\iota}_{t+1}^M + \widehat{\varepsilon}_{t+1}^M \right) = 0$$

to obtain the non-energy import demand schedule

$$\widehat{m}_t \left(\psi^M + \psi^M \beta \Gamma^H + \frac{1}{\varepsilon_v (\iota_{ss}^M)^2} \right) = \psi^M (\widehat{m}_{t-1} - \widehat{\gamma}_t^Z) - \widehat{\varepsilon}_t^M \frac{1}{(\iota_{ss}^M)^2} + \frac{1}{(\iota_{ss}^M)^2} \left(\widehat{mc}_t^{Z^z} - \widehat{p}_t^M + \frac{1}{\varepsilon_v} \widehat{z}_t^z \right) + \psi^M \beta \Gamma^H \mathbf{E}_t \left(\widehat{m}_{t+1} + \widehat{\gamma}_{t+1}^Z \right)$$

$$\widehat{m}_t^z = \frac{\psi^M}{\left(\frac{1}{\varepsilon^v (\Gamma^M \Gamma^Z \Gamma^H)^2} + \psi^M (1+\beta \Gamma^H) \right)} \left(\widehat{m}_{t-1}^z - \widehat{\gamma}_t^z \right) + \frac{\psi^M \beta \Gamma^H}{\left(\frac{1}{\varepsilon^v (\Gamma^M \Gamma^Z \Gamma^H)^2} + \psi^M (1+\beta \Gamma^H) \right)} \mathbf{E}_t \left(\widehat{m}_{t+1}^z + \widehat{\gamma}_{t+1}^z \right)$$
$$+ \frac{1}{\left(\frac{1}{\varepsilon^v} + \psi^M (1+\beta \Gamma_{ss}^H) (\Gamma^M \Gamma^Z \Gamma^H)^2 \right)} \left(\widehat{mc}_t^{Z^z} - \widehat{p}_t^m + \widehat{z}_t^z \frac{1}{\varepsilon^v} \right) - \widehat{\varepsilon}_t^M \tag{A.17}$$

where the import demand shock is rescaled for convenience. Note that the auxiliary term for import adjustment ι_t^M is defined as $\iota_t^M = \Gamma^M \Gamma^H M_t / M_{t-1} \Gamma_t^Z$, $\Leftrightarrow$ $\widehat{\iota}_t^M = \widehat{m}_t - \widehat{m}_{t-1} + \widehat{\gamma}_t^Z$.

Value-added Demand. Taking the derivative with respect to $\widetilde{V}_t$ delivers the value-added demand schedule

$$\widehat{v}_t = \frac{\psi^V}{\left(\frac{1}{\varepsilon^v (\Gamma^Z \Gamma^H)^2} + \psi^V (1+\beta \Gamma^H) \right)} \left(\widehat{v}_{t-1} - \widehat{\gamma}_t^z \right) + \frac{\psi^V \beta \Gamma^H}{\left(\frac{1}{\varepsilon^v (\Gamma^Z \Gamma^H)^2} + \psi^V (1+\beta \Gamma^H) \right)} \mathbf{E}_t \left(\widehat{v}_{t+1} + \widehat{\gamma}_{t+1}^z \right)$$
$$+ \frac{1}{\left(\frac{1}{\varepsilon^v} + \psi^V (1+\beta \Gamma_{ss}^H) (\Gamma^Z \Gamma^H)^2 \right)} \left(\widehat{mc}_t^{Z^z} - \widehat{p}_t^V + \widehat{z}_t^z \frac{1}{\varepsilon^v} \right). \tag{A.18}$$

Final-Output Firms' Energy Import Demand Schedule. The Lagrangian is given by

$$\mathscr{L}_t^Z = -\tau_t^Z \left(P_t^{Z^z} \widetilde{Z}_t^z(i) + P_t^E \widetilde{E}_t^z(i) \left(1 + \Psi_t^E(.)\right) \right) + \widetilde{MC}_t^Z(i) \left(\widetilde{Z}_t(i) - \left(P_t^Z(i)/P_t^Z \right)^{-\frac{\mathscr{M}_Z}{\mathscr{M}_Z - 1}} \widetilde{Z}_t \right)$$

and the Lagrange multiplier $\widetilde{MC}_t^Z(i)$ is the (nominal) shadow cost of producing one more unit of final output $\widetilde{Z}_t$, i.e., the nominal marginal cost, and τ_t^Z is a shock to final output marginal costs that is isomorphic to a markup shock.

$$\begin{aligned} \frac{\partial \mathscr{L}_t^Z}{\partial \widetilde{E}_t(i)} = 0, \quad \Leftrightarrow \widehat{e}_t^z \quad &= \quad \frac{\psi^E}{\left(\frac{1}{\varepsilon^{ez}(\Gamma^Z\Gamma^H)^2} + \psi^E\left(1+\beta\Gamma^H\right) \right)} \left(\widehat{e}_{t-1}^z - \widehat{\gamma}_t^z \right) + \frac{\psi^E \beta \Gamma^H}{\left(\frac{1}{\varepsilon^{ez}(\Gamma^Z\Gamma^H)^2} + \psi^E\left(1+\beta\Gamma^H\right) \right)} \mathbf{E}_t \left(\widehat{e}_{t+1}^z + \widehat{\gamma}_{t+1}^z \right) \\ &+ \frac{1}{\left(\frac{1}{\varepsilon^{ez}} + \psi^E\left(1+\beta\Gamma_{ss}^H\right)(\Gamma^Z\Gamma^H)^2 \right)} \left(\widehat{mc}_t^Z - \widehat{p}_t^E + \widehat{z}_t \frac{1}{\varepsilon^v} \right). \end{aligned} \tag{A.19}$$

Final-Output Firms' Marginal Costs. Note that non-energy final-output firms operate under perfect competition ($p_t^{Z^z} = mc_t^{Z^z}$)

$$\frac{\partial \mathscr{L}_t^Z}{\partial \widetilde{Z}_t^z(i)} = 0 \;\Leftrightarrow\; p_t^{Z^z} = \frac{mc_t^Z(i)}{\tau_t^Z} \left((1-\alpha_{ez}) \frac{Z_t(i)}{Z_t^z(i)} \right)^{\frac{1}{\varepsilon_e}}. \tag{A.20}$$

So, it can be shown that: $\widehat{mc}_t^z = \alpha_{zz}\widehat{mc}_t^{Z^z} + (1-\alpha_{zz})\widehat{p}_t^E$.

Rule-of-Thumb Price Z Setting. The objective of final-output firm i is to maximise its nominal profits

$$\widetilde{D}_t^Z(i) = P_t^Z(i)\widetilde{Z}_t(i) - \left\{ \tau_t^{\mathscr{M}_Z} \left(\widetilde{MC}_t^{Z^z} \widetilde{Z}_t^z(i) + P_t^E \widetilde{E}_t^z (1 + \Psi_t^E(.)) \right) \right\}.$$

A fraction ω_Z of 'rule-of-thumb' firms will set their price based on an index of previous-period and steady-state Z inflation. Only the remaining fraction $(1-\omega_Z)$ attempts to implement the optimal price $P_t^{Z,\#}$. An expression for the aggregate price index

$$P_t^Z \quad = \quad \left(\int_F \left(P_t^{Z,\#}(i) \right)^{\frac{1}{1-\mathscr{M}_Z}} di + \int_{\bar{F}} \left(\left[\left(\Pi_{ss}^Z\right)^{(1-\xi_Z)} \left(\Pi_{t-1}^Z\right)^{\xi_Z} \right] P_{t-1}^Z(i) \right)^{\frac{1}{1-\mathscr{M}_Z}} \mathrm{d}i \right)^{1-\mathscr{M}_Z}$$

where F is the set of those firms who can reoptimise their price

$$1 \quad = \quad (1-\phi_Z) \left\{ \left(p_t^{Z,\#} \right)^{1-\omega_Z} \cdot \left(\zeta_t^Z \right)^{-\omega_Z} \right\}^{\frac{1}{1-\mathscr{M}_Z}} + (\phi_Z) \left(\left[\left(\Pi_{ss}^Z\right)^{(1-\xi_Z)} \left(\Pi_{t-1}^Z\right)^{\xi_Z} \right] \frac{1}{\Pi_t^Z} \right)^{\frac{1}{1-\mathscr{M}_Z}}.$$

With probability $1-\phi_Z$ a non-rule-of-thumb firm can re-optimise its price

$$P_t^Z(i) = \begin{cases} P_t^{Z,\#}(i) & \text{with probability: } 1-\phi_Z \\ P_{t-1}^Z(i) \left(\left(\Pi_{ss}^Z\right)^{1-\xi_Z} \left(\Pi_{t-1}^Z\right)^{\xi_Z} \right) & \text{with probability: } \phi_Z \end{cases}$$

where ξ_Z is the weight attached to the previous period Z inflation, Π_t^Z. Consider a firm who can reset its price in the current period $P_t^Z(i) = P_t^{Z,\#}(i)$ and who is then stuck with its price until future period $t+s$. The price in this case would be

$$P_{t+s}^Z(i) = P_t^{Z,\#}(i)\left(\Pi_{ss}^Z\right)^{s(1-\xi_Z)}\left(\prod_{g=0}^{s-1}\left(\left(\Pi_{t+g}^Z\right)^{\xi_Z}\right)\right) = P_t^{Z,\#}(i)\left[\left(\Pi_{ss}^Z\right)^{s(1-\xi_Z)}\left(\frac{P_{t+s-1}^Z}{P_{t-1}^Z}\right)^{\xi_Z}\right].$$

Subject to the above demand constraint and assuming that a firm i always meets the demand for its product at the current price, firms solve the following optimisation problem

$$\max_{P_t^{Z,\#}(i)} \mathbf{E}_t \sum_{s=0}^{\infty} (\phi_Z)^s \Lambda_{t,t+s}^{C,o} P_{t+s}^Z \widetilde{\chi}_{t+s}^Z \left[\left(\frac{P_{t+s}^Z(i)}{\widetilde{\chi}_{t+s}^Z P_{t+s}^Z} - mc_{t+s}^Z\right)\widetilde{Z}_{t+s}(i)\right]$$

$$\text{s.t. } \widetilde{Z}_{t+s}(i) = \left(\frac{P_t^{Z,\#}(i)}{P_{t+s}^Z}\right)^{-\frac{\mathscr{M}_Z}{\mathscr{M}_Z-1}} \widetilde{Z}_{t+s}.$$

Taking the derivative with respect to $P_t^{Z,\#}(i)$ delivers the familiar price inflation schedule

$$\begin{aligned}
1 &= \left(\frac{1-(\phi_Z)\left(\zeta_t^Z\right)^{\frac{-1}{1-\mathscr{M}_Z}}}{1-\phi_Z}\right)^{1-\mathscr{M}_Z}\left[\left(\frac{f_t^{Z,1}\mathscr{M}_Z}{f_t^{Z,2}}\right)^{1-\omega_Z}\cdot\left(\zeta_t^Z\right)^{-\omega_Z}\right]^{-1} \\
f_t^{Z,1} &= Z_t mc_t^Z + \phi_Z \mathbf{E}_t\left[\mathbf{B}_{t+1}\frac{U_{u,t+1}^{C,o}}{U_{u,t}^{C,o}}\frac{1}{\Gamma_{t+1}^Z}\frac{\Pi_{t+1}^Z}{\Pi_{t+1}^{CPI}}\left(\zeta_{t+1}^Z\right)^{\frac{\mathscr{M}_Z}{\mathscr{M}_Z-1}} f_{t+1}^{Z,1}\right] \\
f_t^{Z,2} &= Z_t + \phi_Z \mathbf{E}_t\left[\mathbf{B}_{t+1}\frac{U_{u,t+1}^{C,o}}{U_{u,t}^{C,o}}\frac{1}{\Gamma_{t+1}^Z}\frac{\Pi_{t+1}^Z}{\Pi_{t+1}^{CPI}}\left(\zeta_{t+1}^Z\right)^{\frac{1}{\mathscr{M}_Z-1}} f_{t+1}^{Z,2}\right],\ \zeta_t^Z \equiv \frac{\Pi_t^Z}{(\Pi_{ss}^Z)^{1-\xi_Z}(\Pi_{t-1}^Z)^{\xi_Z}} \\
\mathscr{D}_t^Z &= (1-\phi_Z)\left(\left(1-\phi_Z\left(\zeta_t^Z\right)^{\frac{1}{\mathscr{M}_Z-1}}\right)/(1-\phi_Z)\right)^{\mathscr{M}_Z} + \phi_Z\left(\zeta_t^Z\right)^{\frac{\mathscr{M}_Z}{\mathscr{M}_Z-1}}\mathscr{D}_{t-1}^Z.
\end{aligned}$$

Aggregation implies $\int_0^1 Z_t(i)\mathrm{d}i = \int_0^1\left(\frac{P_t(i)}{P_t}\right)^{-\frac{\mathscr{M}_Z}{\mathscr{M}_Z-1}} Z_t\mathrm{d}i = Z_t\int_0^1\left(\frac{P_t(i)}{P_t}\right)^{-\frac{\mathscr{M}_Z}{\mathscr{M}_Z-1}}\mathrm{d}i$ where we define price dispersion as $\mathscr{D}_t^Z \equiv \int_0^1\left(\frac{P_t(i)}{P_t}\right)^{-\frac{\mathscr{M}_Z}{\mathscr{M}_Z-1}}\mathrm{d}i$ which can be written recursively.

Log-linear Final-Output Inflation Equation. This is given by

$$\begin{aligned}
\widehat{\pi}_t^Z &= \widehat{\mu}_t^Z + \widehat{\mu}_t^{Z,temp} + \frac{(1-\phi_Z)\left(1-\omega_Z-(1-\omega_Z)\,\Gamma_{ss}^H\beta\,\phi_Z\right)}{\phi_Z\,(1+\Gamma_{ss}^H\beta\,\xi_Z)+(1-\phi_Z)\,\omega_Z}\widehat{mc}_t^Z \\
&\quad + \frac{\xi_Z\,(\phi_Z+(1-\phi_Z)\,\omega_Z)}{\phi_Z\,(1+\Gamma_{ss}^H\beta\,\xi_Z)+(1-\phi_Z)\,\omega_Z}\,\widehat{\pi}_{t-1}^Z \\
&\quad + \mathbf{E}_t\widehat{\pi}_{t+1}^Z\,\frac{\beta\,\Gamma_{ss}^H\,\phi_Z}{\phi_Z\,(1+\Gamma_{ss}^H\beta\,\xi_Z)+(1-\phi_Z)\,\omega_Z}.
\end{aligned} \tag{A.21}$$

Note that the final-output price markup shock consists of two terms.

A.4 Value-added Firms

As described above, value-added goods are purchased by non-energy final-output producers and used as production inputs. Value-added goods production involves two types of agents: (i) perfectly competitive value-added output packers and (ii) monopolistically competitive value-added output producers.

Value-added Output Packers. Value-added output packers demand and aggregate infinitely many varieties of value-added output goods $\widetilde{V}_t(i),\ i \in [0,1]$ into an aggregate value-added output good $\widetilde{V}_t$. $\widetilde{V}_t(i)$ denotes the demand for a specific variety i of the value-added output and $\widetilde{V}_t$ denotes the aggregate demand of the value-added output. ε_v is the elasticity of substitution and $\mathscr{M}_V = \varepsilon_V/(\varepsilon_V - 1)$ is the corresponding gross markup of monopolistically competitive value-added output producers. Value-added output packers purchase a single variety at given prices $P_t^V(i)$ and sell the value-added output $\widetilde{V}_t$ at price P_t^V to the final non-energy producer. The packers' CES production function, and the implied demand schedule associated with the cost minimisation are

$$\widetilde{V}_t = \left[\int_0^1 \left(\widetilde{V}_t(i)\right)^{1-\frac{1}{\varepsilon_V}} \mathrm{d}i\right]^{\frac{\varepsilon_V}{\varepsilon_V - 1}}, \quad \widetilde{V}_t(i) = \left(\frac{P_t^V(i)}{P_t^V}\right)^{\frac{\mathscr{M}_V}{1-\mathscr{M}_V}} \widetilde{V}_t, \quad P_t^V \equiv \left(\int_0^1 \left(P_t^V(i)\right)^{\frac{1}{1-\mathscr{M}_V}} \mathrm{d}i\right)^{1-\mathscr{M}_V}$$

where P_t^V is the price index and optimal behaviour implies $P_t^V \widetilde{V}_t = \int_0^1 P_t^V(i)\widetilde{V}_t(i)\mathrm{d}i$.

Value-added Output Producers. Each variety $\widetilde{V}_t(i)$ that the value-added output packer demands and assembles is produced and supplied by a single *monopolistically competitive* value-added output producer $i \in [0,1]$ according to the value-added output production function which combines capital and labour and which is subject to a TFP shock (ε_t^{TFP})

$$\begin{aligned} \widetilde{V}_t(i) &= \varepsilon_t^{TFP}\left(\widetilde{K}_{t-1}(i)\right)^{1-\alpha_L}\left(\widetilde{\chi}_t^{LAP,d} L_t(i)\right)^{\alpha_L}, \qquad \widetilde{\chi}_t^V = \left(\widetilde{\chi}_t^Z \widetilde{\chi}_t^I\right)^{1-\alpha_L}\left(\widetilde{\chi}_t^{LAP,d}\right)^{\alpha_L} \\ V_t(i) &= \varepsilon_t^{TFP}\left(\frac{K_{t-1}(i)}{\Gamma^I \Gamma_t^Z}\right)^{1-\alpha_L}\left(L_t(i)\right)^{\alpha_L}, \quad \widehat{v}_t = \widehat{\varepsilon}_t^{TFP} + (1-\alpha_L)(\widehat{k}_{t-1} - \widehat{\gamma}_t) + \alpha_L \widehat{l}_t \end{aligned} \tag{A.22}$$

The production inputs demanded by a specific firm i are capital $\widetilde{K}_{t-1}(i)$ and labour $L_t(i)$. α_L denotes the share of labour in value-added production. The Lagrangian for the cost minimisation problem is

$$\mathscr{L}_t^V = -\tau_t^{\mathscr{M}_V}\left(\widetilde{W}_t L_t(i)\left(1 + \psi^L\left(\frac{L_t(i)}{L_{t-1}(i)} - \Gamma_{ss}^L\right)^2\right) + \widetilde{R}_t^K \widetilde{K}_{t-1}(i)\right) + \widetilde{MC}_t^V(i)\left((\varepsilon_t^{TFP})\left(\widetilde{K}_{t-1}(i)\right)^{1-\alpha_L}\left(\widetilde{\chi}_t^{LAP,d} L_t(i)\right)^{\alpha_L} - \left(\frac{P_t^V(i)}{P_t^V}\right)^{-\frac{\mathscr{M}_V}{\mathscr{M}_V - 1}} \widetilde{V}_t\right), \quad \Gamma_{ss}^L = 1$$

and the Lagrange multiplier $\widetilde{MC}_t^V(i)$ is the (nominal) shadow cost of producing one more unit of value-added output, i.e., the nominal marginal cost and $\tau_t^{\mathscr{M}_V}$ is a shock to value-added marginal costs that is isomorphic to a markup shock.

Labour Demand. Firm i purchases labour services from the union. Note that $mc_t^V \equiv \widetilde{MC}_t^V/(P_t^V \tau_t^{\mathscr{M}_V})$, $p_t^V \equiv P_t^V/P_t^Z$ and $w_t \equiv \widetilde{W}_t/(P_t^Z \widetilde{\chi}_t^Z)$, under $\psi^L = 0$, in the absence of labour adjustment costs we would get:

$$w_t/p_t^V = \alpha_L mc_t^V V_t(i)/L_t(i), \quad \Leftrightarrow \quad \widehat{w}_t = \widehat{p}_t^v + \widehat{mc}_t^v + \widehat{v}_t - \widehat{l}_t,$$

which, in the presence of quadratic labour adjustment costs, generalises to:

$$\widehat{l}_t = \psi^L \frac{\widehat{l}_{t-1} - \widehat{\gamma}_t^L}{1+\psi^L(1+\beta\Gamma^H)} + \psi^L \beta \Gamma^H m^f \frac{E_t \widehat{l}_{t+1} + E_t \widehat{\gamma}_{t+1}^L}{1+\psi^L(1+\beta\Gamma^H)} + \frac{\widehat{v}_t + \widehat{p}_t^v + \widehat{mc}_t^v - \widehat{w}_t}{1+\psi^L(1+\beta\Gamma^H)} \tag{A.23}$$

Capital Demand. Firm i rents capital services from optimising households who own capital

$$\tau_t^{\mathscr{M}_V} \widetilde{R}_t^K = (1-\alpha_L) \widetilde{MC}_t^V (\varepsilon_t^{TFP}) \left(\widetilde{\chi}_t^{LAP}\right)^{\alpha_L} \left(\widetilde{K}_{t-1}(i)/L_t(i)\right)^{\alpha_L} \tag{A.24}$$

$$\widehat{r}_t^k = \widehat{p}_t^v + \widehat{mc}_t^v + \widehat{v}_t - (\widehat{k}_{t-1} - \widehat{\gamma}_t^z). \tag{A.25}$$

Rule-of-Thumb Price V Setting. The objective of value-added firm i is to maximise its nominal profits

$$\widetilde{D}_t^V(i) = P_t^V(i) \widetilde{V}_t(i) - \left\{ \tau_t^{\mathscr{M}_V} \left(\widetilde{W}_t L_t(i) + \widetilde{R}_t^K \widetilde{K}_{t-1}(i) \right) \right\}.$$

A fraction ω_V of 'rule-of-thumb' firms will set their price based on an index of previous-period and steady-state V inflation. Only the remaining fraction $(1-\omega_V)$ attempts to implement the optimal price $P_t^{V,\#}$. An expression for the aggregate price index

$$P_t^V = \left(\int_F \left(P_t^{V,\#}(i)\right)^{\frac{1}{1-\mathscr{M}_V}} \mathrm{d}i + \int_{\bar{F}} \left(\left[\left(\Pi_{ss}^V\right)^{(1-\xi_V)} \left(\Pi_{t-1}^V\right)^{\xi_V} \right] P_{t-1}^V(i) \right)^{\frac{1}{1-\mathscr{M}_V}} \mathrm{d}i \right)^{1-\mathscr{M}_V}$$

where F is the set of those firms who can reoptimise their price

$$1 = (1-\phi_V) \left\{ \left(p_t^{V,\#}\right)^{1-\omega_V} \cdot \left(\zeta_t^V\right)^{-\omega_V} \right\}^{\frac{1}{1-\mathscr{M}_V}} + (\phi_V) \left(\left[\left(\Pi_{ss}^V\right)^{(1-\xi_V)} \left(\Pi_{t-1}^V\right)^{\xi_V} \right] \frac{1}{\Pi_t^V} \right)^{\frac{1}{1-\mathscr{M}_V}}.$$

With probability $1-\phi_V$ a non-rule-of-thumb firm can re-optimise its price

$$P_t^V(i) = \begin{cases} P_t^{V,\#}(i) & \text{with probability: } 1-\phi_V \\ P_{t-1}^V(i) \left(\left(\Pi_{ss}^V\right)^{1-\xi_V} \left(\Pi_{t-1}^V\right)^{\xi_V} \right) & \text{with probability: } \phi_V \end{cases}$$

where ξ_V is the weight attached to the previous period V inflation, $\Pi_t^V = P_t^V / P_{t-1}^V$. Consider a firm who can reset its price in the current period $P_t^V(i) = P_t^{V,\#}(i)$ and who is then stuck with its price until future period $t+s$. The price in this case would be

$$P_{t+s}^V(i) = P_t^{V,\#}(i) \left(\Pi_{ss}^V\right)^{s(1-\xi_V)} \left(\prod_{g=0}^{s-1} \left(\left(\Pi_{t+g}^V\right)^{\xi_V} \right) \right) = P_t^{V,\#}(i) \left[\left(\Pi_{ss}^V\right)^{s(1-\xi_V)} \left(\frac{P_{t+s-1}^V}{P_{t-1}^V} \right)^{\xi_V} \right].$$

Subject to the above demand constraint and assuming that a firm i always meets the demand for its product at the current price, firms solve the following optimisation problem

$$\max_{P_t^{V,\#}(i)} \mathbf{E}_t \sum_{s=0}^{\infty} (\phi_V)^s \Lambda_{t,t+s}^{C,o} P_{t+s}^Z \widetilde{\chi}_{t+s}^Z \left[\left(P_{t+s}^V(i) - \widetilde{MC}_{t+s}^V \right) \frac{\widetilde{V}_{t+s}(i)}{\widetilde{\chi}_{t+s}^Z P_{t+s}^Z} \right] \text{ s.t. } \widetilde{V}_{t+s}(i) = \left(\frac{P_t^{V,\#}(i)}{P_{t+s}^V} \right)^{-\frac{\mathscr{M}_V}{\mathscr{M}_V - 1}} \widetilde{V}_{t+s}.$$

Taking the derivative with respect to $P_t^{V,\#}(i)$ delivers the familiar price inflation schedule

$$\begin{aligned}
1 &= \left(\frac{1 - (\phi_V) \left(\zeta_t^V \right)^{\frac{-1}{1-\mathscr{M}_V}}}{1 - \phi_V} \right)^{1-\mathscr{M}_V} \left[\left(\frac{f_t^{V,1} \mathscr{M}_V}{f_t^{V,2}} \right)^{1-\omega_V} \cdot \left(\zeta_t^V \right)^{-\omega_V} \right]^{-1}, \quad \zeta_t^V \equiv \frac{\Pi_t^V}{(\Pi_{ss}^V)^{1-\xi_V} (\Pi_{t-1}^V)^{\xi_V}} \\
f_t^{V,1} &= V_t mc_t^V + \phi_V \Gamma^H \mathbf{E}_t \left[\mathbf{B}_{t+1} \frac{U_{u,t+1}^{C,o}}{U_{u,t}^{C,o}} \frac{\Pi_{t+1}^V}{\Pi_{t+1}^{CPI}} \left(\zeta_{t+1}^V \right)^{\frac{\mathscr{M}_V}{\mathscr{M}_V - 1}} f_{t+1}^{V,1} \right] \\
f_t^{V,2} &= V_t + \phi_V \Gamma^H \mathbf{E}_t \left[\mathbf{B}_{t+1} \frac{U_{u,t+1}^{C,o}}{U_{u,t}^{C,o}} \frac{\Pi_{t+1}^V}{\Pi_{t+1}^{CPI}} \left(\zeta_{t+1}^V \right)^{\frac{1}{\mathscr{M}_V - 1}} f_{t+1}^{V,2} \right] \\
\mathscr{D}_t^V &= (1 - \phi_V) \left(\left(1 - \phi_V \left(\zeta_t^V \right)^{\frac{1}{\mathscr{M}_V - 1}} \right) / (1 - \phi_V) \right)^{\mathscr{M}_V} + \phi_V \left(\zeta_t^V \right)^{\frac{\mathscr{M}_V}{\mathscr{M}_V - 1}} \mathscr{D}_{t-1}^V.
\end{aligned}$$

Aggregation implies $\int_0^1 V_t(i) \mathrm{d}i = \int_0^1 \left(\frac{P_t^V(i)}{P_t^V} \right)^{-\frac{\mathscr{M}_V}{\mathscr{M}_V - 1}} V_t \mathrm{d}i = V_t \int_0^1 \left(\frac{P_t^V(i)}{P_t^V} \right)^{-\frac{\mathscr{M}_V}{\mathscr{M}_V - 1}} \mathrm{d}i$ where we define price dispersion as $\mathscr{D}_t^V \equiv \int_0^1 \left(\frac{P_t^V(i)}{P_t^V} \right)^{-\frac{\mathscr{M}_V}{\mathscr{M}_V - 1}} \mathrm{d}i$ which can be written recursively. Also note that

$$\Pi_t^V = p_t^V / p_{t-1}^V \Pi_t^Z, \quad \Leftrightarrow \quad \widehat{\pi}_t^V = \widehat{p}_t^V - \widehat{p}_{t-1}^V + \widehat{\pi}_t^Z. \tag{A.26}$$

Log-linear Value-added Inflation Equation. This is given by

$$\begin{aligned}
\widehat{\pi}_t^V = \widehat{\mu}_t^V &+ \frac{(1 - \phi_V) \left(1 - \omega_V - (1 - \omega_V) \Gamma_{ss}^H \beta \phi_V \right)}{\phi_V (1 + \Gamma_{ss}^H \beta \xi_V) + (1 - \phi_V) \omega_V} \widehat{mc}_t^V + \frac{\xi_V (\phi_V + (1 - \phi_V) \omega_V)}{\phi_V (1 + \Gamma_{ss}^H \beta \xi_V) + (1 - \phi_V) \omega_V} \widehat{\pi}_{t-1}^V \\
&+ \frac{\beta \Gamma_{ss}^H \phi_V}{\phi_V (1 + \Gamma_{ss}^H \beta \xi_V) + (1 - \phi_V) \omega_V} \mathbf{E}_t \widehat{\pi}_{t+1}^V.
\end{aligned} \tag{A.27}$$

A.5 Non-Energy Import Firms

Non-energy imports are purchased by non-energy fina- output producers and used as production inputs. Non-energy import 'production' involves two types of agents: (i) perfectly competitive packers and (ii) monopolistically competitive import firms.

Non-Energy Import Packers. Non-energy import packers demand and aggregate infinitely many varieties of non-energy imports $\widetilde{M}_t(i),\ i \in [0,1]$ into an aggregate import good $\widetilde{M}_t$. $\widetilde{M}_t(i)$ denotes the demand for a specific variety i of the import and $\widetilde{M}_t$ denotes the aggregate demand. ε_M is the elasticity of substitution and $\mathscr{M}_M = \varepsilon_M / (\varepsilon_M - 1)$ is the corresponding gross markup of monopolistically competitive import firms. Import packers purchase a single variety at given prices $P_t^M(i)$ and sell the import good $\widetilde{M}_t$ at price P_t^M to the final non-energy producer. The packers' CES production function, and the implied demand schedule associated with the cost minimisation are

$$\widetilde{M}_t = \left[\int_0^1 \left(\widetilde{M}_t(i)\right)^{1-\frac{1}{\varepsilon_M}} \mathrm{d}i\right]^{\frac{\varepsilon_M}{\varepsilon_M-1}}, \quad \widetilde{M}_t(i) = \left(\frac{P_t^M(i)}{P_t^M}\right)^{\frac{\mathscr{M}_M}{1-\mathscr{M}_M}} \widetilde{M}_t, \quad P_t^M \equiv \left(\int_0^1 \left(P_t^M(i)\right)^{\frac{1}{1-\mathscr{M}_M}} \mathrm{d}i\right)^{1-\mathscr{M}_M}$$

where P_t^M is the price index and optimal behaviour implies $P_t^M\widetilde{M}_t = \int_0^1 P_t^M(i)\widetilde{M}_t(i)di$.

Non-Energy Import Producers. Each non-energy import variety $\widetilde{M}_t(i)$ that the packer demands and assembles is supplied by a single *monopolistically competitive* import firm $i \in [0,1]$. They buy a homogenous tradeable non-energy good on the world market from foreign non-energy exporters at the world non-energy export price $P_t^{X^F}$ which is denominated in foreign currency. In order to transform that price into domestic currency units one has to divide by the nominal exchange rate. The importers then transform and differentiate the homogenous good they purchased. This transformation can be described by the following 'production' function

$$\widetilde{M}_t(i) = \widetilde{X}_t^F(i).$$

The Lagrangian for the cost minimisation problem is

$$\mathscr{L}_t^M = -\tau_t^{\mathscr{M}_M}\left(P_t^{X^F}/\mathscr{E}_t\widetilde{X}_t^F(i)\right) + \widetilde{MC}_t^M(i)\left(\widetilde{X}_t^F(i)\right)$$

and the Lagrange multiplier $\widetilde{MC}_t^M(i)$ is the (nominal) shadow cost of producing one more unit of imports, e.g. the nominal marginal cost and $\tau_t^{\mathscr{M}_M}$ is a shock to non-energy import marginal costs that is isomorphic to a markup shock.

Domestic Demand for World Non-Energy Exports. Recall that $P_t^Z/\widetilde{\chi}_t^M p_t^M = P_t^M$ and also note that $mc_t^M \equiv \widetilde{MC}_t^M/(P_t^M\tau_t^{\mathscr{M}_M})$. Under the assumption that imports and world-level exports have the same trend we get

$$P_t^M mc_t^M = \frac{P_t^{X^F}}{\mathscr{E}_t}, \; mc_t^M = \frac{\frac{P_t^{V^F}}{\widetilde{\chi}_t^{X^F}} p_t^{X^F}}{\frac{P_t^Z}{\widetilde{\chi}_t^M} p_t^M} \frac{1}{Q_t \frac{P_t^{V^F}}{P_t^Z}}, \; mc_t^M = \frac{p_t^{X^F}}{p_t^M}\frac{1}{Q_t} \Leftrightarrow \widehat{mc}_t^m = \widehat{p}_t^{xf} - \widehat{q}_t - \widehat{p}_t^m \quad \text{(A.28)}$$

Moreover, we assume that the foreign export price level follows the exogenous process

$$p_t^{X^F} = \left(p_{ss}^{X^F}\right)^{1-\rho_{pXF}}\left(p_{t-1}^{X^F}\right)^{\rho_{pXF}} \varepsilon_t^{pxf}, \quad \log\varepsilon_t^{pxf} = (1-\rho_{pxf})^2\sigma_{pxf}\eta_t^{pxf}, \; \eta_t^{pxf} \sim N(0,1).$$

Also, note that

$$P_t^{X^F} = \frac{P_t^{V^F}}{\widetilde{\chi}_t^{X^F}} P_t^{X^F}, \quad p_t^{X^F} = \Gamma^{X^F}\frac{\Pi_t^{X^F}}{\Pi_t^{Z^F}} p_{t-1}^{X^F}, \quad \Leftrightarrow \quad \widehat{p}_t^{xf} = \widehat{\pi}_t^{xf} + \widehat{p}_{t-1}^{xf} - \widehat{\pi}_t^{vf} \quad \text{(A.29)}$$

Rule-of-Thumb Price M Setting. The objective of non-energy importer i is to maximise nominal profits

$$\widetilde{D}_t^M(i) = P_t^M(i)\widetilde{M}_t(i) - \left\{ \tau_t^{\mathscr{M}_M} \left(P_t^{X^F} / \mathscr{E}_t \widetilde{M}_t(i) \right) \right\}.$$

A fraction ω_M of 'rule-of-thumb' firms will set their price based on an index of previous-period and steady-state M inflation. Only the remaining fraction $(1-\omega_M)$ attempts to implement the optimal price $P_t^{M,\#}$. An expression for the aggregate price index is given by

$$P_t^M = \left(\int_F \left(P_t^{M,\#}(i) \right)^{\frac{1}{1-\mathscr{M}_M}} di + \int_{\bar{F}} \left(\left[\left(\Pi_{ss}^M \right)^{(1-\xi_M)} \left(\Pi_{t-1}^M \right)^{\xi_M} \right] P_{t-1}^M(i) \right)^{\frac{1}{1-\mathscr{M}_M}} di \right)^{1-\mathscr{M}_M}$$

where F is the set of those firms who can reoptimise their price

$$1 = (1-\phi_M) \left\{ \left(p_t^{M,\#} \right)^{1-\omega_M} \cdot \left(\zeta_t^M \right)^{-\omega_M} \right\}^{\frac{1}{1-\mathscr{M}_M}} + (\phi_M) \left(\left[\left(\Pi_{ss}^M \right)^{(1-\xi_M)} \left(\Pi_{t-1}^M \right)^{\xi_M} \right] \frac{1}{\Pi_t^M} \right)^{\frac{1}{1-\mathscr{M}_M}}.$$

With probability $1-\phi_M$ a non-rule-of-thumb firm can reoptimise its price

$$P_t^M(i) = \begin{cases} P_t^{M,\#}(i) & \text{with probability: } 1-\phi_M \\ P_{t-1}^M(i) \left(\left(\Pi_{ss}^M \right)^{1-\xi_M} \left(\Pi_{t-1}^M \right)^{\xi_M} \right) & \text{with probability: } \phi_M \end{cases}$$

where ξ_M is the weight attached to the previous period M inflation, $\Pi_t^M = P_t^M / P_{t-1}^M$. Consider a firm who can reset its price in the current period $P_t^M(i) = P_t^{M,\#}(i)$ and who is then stuck with its price until future period $t+s$. The price in this case would be

$$P_{t+s}^M(i) = P_t^{M,\#}(i) \left(\Pi_{ss}^M \right)^{s(1-\xi_M)} \left(\prod_{g=0}^{s-1} \left(\left(\Pi_{t+g}^M \right)^{\xi_M} \right) \right) = P_t^{M,\#}(i) \left[\left(\Pi_{ss}^M \right)^{s(1-\xi_M)} \left(\frac{P_{t+s-1}^M}{P_{t-1}^M} \right)^{\xi_M} \right].$$

Subject to the above derived demand constraint and assuming that a firm i always meets the demand for its product at the current price, firms solve the following optimisation problem

$$\max_{P_t^{M,\#}(i)} \mathbf{E}_t \sum_{s=0}^{\infty} (\phi_M)^s \Lambda_{t,t+s}^{C,o} P_{t+s}^Z \widetilde{\chi}_{t+s}^Z \left[\left(P_{t+s}^M(i) - \widetilde{MC}_{t+s}^M \right) \frac{\widetilde{M}_{t+s}(i)}{\widetilde{\chi}_{t+s}^Z P_{t+s}^Z} \right]$$

$$\text{s.t. } \widetilde{M}_{t+s}(i) = \left(\frac{P_t^{M,\#}(i)}{P_{t+s}^M} \right)^{-\frac{\mathscr{M}_M}{\mathscr{M}_M - 1}} \widetilde{M}_{t+s}.$$

Taking the derivative with respect to $P_t^{M,\#}(i)$ delivers the familiar price inflation schedule

$$
\begin{aligned}
1 &= \left(\frac{1-(\phi_M)\left(\zeta_t^M\right)^{\frac{-1}{1-\mathscr{M}_M}}}{1-\phi_M}\right)^{1-\mathscr{M}_M} \left[\left(\frac{f_t^{M,1}\mathscr{M}_M}{f_t^{M,2}}\right)^{1-\omega_M} \cdot \left(\zeta_t^M\right)^{-\omega_M}\right]^{-1}, \quad \zeta_t^M \equiv \frac{\Pi_t^M}{(\Pi_{ss}^M)^{1-\xi_M}(\Pi_{t-1}^M)^{\xi_M}} \\
f_t^{M,1} &= M_t mc_t^M + \phi_M \Gamma^H \Gamma^M \mathbf{E}_t \left[\mathbf{B}_{t+1} U_{u,t+1}^{C,o}/U_{u,t}^{C,o} \Pi_{t+1}^M / \Pi_{t+1}^{CPI} \left(\zeta_{t+1}^M\right)^{\frac{\mathscr{M}_M}{\mathscr{M}_M-1}} f_{t+1}^{M,1}\right] \\
f_t^{M,2} &= M_t + \phi_M \Gamma^H \Gamma^M \mathbf{E}_t \left[\mathbf{B}_{t+1} U_{u,t+1}^{C,o}/U_{u,t}^{C,o} \Pi_{t+1}^M / \Pi_{t+1}^{CPI} \left(\zeta_{t+1}^M\right)^{\frac{1}{\mathscr{M}_M-1}} f_{t+1}^{M,2}\right] \\
\mathscr{D}_t^M &= (1-\phi_M)\left(\left(1-\phi_M\left(\zeta_t^M\right)^{\frac{1}{\mathscr{M}_M-1}}\right)/(1-\phi_M)\right)^{\mathscr{M}_M} + \phi_M \left(\zeta_t^M\right)^{\frac{\mathscr{M}_M}{\mathscr{M}_M-1}} \mathscr{D}_{t-1}^M.
\end{aligned}
$$

Aggregation implies $\int_0^1 M_t(i)\mathrm{d}i = \int_0^1 \left(\frac{P_t^M(i)}{P_t^M}\right)^{-\frac{\mathscr{M}_M}{\mathscr{M}_M-1}} M_t \mathrm{d}i = M_t \int_0^1 \left(\frac{P_t^M(i)}{P_t^M}\right)^{-\frac{\mathscr{M}_M}{\mathscr{M}_M-1}} \mathrm{d}i$ where we define price dispersion as $\mathscr{D}_t^M \equiv \int_0^1 \left(\frac{P_t^M(i)}{P_t^M}\right)^{-\frac{\mathscr{M}_M}{\mathscr{M}_M-1}} \mathrm{d}i$ which can be written recursively. Also note that

$$
\Pi_t^M = p_t^M / p_{t-1}^M \Pi_t^Z, \quad \Leftrightarrow \quad \widehat{\pi}_t^M = \widehat{p}_t^M - \widehat{p}_{t-1}^M + \widehat{\pi}_t^Z. \tag{A.30}
$$

Log-linear Non-Eenergy Import Inflation Equation. This is given by

$$
\begin{aligned}
\widehat{\pi}_t^M &= \widehat{\mu}_t^M + \frac{(1-\phi_M)\left(1-\omega_M-(1-\omega_M)\,\Gamma_{ss}^H\,\beta\,\phi_M\right)}{\phi_M\,(1+\Gamma_{ss}^H\,\beta\,\xi_M)+(1-\phi_M)\,\omega_M}\widehat{mc}_t^M + \frac{\xi_M\,(\phi_M+(1-\phi_M)\,\omega_M)}{\phi_M\,(1+\Gamma_{ss}^H\,\beta\,\xi_M)+(1-\phi_M)\,\omega_M}\widehat{\pi}_{t-1}^M \\
&\quad + \frac{\beta\,\Gamma_{ss}^H\,\phi_M}{\phi_M\,(1+\Gamma_{ss}^H\,\beta\,\xi_M)+(1-\phi_M)\,\omega_M}\mathbf{E}_t\,\widehat{\pi}_{t+1}^M.
\end{aligned} \tag{A.31}
$$

A.6 Energy Import Firms

Energy imports are purchased by final output firms for production and by energy retailers for consumption. Energy import ‘production’ involves two types of agents: (i) perfectly competitive packers and (ii) monopolistically competitive import firms.

Energy Import Packers. Energy import packers demand and aggregate infinitely many varieties of energy imports $\widetilde{E}_t(i),\ i \in [0,1]$ into an aggregate energy import good $\widetilde{E}_t$. $\widetilde{E}_t(i)$ denotes the demand for a specific variety i of the energy import and $\widetilde{E}_t$ denotes the aggregate demand. ε_e is the elasticity of substitution and $\mathscr{M}_E = \varepsilon_e/(\varepsilon_e - 1)$ is the corresponding gross markup of monopolistically competitive import firms. Import packers purchase a single variety at given prices $P_t^E(i)$ and sell the energy import good at price P_t^E to the final output producer and retailer. The packers’ CES production function, and the implied demand schedule associated with the cost minimisation are

$$
\widetilde{E}_t = \left[\int_0^1 \left(\widetilde{E}_t(i)\right)^{1-\frac{1}{\varepsilon_e}} \mathrm{d}i\right]^{\frac{\varepsilon_e}{\varepsilon_e-1}}, \quad \widetilde{E}_t(i) = \left(\frac{P_t^E(i)}{P_t^E}\right)^{\frac{\mathscr{M}_E}{1-\mathscr{M}_E}} \widetilde{E}_t, \quad P_t^E \equiv \left(\int_0^1 \left(P_t^E(i)\right)^{\frac{1}{1-\mathscr{M}_E}} \mathrm{d}i\right)^{1-\mathscr{M}_E}
$$

where P_t^E is the price index and optimal behaviour implies $P_t^E \widetilde{E}_t = \int_0^1 P_t^E(i) \widetilde{E}_t(i) \mathrm{d}i$. Note that $\widetilde{E}_t^c + \widetilde{E}_t^z = \widetilde{E}_t$.

Energy Import Producers. Each energy import variety $\widetilde{E}_t(i)$ that the energy packer demands and assembles is supplied by a single *monopolistically competitive* energy import firm $i \in [0,1]$. They buy a homogenous tradeable energy good on the world market from foreign energy exporters at the global energy export price $P_t^{E,F}$ which is denominated in foreign currency. In order to transform that price into domestic currency units one has to divide by the nominal exchange rate. The energy importers then transform and differentiate the homogenous good they purchased. This transformation can be described by the following 'production' function

$$\widetilde{E}_t(i) = \widetilde{E}_t^F(i).$$

The Lagrangian for the cost minimisation problem is

$$\mathscr{L}_t^E = -\left(P_t^{E,F}/\mathscr{E}_t \widetilde{E}_t^F(i)\right) + \widetilde{MC}_t^E(i)\left(\widetilde{E}_t^F(i)\right)$$

and the Lagrange multiplier $\widetilde{MC}_t^E(i)$ is the (nominal) shadow cost of producing one more unit of imports, e.g. the nominal marginal cost.

Domestic Demand for World Energy Exports. Note that $p_t^E \equiv P_t^E/P_t^Z$ and also note that $mc_t^E \equiv \widetilde{MC}_t^E/P_t^E$, so

$$P_t^E mc_t^E = \frac{P_t^{E^F}}{\mathscr{E}_t}, \;\; mc_t^E = \frac{P_t^{V^F} p_t^{E^F}}{P_t^Z p_t^E} \frac{1}{Q_t \frac{P_t^{V^F}}{P_t^Z}} \;, \;\; mc_t^E = \frac{p_t^{E^F}}{p_t^E} \frac{1}{Q_t} \;\Leftrightarrow\; \widehat{mc}_t^e = \widehat{p}_t^{ef} - \widehat{q}_t - \widehat{p}_t^e \quad \text{(A.32)}$$

We assume that the global energy export price level follows the exogenous process

$$p_t^{E,F} \;=\; \left(p_{ss}^{E,F}\right)^{1-\rho_E} \left(p_{t-1}^{E,F}\right)^{\rho_E} \varepsilon_t^E \qquad \text{(A.33)}$$

The log-linear energy-price inflation equation that results from the dynamic price setting problem under Calvo frictions is give by:

$$\begin{aligned}\widehat{\pi}_t^E \;=\; & +\frac{(1-\phi_E)\left(1-\omega_E-(1-\omega_E)\,\Gamma_{ss}^H\,\beta\,\phi_E\right)}{\phi_E\,(1+\Gamma_{ss}^H\,\beta\,\xi_E)+(1-\phi_E)\,\omega_E}\widehat{mc}_t^E + \frac{\xi_E\,(\phi_E+(1-\phi_E)\,\omega_E)}{\phi_E\,(1+\Gamma_{ss}^H\,\beta\,\xi_E)+(1-\phi_E)\,\omega_E}\widehat{\pi}_{t-1}^E \\ & +\frac{\beta\,\Gamma_{ss}^H\,\phi_E}{\phi_E\,(1+\Gamma_{ss}^H\,\beta\,\xi_E)+(1-\phi_E)\,\omega_E}\mathbf{E}_t\widehat{\pi}_{t+1}^E. \qquad \text{(A.34)}\end{aligned}$$

A.7 Export Firms

The export sector involves two types of agents: (i) perfectly competitive packers and (ii) monopolistically competitive export firms.

Export Packers. Export packers demand and aggregate infinitely many varieties of domestic exports $\widetilde{X}_t(i),\ i \in [0,1]$ into an aggregate export good $\widetilde{X}_t$. $\widetilde{X}_t(i)$ denotes the demand for a specific variety i of the export and $\widetilde{X}_t$ denotes the aggregate demand. ε_X is the elasticity of substitution and $\mathscr{M}_X = \varepsilon_X/(\varepsilon_X - 1)$ is the corresponding gross markup of monopolistically competitive export firms. Export packers purchase a single variety at given prices $P_t^{EXP}(i)$ and sell the export good $\widetilde{X}_t$ to the rest of the world at price P_t^{EXP}. The packers' CES production function, and the implied demand schedule associated with the cost minimisation are

$$\widetilde{X}_t = \left[\int_0^1 \left(\widetilde{X}_t(i)\right)^{1-\frac{1}{\varepsilon_X}} \mathrm{d}i\right]^{\frac{\varepsilon_X}{\varepsilon_X-1}}, \quad \widetilde{X}_t(i) = \left(\frac{P_t^X(i)}{P_t^X}\right)^{\frac{\mathscr{M}_X}{1-\mathscr{M}_X}} \widetilde{X}_t, \quad P_t^{EXP} \equiv \left(\int_0^1 \left(P_t^{EXP}(i)\right)^{\frac{1}{1-\mathscr{M}_X}} \mathrm{d}i\right)^{1-\mathscr{M}_X}$$

where P_t^{EXP} is the price index and optimal behaviour implies $P_t^{EXP}\widetilde{X}_t = \int_0^1 P_t^{EXP}(i)\widetilde{X}_t(i)\mathrm{d}i$.

Export Producers. Each export variety $\widetilde{X}_t(i)$ that the packer demands and assembles is supplied by a single *monopolistically competitive* export firm $i \in [0,1]$. They buy a homogenous tradeable export good from the domestic sectoral retailer at the domestic currency export price P_t^X. The exporters then transform and differentiate the homogenous good they purchased. This transformation can be described by the following 'production' function $\widetilde{X}_t(i) = \widetilde{Z}_t^X(i)$. The Lagrangian for the cost minimisation problem is $\mathscr{L}_t^X = -\tau_t^{\mathscr{M}_X}\left(P_t^X \widetilde{Z}_t^X(i)\right) + \widetilde{MC}_t^X(i)\left(\widetilde{Z}_t^X(i)\right)$ and the Lagrange multiplier $\widetilde{MC}_t^X(i)$ is the (nominal) shadow cost of producing one more unit of exports, i.e., the nominal marginal cost, and $\tau_t^{\mathscr{M}_X}$ is a shock to export marginal costs that is isomorphic to a markup shock. Note that

$$p_t^{EXP} = \Gamma^X \Pi_t^{EXP}/\Pi_t^{V^F} p_{t-1}^{EXP}, \quad \Leftrightarrow \quad \widehat{p}_t^{exp} = \widehat{\pi}_t^{exp} - \widehat{\pi}_t^{cf} + \widehat{p}_{t-1}^{exp} \tag{A.35}$$

Rule-of-Thumb Price X Setting. The objective of exporter i is to maximise nominal profits

$$\widetilde{D}_t^X(i) = P_t^{EXP}(i)/\mathscr{E}_t\widetilde{X}_t(i) - \left\{\tau_t^{\mathscr{M}_X}\left(P_t^X \widetilde{X}_t(i)\right)\right\}.$$

A fraction ω_X of 'rule-of-thumb' firms will set their price based on an index of previous-period and steady state X inflation. Only the remaining fraction $(1-\omega_X)$ attempts to implement the optimal price $P_t^{EXP,\#}$. An expression for the aggregate price index is given by

$$P_t^{EXP} = \left(\int_F \left(P_t^{EXP,\#}(i)\right)^{\frac{1}{1-\mathscr{M}_X}} \mathrm{d}i + \int_{\bar{F}} \left(\left[\left(\Pi_{ss}^{EXP}\right)^{(1-\xi_X)} \left(\Pi_{t-1}^{EXP}\right)^{\xi_X}\right] P_{t-1}^{EXP}(i)\right)^{\frac{1}{1-\mathscr{M}_X}} di\right)^{1-\mathscr{M}_X}$$

where F is the set of those firms who can reoptimise their price

$$1 = (1-\phi_X)\left\{\left(p_t^{EXP,\#}\right)^{1-\omega_X} \cdot \left(\zeta_t^X\right)^{-\omega_X}\right\}^{\frac{1}{1-\mathscr{M}_X}} + (\phi_X)\left(\left[\left(\Pi_{ss}^{EXP}\right)^{(1-\xi_X)} \left(\Pi_{t-1}^{EXP}\right)^{\xi_X}\right] \frac{1}{\Pi_t^{EXP}}\right)^{\frac{1}{1-\mathscr{M}_X}}.$$

With probability $1-\phi_X$ a non-rule-of-thumb firm can re-optimise its price

$$P_t^{EXP}(i) = \begin{cases} P_t^{EXP,\#}(i) & \text{with probability: } 1-\phi_X \\ P_{t-1}^{EXP}(i)\left(\left(\Pi_{ss}^{EXP}\right)^{1-\xi_X}\left(\Pi_{t-1}^{EXP}\right)^{\xi_X}\right) & \text{with probability: } \phi_X \end{cases}$$

where ξ_X is the weight attached to the previous period X inflation, $\Pi_t^{EXP} = P_t^{EXP}/P_{t-1}^{EXP}$. Consider a firm who can reset its price in the current period $P_t^{EXP}(i) = P_t^{EXP,\#}(i)$ and who is then stuck with its price until future period $t+s$. The price in this case would be

$$P_{t+s}^{EXP}(i) = P_t^{EXP,\#}(i)\left(\Pi_{ss}^{EXP}\right)^{s(1-\xi_X)}\left(\prod_{g=0}^{s-1}\left(\left(\Pi_{t+g}^{EXP}\right)^{\xi_X}\right)\right) = P_t^{EXP,\#}(i)\left[\left(\Pi_{ss}^{EXP}\right)^{s(1-\xi_X)}\left(\frac{P_{t+s-1}^{EXP}}{P_{t-1}^{EXP}}\right)^{\xi_X}\right].$$

Subject to the above derived demand constraint and assuming that a firm i always meets the demand for its product at the current price, firms solve the following optimisation problem

$$\max_{P_t^{EXP,\#}(i)} \mathbf{E}_t \sum_{s=0}^{\infty} (\phi_X)^s \Lambda_{t,t+s}^{C,o} P_{t+s}^Z \widetilde{\chi}_{t+s}^Z \left[\left(P_{t+s}^{EXP}(i) - \widetilde{MC}_{t+s}^X\right)\frac{\widetilde{X}_{t+s}(i)}{\widetilde{\chi}_{t+s}^Z P_{t+s}^Z}\right]$$

$$\text{s.t. } \widetilde{X}_{t+s}(i) = \left(\frac{P_t^{EXP,\#}(i)}{P_{t+s}^{EXP}}\right)^{-\frac{\mathscr{M}_X}{\mathscr{M}_X-1}} \widetilde{X}_{t+s}.$$

Taking the derivative with respect to $P_t^{EXP,\#}(i)$ delivers the familiar price inflation schedule

$$\begin{aligned}
1 &= \left(\frac{1-(\phi_X)\left(\zeta_t^{EXP}\right)^{\frac{-1}{1-\mathscr{M}_X}}}{1-\phi_X}\right)^{1-\mathscr{M}_X}\left[\left(\frac{f_t^{X,1}\mathscr{M}_X}{f_t^{X,2}}\right)^{1-\omega_X}\cdot\left(\zeta_t^{EXP}\right)^{-\omega_X}\right]^{-1}, \quad \zeta_t^{EXP} \equiv \frac{\Pi_t^{EXP}}{(\Pi_{ss}^{EXP})^{1-\xi_X}(\Pi_{t-1}^{EXP})^{\xi_X}} \\
f_t^{X,1} &= X_t \frac{p_t^X}{p_t^{EXP}} \tau_t^{\mathscr{M}_X} Q_t + \phi_X \Gamma^H \Gamma^X \mathbf{E}_t\left[\mathbf{B}_{t+1}\frac{U_{u,t+1}^{C,o}}{U_{u,t}^{C,o}}\frac{\Pi_{t+1}^{EXP}}{\Pi_{t+1}^{CPI}}\left(\zeta_{t+1}^{EXP}\right)^{\frac{\mathscr{M}_X}{\mathscr{M}_X-1}} f_{t+1}^{X,1}\right] \\
f_t^{X,2} &= X_t + \phi_X \Gamma^H \Gamma^X \mathbf{E}_t\left[\mathbf{B}_{t+1}\frac{U_{u,t+1}^{C,o}}{U_{u,t}^{C,o}}\frac{\Pi_{t+1}^{EXP}}{\Pi_{t+1}^{CPI}}\left(\zeta_{t+1}^{EXP}\right)^{\frac{1}{\mathscr{M}_X-1}} f_{t+1}^{X,2}\right] \\
\mathscr{D}_t^X &= (1-\phi_X)\left(\left(1-\phi_X\left(\zeta_t^{EXP}\right)^{\frac{1}{\mathscr{M}_X-1}}\right)/(1-\phi_X)\right)^{\mathscr{M}_X} + \phi_X\left(\zeta_t^{EXP}\right)^{\frac{\mathscr{M}_X}{\mathscr{M}_X-1}}\mathscr{D}_{t-1}^X.
\end{aligned}$$

Aggregation implies $\int_0^1 X_t(i)\mathrm{d}i = \int_0^1\left(\frac{P_t^{EXP}(i)}{P_t^{EXP}}\right)^{-\frac{\mathscr{M}_X}{\mathscr{M}_X-1}} X_t \mathrm{d}i = X_t \int_0^1\left(\frac{P_t^{EXP}(i)}{P_t^{EXP}}\right)^{-\frac{\mathscr{M}_X}{\mathscr{M}_X-1}} \mathrm{d}i$ where we define price dispersion as $\mathscr{D}_t^X \equiv \int_0^1\left(\frac{P_t^{EXP}(i)}{P_t^{EXP}}\right)^{-\frac{\mathscr{M}_X}{\mathscr{M}_X-1}} \mathrm{d}i$ which can be written recursively.

Log-linear Export Inflation Equation. This is given by

$$\begin{aligned}
\widehat{\pi}_t^{EXP} &= \frac{(1-\phi_X)\left(1-\omega_X-(1-\omega_X)\Gamma_{ss}^H\beta\,\phi_X\right)}{\phi_X\,(1+\Gamma_{ss}^H\beta\,\xi_X)+(1-\phi_X)\,\omega_X}\left(\underbrace{\widehat{p}_t^X}_{=0}+\underbrace{\widehat{q}_t-\widehat{p}_t^{EXP}}_{\widehat{mc}_t^x}+\underbrace{\widehat{\tau}_t^{\mathscr{M}_X}}_{\equiv\widehat{\mu}_t^x/s}\right) \\
&\quad + \frac{\xi_X\,(\phi_X+(1-\phi_X)\,\omega_X)}{\phi_X\,(1+\Gamma_{ss}^H\beta\,\xi_X)+(1-\phi_X)\,\omega_X}\widehat{\pi}_{t-1}^{EXP} + \frac{\beta\,\Gamma_{ss}^H\,\phi_X}{\phi_X\,(1+\Gamma_{ss}^H\beta\,\xi_X)+(1-\phi_X)\,\omega_X}\mathbf{E}_t\,\widehat{\pi}_{t+1}^{EXP} && \text{(A.36)} \\
\widehat{mc}_t^x &= \widehat{q}_t - \widehat{p}_t^{exp}. && \text{(A.37)}
\end{aligned}$$

A.8 Retail Firms

There is a continuum of perfectly-competitive retailers defined on the unit interval, who buy final output goods from the final-output good packers at price P_t^Z and convert them into differentiated goods representing each expenditure component (non-energy consumption goods, investment goods, government consumption goods and export goods). Retailer i in sector N converts final output using the following linear technology

$$\widetilde{N}_t(i) = \widetilde{\chi}_t^N \widetilde{Z}_t^N(i), \quad \text{for } N = C^z, G, X, I, I^O,$$

where the input $\widetilde{Z}_t^N(i)$ is the per capita amount of the final output $\widetilde{Z}_t$ demanded by firm i in expenditure sector N and where final output $\widetilde{Z}_t$, is defined by its above stated the CES aggregator.

The term $\widetilde{\chi}_t^N$, common to all retailers in sector N, is the rate at which the final-output good bundle can be converted into the expenditure component, N. Hence, $\widetilde{\chi}_t^N$ is a measure of sector-specific productivity in sector N. Below we will normalise so that $\widetilde{\chi}_t^{Cz} = 1$, implying that the final-output production function could be interpreted as a production function for a 'generic final non-energy consumption good', which could be used directly for consumption, or which could be transformed by 'retailers' into investment, government spending or export goods. Each retailer i in sector N chooses its input $\widetilde{Z}_t^N(i)$ to maximise profits, taking the price of its output, P_t^N, and the price of the final-output good, P_t^Z, as given

$$\max_{\widetilde{Z}_t^N(i)} P_t^N \widetilde{\chi}_t^N \widetilde{Z}_t^N(i) - P_t^Z \widetilde{Z}_t^N(i), \qquad p_t^N = P_t^N \widetilde{\chi}_t^N / P_t^Z, \quad \text{for } N = C^z, G, X, I, I^O.$$

Energy in the Consumption Basket. Perfectly competitive consumption retailers purchase non-energy consumption ($\widetilde{C}^z$) from the respective non-energy consumption retailer and energy for consumption ($\widetilde{E}^c$) from the energy import firm. They produce a consumption bundle $\widetilde{C}_t$ and sell it to households at price P_t^{CPI}

$$\max_{\widetilde{C}_t^z, \widetilde{E}_t^c} \left\{ P_t^{CPI} \widetilde{C}_t - P_t^{Cz} \widetilde{C}_t^z - P_t^E \widetilde{E}_t^c \right\} \quad \text{s.t.} \quad \widetilde{C}_t = \left((1-\alpha_{ec})^{\frac{1}{\varepsilon_{ec}}} (\widetilde{C}_t^z)^{\frac{\varepsilon_{ec}-1}{\varepsilon_{ec}}} + \alpha_{ec}^{\frac{1}{\varepsilon_{ec}}} \left(\widetilde{E}_t^c\right)^{\frac{\varepsilon_{ec}-1}{\varepsilon_{ec}}} \right)^{\frac{\varepsilon_{ec}}{\varepsilon_{ec}-1}}$$

which implies $\frac{\partial \widetilde{C}_t}{\partial \widetilde{C}_t^z} = \frac{P_t^{Cz}}{P_t^{CPI}}$ and $\frac{\partial \widetilde{C}_t}{\partial \widetilde{E}_t^c} = \frac{P_t^E}{P_t^{CPI}}$ so that the relative-demand schedules are given by

$$\widetilde{C}_t^z = \left(\frac{p_t^{Cz}}{p_t^{CPI}}\right)^{-\varepsilon_{ec}} (1-\alpha_{ec})\widetilde{C}_t \quad \Leftrightarrow \quad \widehat{c}_t^z = \widehat{c}_t + \varepsilon_{ce}(\widehat{p}_t^c), \qquad \widehat{p}_t^c = \log(p_t^{CPI}) \tag{A.38}$$

$$\widetilde{E}_t^c = \left(\frac{p_t^E}{p_t^{CPI}}\right)^{-\varepsilon_{ec}} (\alpha_{ec})\widetilde{C}_t \quad \Leftrightarrow \quad \widehat{c}_t^e = \widehat{c}_t + \varepsilon_{ce}(\widehat{p}_t^c - \widehat{p}_t^e). \tag{A.39}$$

Optimality also implies

$$p_t^{CPI} \widetilde{C}_t = p_t^{Cz} \widetilde{C}_t^z + p_t^E \widetilde{E}_t^c, \qquad P_t^{CPI}/P_t^Z = p_t^{CPI}, \quad P_t^{Cz}/P_t^Z = p_t^{Cz}, \quad P_t^E/P_t^Z = p_t^E$$

$$p_t^{CPI} = \left[(1-\alpha_{ec}) \left(p_t^{Cz}\right)^{1-\varepsilon_{ec}} + \alpha_{ec} \left(p_t^E\right)^{1-\varepsilon_{ec}} \right]^{\frac{1}{1-\varepsilon_{ec}}} \tag{A.40}$$

where P_t^Z is the domestic final output price level and P_t^{CPI} is the price of the CES consumption bundle (the 'consumer price index').

A.9 Policy

Fiscal Policy. The government purchases goods from retailers and finances its expenditure by raising lump-sum taxes from optimising households.[29] Real government spending growth follows an exogenous rule, where, for simplicity, growth in government spending is assumed to exhibit 'error correction' to its long-run trend:

$$\frac{\widetilde{G}_t}{\widetilde{G}_{t-1}} = \left(\frac{\widetilde{G}_{t-1}}{\widetilde{\chi}^Z_{t-1}\widetilde{\chi}^G_{t-1}}\right)^{\rho_G-1} \varepsilon^G_t, \quad \Leftrightarrow \quad \widehat{g}_t - \widehat{g}_{t-1} + \widehat{\gamma}^z_t = (\rho^g - 1)\widehat{g}_{t-1} + \widehat{\varepsilon}^g_t \quad \text{(A.41)}$$

where $\widetilde{G}_t$ is real per capita government spending, $\widetilde{\chi}^Z_{t-1}\widetilde{\chi}^G_{t-1}$ is the trend in government spending and ε^G_t is a disturbance to government spending which follows

$$\log \varepsilon^G_t \;\; = \left(1-\rho^2_G\right)^{\frac{1}{2}} \sigma_G \eta^G_t, \quad \eta^G_t \sim N(0,1).$$

The government budget constraint is given by:

$$P^G_t \widetilde{G}_t + \frac{\widetilde{B}_{t-1}R_{t-1} + \widetilde{Mon}_{t-1}}{\Gamma^H} + P^Z_t \widetilde{\chi}^Z_t \mathscr{T}_i = \widetilde{T}^G_t + \widetilde{B}_t + \widetilde{Mon}_t, \quad \widetilde{B}_t = 0, \quad P^G_t \widetilde{\chi}^G_t / P^Z_t = 1.$$

which shows that the government finances government spending and net government debt B^G_t using the lump-sum tax. Since the lump sum T^G is levied on optimising households, the model exhibits so-called 'Ricardian equivalence' in the sense that, in equilibrium, the present value of the lump-sum tax payments offsets the value of the government debt in optimising households' lifetime budget constraints. As a result, the debt issuance (and tax financing) decisions of the government have no effect on the consumption decisions of households or any other variables. In light of this observation, we choose the simplest possible assumptions for government debt issuance and tax financing. We assume that debt issuance is zero in each period and that the lump sum taxes adjusts to ensure that the budget holds.

Monetary Policy. Monetary policy follows a simple rule for the nominal interest rate in which it responds to deviations of annual CPI inflation, $\Pi^{CPI,annual}_t$, from its target, $\Pi^{CPI*,annual}$, and a measure of the output gap, $\mathscr{Y}_t$

$$R_t \;\; = \;\; R^{1-\theta_R}_{ss} R^{\theta_R}_{t-1} \left(\frac{\Pi^{CPI,annual}_t}{\Pi^{CPI*,annual}}\right)^{\frac{(1-\theta_R)\theta_\Pi}{4}} (\mathscr{Y}_t)^{(1-\theta_R)\theta_Y} \varepsilon^R_t$$

[29]As described above, the government also makes transfers between optimising and rule-of-thumb households to ensure that per capita consumption in the two groups are equalised in steady state.

with

$$
\begin{aligned}
\Pi_t^{CPI,annual} &= \frac{P_t^{CPI}}{P_{t-4}^{CPI}} = \frac{P_t^{CPI}}{P_{t-1}^{CPI}}\frac{P_{t-1}^{CPI}}{P_{t-2}^{CPI}}\frac{P_{t-2}^{CPI}}{P_{t-3}^{CPI}}\frac{P_{t-3}^{CPI}}{P_{t-4}^{CPI}} = \Pi_t^{CPI}\Pi_t^{CPI,lag1}\Pi_t^{CPI,lag2}\Pi_{t-1}^{CPI,lag2} && \text{(A.42)}\\
\Pi_t^{CPI,lag1} &= \Pi_{t-1}^{CPI} && \text{(A.43)}\\
\Pi_t^{CPI,lag2} &= \Pi_{t-1}^{CPI,lag1} && \text{(A.44)}
\end{aligned}
$$

and where $\Pi^{CPI*,annual} = \left(\Pi^{CPI*}\right)^4$ and

$$
\mathscr{Y}_t \equiv \widetilde{V}_t/\widetilde{V}_t^{flex} \tag{A.45}
$$

where $\widetilde{Z}_t^{flex}$ is the level of final output that would be observed if all prices and wages were flexible (to be defined below), R is the steady state nominal interest rate consistent with steady-state inflation being at target, and ε_t^R is an interest rate shock which follows

$$
\log \varepsilon_t^R = \sigma_R \eta_t^R, \qquad \eta_t^R \sim N(0,1)
$$

This implies the following log-linear relationships

$$
\begin{aligned}
\widehat{r}_t &= \widehat{rule}_t + \widehat{\varepsilon}_t^r && \text{(A.46)}\\
\widehat{rule}_t &= (1-\theta_r)\left(\frac{\theta_\pi}{4}(\widehat{\pi}_t^{CPI,annual} - (1-\theta_E)\times \widehat{\text{econt}}_t - \widehat{\mu}_t^{z,temp,ann}) + \theta_y \widehat{y}_t^{gap}\right) + \theta_r \widehat{r}_{t-1} && \text{(A.47)}
\end{aligned}
$$

where we parametrise the possibility that the central bank only targets core ($\theta_E = 0$) or instead targets full headline CPI inflation ($\theta_E = 1$). $econt_t$ is the annual contribution of energy to CPI inflation.

A.10 The Rest of the World

The model is closed by specifying global demand for the domestic export good $\widetilde{X}_t$.

Foreign Households. The problem of the foreign household i is to maximise lifetime utility $\mathscr{U}_{is}^F$

$$
\mathscr{U}_{is}^F = E_s \sum_{t=s}^{\infty} (\beta^F)^t \widetilde{\chi}_t^{H^F} \left\{ \varepsilon_t^{C^F} \left[\frac{\left(\frac{\widetilde{C}_{i,t}^F}{\widetilde{\chi}_t^{V^F}} - \psi_C^F \frac{\widetilde{C}_{t-1}^F}{\widetilde{\chi}_{t-1}^{V^F}} \right)^{1-\varepsilon_C^F}}{1-\varepsilon_C^F} - \nu_L^F \frac{\left(L_{i,t}^F\right)^{1+\varepsilon_L^F}}{1+\varepsilon_L^F} \right] \right\} \; s.t. \;\; P_t^{C^F} \widetilde{C}_{i,t}^F + \widetilde{B}_{i,t}^F + \widetilde{T}_{i,t}^F = \widetilde{W}_t^F L_{i,t}^{F,s} + R_{t-1}^F \widetilde{B}_{i,t-1}^F / \Gamma^{H^F} + \widetilde{D}_{i,t}^F
$$

where $\varepsilon_t^{C^F}$ denotes a foreign consumption preference shock. The intertemporal optimality condition is

$$
\Lambda_t^{C^F} = \beta^F R_t^F \mathbf{E}_t \left\{ \frac{1}{\Pi_{t+1}^{C^F} \Gamma_{t+1}^{V^F}} \Lambda_{t+1}^{C^F} \right\}, \quad \Lambda_t^{C^F} = \left(C_t^F - \psi_C^F C_{t-1}^F\right)^{-\varepsilon_c^F} \varepsilon_t^{C^F}.
$$

The intra-temporal optimality condition is given by

$$
U_t^{L^F} = -\varepsilon_t^{C^F} \nu_L^F \left(L_t^F\right)^{\varepsilon_L^F}, \quad W_t^F = -U_t^{L^F}/\Lambda_t^{C^F}, \quad W_t^F \equiv \widetilde{W}_t^F / (P_t^{V^F} \widetilde{\chi}_t^{V^F})
$$

Note that on the world level, we abstract from investment and government spending so that market clearing implies $C_t = V_t^F$ and $\Pi_t^{C^F} = \Pi_t^{V^F}$. The log-linearised consumption Euler equation is given by

$$\widehat{v}_t^f = \left(\frac{1}{1+\psi_C^F}\right)\mathbf{E}_t\widehat{v}_{t+1}^f + \left(\frac{\psi_C^F}{1+\psi_C^F}\right)\widehat{v}_{t-1}^f - \left(\frac{1-\psi_C^F}{\varepsilon_C^F(1+\psi_C^F)}\right)\left(\widehat{r}_t^f - \mathbf{E}_t\widehat{\pi}_{t+1}^{cf} - \mathbf{E}_t\widehat{\gamma}_{t+1}^{vf}\right) + (1-\rho^{cf})\widehat{\varepsilon}_t^{cf} \tag{A.48}$$

Foreign Firms. We assume that in the rest of the world production only depends on labour, such that world output produced by a firm i is $\widetilde{V}_t^F(i) = \left(\widetilde{K}_{t-1}^F(i)\right)^{1-\alpha_{L^F}} \left(\widetilde{\chi}_t^{LAP} L_t^F(i)\right)^{\alpha_{L^F}}$. Combining the labour demand schedule with the household labour supply schedule and log-linearising around the steady state yields

$$\widehat{mc}_t^{vf} = \left(\frac{\varepsilon_L^F + 1 - \alpha_L^F}{\alpha_L^F} + \frac{\varepsilon_C^F}{1-\psi_C^F}\right)\widehat{v}_t^f - \left(\varepsilon_C^F \frac{\psi_C^F}{1-\psi_C^F}\right)\widehat{v}_{t-1}^f. \tag{A.49}$$

We assume that foreign firms are subject to nominal rigidities which gives rise to a world price inflation equation

$$\widehat{\pi}_t^{cf} = \left(\frac{(1-\phi_{vf})(1-\phi_{vf}\beta\Gamma_{ss}^H)}{\phi_{vf}(1+\xi_{vf}\beta\Gamma_{ss}^H)}\right)\widehat{mc}_t^{vf} + \frac{\xi_{vf}}{1+\xi_{vf}\beta\Gamma_{ss}^H}\widehat{\pi}_{t-1}^{cf} + \frac{\beta\Gamma_{ss}^H}{1+\xi_{vf}\beta\Gamma_{ss}^H}\mathbf{E}_t\widehat{\pi}_{t+1}^{cf} + \widehat{\mu}_t^{vf} \tag{A.50}$$

Export Demand and Trade. We assume that world trade $\widetilde{Z}_t^F$ is a simple mapping of world GDP $\widetilde{V}_t^F$ such that

$$\widetilde{Z}_t^F = \left(\varepsilon_t^{Z^F}\right)^{\phi^{Z^F}} \widetilde{V}_t^F \quad \Leftrightarrow \quad \widehat{z}_t^f = \widehat{v}_t^f + \phi^{Z^F}\widehat{\varepsilon}_t^{zf} \tag{A.51}$$

Note that $\Gamma_t^{Z^F} = \Gamma_t^{V^F}$. The 'world trade shock' $\varepsilon_t^{Z^F}$ disturbs the relationship between world trade and world GDP

$$\log\varepsilon_t^{Z^F} = (1-\rho_{Z^F})\log\varepsilon^{Z^F} + \rho_{Z^F}\log\varepsilon_{t-1}^{Z^F} + \left(1-\rho_{Z^F}^2\right)^{1/2}\sigma_{Z^F}\eta_t^{Z^F}, \qquad \eta_t^{Z^F} \sim N(0,1).$$

Demand for the bundle of domestic exports depends on the foreign currency price of domestic exports relative to the world export price, $\frac{P_t^{EXP}}{P_t^{X^F}}$, and on the world trade volume Z_t^F:

$$\widetilde{X}_t = \left(\frac{P_t^{EXP}}{P_t^{X^F}}\right)^{-\varepsilon_F} \widetilde{Z}_t^F \kappa_t^F \frac{\widetilde{\chi}_t^{H^F}}{\widetilde{\chi}_t^H}\widetilde{\chi}_t^X, \quad \kappa_t^F = \kappa_{ss}^F\varepsilon_t^{\kappa^F}$$

where the parameter ε_F is the elasticity of substitution between differentiated export goods in the rest of the world. Total world demand for exports is made up of four terms: $\widetilde{Z}_t^F$ is the total world trade volume expressed in foreign per capita terms; κ_t^F can be interpreted as a preference shifter of the world's demand for domestic exports; $\widetilde{\chi}_t^{H^F}/\widetilde{\chi}_t^H$ converts world output in foreign per capita terms into a measure expressed in domestic per capita terms; $\widetilde{\chi}_t^X$ is the trend productivity growth in the export retail sector, which is assumed to be mirrored in the world economy consistent with a balanced growth path (as discussed below). The export demand preference shifter satisfies:

$$\log \varepsilon_t^{\kappa^F} = (1-\rho_{\kappa^F})\log \varepsilon^{\kappa^F} + \rho_{\kappa^F}\log \varepsilon_{t-1}^{\kappa^F} + \left(1-\rho_{\kappa^F}^2\right)^{1/2}\sigma_{\kappa^F}\eta_t^{\kappa^F}, \qquad \eta_t^{\kappa^F} \sim N(0,1)$$

Detrending $\widetilde{X}_t$.

$$\frac{\widetilde{X}_t}{\widetilde{\chi}_t^Z\widetilde{\chi}_t^X} = \varepsilon_t^{\kappa^F}\left(\frac{\widetilde{\chi}_t^{P^{EXP}}P_t^{EXP}}{\widetilde{\chi}_t^{P^{XF}}P_t^{XF}}\right)^{-\varepsilon^F}\underbrace{\frac{\widetilde{\chi}_t^{V^F}}{\widetilde{\chi}_t^Z}}_{\equiv\Omega_t^F}Z_t^{F,d}\frac{\widetilde{\chi}_t^{H^F}}{\widetilde{\chi}_t^H}, \quad \frac{\widetilde{\chi}_t^{V^F}}{\widetilde{\chi}_t^Z}\equiv\Omega_t^F \quad \frac{\frac{\widetilde{\chi}_t^{V^F}}{\widetilde{\chi}_t^Z}}{\frac{\widetilde{\chi}_{t-1}^{V^F}}{\widetilde{\chi}_{t-1}^Z}} = \frac{\Gamma_t^{V^F}}{\Gamma_t^Z} = \frac{\Omega_t^F}{\Omega_{t-1}^F}$$

$$X_t = \kappa_t^F\left(\frac{P_t^{EXP}}{P_t^{XF}}\right)^{-\varepsilon^F}\Omega_t^F Z_t^{F,d}, \quad \Leftrightarrow \widehat{x}_t = \widehat{v}_t^f + \phi^{Z^F}\widehat{\varepsilon}_t^{Z^F} + \widehat{\varepsilon}_t^{\kappa^F} - \varepsilon^f(\widehat{p}_t^{exp} - \widehat{p}_t^{xf}) + \widehat{\omega}_t^F \tag{A.52}$$

which implies

$$\Omega_t^F = \Omega^F\varepsilon_t^{\Omega^F}, \log \varepsilon_t^{\Omega^F} = (1-\rho_{\Omega^F})\log \varepsilon^{\Omega^F} + \rho_{\Omega^F}\log \varepsilon_{t-1}^{\Omega^F} + \left(1-\rho_{\Omega^F}^2\right)^{\frac{1}{2}}\sigma_{\Omega^F}\eta_t^{\Omega^F}, \quad \eta_t^{\Omega^F} \sim N(0,1)$$

World Monetary Policy. We assume that world interest rates follow a Taylor rule

$$\widehat{r}_t^f = \theta^{rf}\widehat{r}_{t-1}^f + (1-\theta^{rf})\left((\theta^{\pi^f}/4)\widehat{\pi}_t^{cf,annual} + \theta^{vf}(\widehat{v}_t^f - \widehat{v}_t^{f,flex})\right) + \widehat{\varepsilon}_t^{rf} \tag{A.53}$$

$$\widehat{\pi}_t^{cf,annual} = \widehat{\pi}_t^{cf} + \widehat{\pi}_t^{cf,l1} + \widehat{\pi}_t^{cf,l2} + \widehat{\pi}_{t-1}^{cf,l2}, \tag{A.54}$$

$$\widehat{\pi}_t^{cf,l1} = \widehat{\pi}_{t-1}^{cf}, \tag{A.55}$$

$$\widehat{\pi}_t^{cf,l2} = \widehat{\pi}_{t-1}^{cf,l1} \tag{A.56}$$

A.11 Growth and Detrending Factors

The model has a well-defined balanced growth path along which variables grow at constant rates in the steady state. Here, we define the steady-state balanced growth path, discuss the assumptions underpinning it and summarise the factors that were obtained from detrending the model.

Growth in the model arises from labour-augmenting productivity (LAP) growth, which is stochastic, as well as deterministic population growth and deterministic retail sector-specific productivity growth. LAP follows a stochastic process

$$\widetilde{\chi}_t^{LAP} = \Gamma^{LAP}\widetilde{\chi}_{t-1}^{LAP}\left(\varepsilon_t^{LAP}\right), \log \varepsilon_t^{LAP} = (1-\rho_{LAP})\log \varepsilon^{LAP} + \rho_{LAP}\log \varepsilon_{t-1}^{LAP} + \left(1-\rho_{LAP}^2\right)^{1/2}\sigma_{LAP}\eta_t^{LAP}.$$

where Γ^{LAP} defines the steady-state growth rate of LAP. This implies that $\Gamma_t^{LAP} = \Gamma_{ss}^{LAP}\varepsilon_t^{LAP}$. Note that $\eta_t^L \sim N(0,1)$.

Trend Growth Rates.

We now derive the trend growth rates for value-added and final output, which in turn are a function of LAP growth. Recall the final-output CES production function structure, which depends on value added and non-energy imports

$$
\begin{aligned}
\left(\widetilde{\chi}_t^Z\right)^{\frac{\varepsilon_v-1}{\varepsilon_v}} &= (\alpha_v)^{\frac{1}{\varepsilon_v}} \left(\widetilde{\chi}_t^V/c_{vz}\right)^{\frac{\varepsilon_v-1}{\varepsilon_v}} + (1-\alpha_v)^{\frac{1}{\varepsilon_v}} \left(\widetilde{\chi}_t^Z \widetilde{\chi}_t^M / \widetilde{\chi}_t^M\right)^{\frac{\varepsilon_v-1}{\varepsilon_v}} \\
1 &= (\alpha_v)^{\frac{1}{\varepsilon_v}} \left(\frac{\widetilde{\chi}_t^V}{\widetilde{\chi}_t^Z}\frac{1}{c_{vz}}\right)^{\frac{\varepsilon_v-1}{\varepsilon_v}} + (1-\alpha_v)^{\frac{1}{\varepsilon_v}} \\
\frac{\widetilde{\chi}_t^V}{\widetilde{\chi}_t^Z} &= 1, \qquad c_{vz} = \left(\frac{1}{(\alpha_v)^{\frac{1}{\varepsilon_v}}} - \frac{(1-\alpha_v)^{\frac{1}{\varepsilon_v}}}{(\alpha_v)^{\frac{1}{\varepsilon_v}}}\right)^{-\frac{\varepsilon_v}{\varepsilon_v-1}}
\end{aligned}
$$

such that

$$
\Gamma_t^Z = \Gamma_t^V, \quad \Leftrightarrow \quad \widehat{\gamma}_t^Z = \widehat{\gamma}_t^V. \tag{A.57}
$$

Next, consider the value-added production function, which states that value added is a function of capital services and effective labour

$$
\begin{aligned}
\widetilde{\chi}_t^V V_t &= \varepsilon_t^{TFP} \left(\widetilde{\chi}_t^Z \widetilde{\chi}_t^I \frac{\widetilde{\chi}_{t-1}^Z \widetilde{\chi}_{t-1}^I K_{t-1}}{\widetilde{\chi}_t^Z \widetilde{\chi}_t^I}\right)^{1-\alpha_L} \left(\widetilde{\chi}_t^L \widetilde{\chi}_t^{LAP,d}\right)^{\alpha_L} (L_t)^{\alpha_L}, \widetilde{\chi}_t^V = \left(\widetilde{\chi}_t^Z \widetilde{\chi}_t^I\right)^{1-\alpha_L} \left(\widetilde{\chi}_t^L \widetilde{\chi}_t^{LAP,d}\right)^{\alpha_L} \\
\Gamma_t^V &= \left(\Gamma_t^Z \Gamma^I\right)^{1-\alpha_L} \left(\Gamma_t^L \Gamma_t^{LAP,d}\right)^{\alpha_L}, \Gamma_t^Z = \left(\Gamma^I\right)^{\frac{1-\alpha_L}{\alpha_L}} \Gamma_t^L \Gamma_t^{LAP,d} \Leftrightarrow \widehat{\gamma}_t^Z - \widehat{\gamma}_t^L = \widehat{\gamma}_t^{LAP,d}
\end{aligned}
$$

The same equation holds for the world block: $\widehat{\gamma}_t^{V,F} - \widehat{\gamma}_t^{L,F} = \widehat{\gamma}_t^{LAP}$. Domestic LAP growth is a function of world LAP and omega

$$
\frac{\Omega_t^F}{\Omega_{t-1}^F} = \frac{\Gamma_t^{LAP}}{\Gamma_t^{LAP,d}}, \Leftrightarrow \widehat{\gamma}_t^{LAP,d} = \widehat{\gamma}_t^{LAP} + \widehat{\omega}_{t-1} - \widehat{\omega}_t \Leftrightarrow \widehat{\gamma}_t^Z - \widehat{\gamma}_t^L = \widehat{\gamma}_t^{LAP} - (\omega_t^F - \omega_{t-1}^F) \tag{A.58}
$$

and

$$
\widehat{\gamma}_t^{v,f} - \widehat{\gamma}_t^{L,F} = \widehat{\varepsilon}_t^{LAP}, \quad \widehat{\gamma}_t^{L,F} = \widehat{\gamma}_t^L \tag{A.59}
$$

A.12 Potential Output and the Flexible-Price Economy

We define potential output as the level of output that would prevail if all prices were flexible and only a subset of shocks (detailed below) affect the economy.[30] In this economy, only real factors affect the paths for real variables. Although relative prices move in response to the real shocks, the absence of nominal rigidities implies that, as long as policymakers set interest rates appropriately, inflation is always at target.

So potential output is defined using a variant of the model in which there are no costs of adjusting prices or wages and where nominal shocks do not exist. This means that it is a version of the model where the monetary policy shock and mark-up shocks do not exist. As a result, the equations of the flexible-price model are equivalent to those just presented with the exception of those associated with price- and wage-setting and the Taylor rule. For the price- and wage-setting equations, the flexible-price model equations are obtained by setting the Calvo-probabilities of not being able to re-optimise

[30] The flexible-price model is derived under the assumption that prices have always been flexible, and will remain flexible in the future.

prices to zero, $\phi_J = 0$ for $J = W, Z, V, M, X, V^F$. There is no explicit monetary policy rule in the flexible-price model. Implicitly, the rule is that inflation is at target in all periods.

A.13 Summary of Log-Linearised Equations

Households

$$\begin{aligned}
\widehat{c}_t^{opt} &= \frac{1}{1+\psi_C+\varepsilon_\beta \frac{1-\psi_C}{\varepsilon_C}} m^h \mathbf{E}_t \widehat{c}_{t+1}^{opt} + \frac{\psi_C}{1+\psi_C+\varepsilon_\beta \frac{1-\psi_C}{\varepsilon_C}} \widehat{c}_{t-1}^{opt} \\
&\quad - \left(\frac{(1-\psi_C)}{\varepsilon_C(1+\psi_C)+\varepsilon_\beta(1-\psi_C)} \right) \left(\widehat{r}_t - m^h \mathbf{E}_t \widehat{\pi}_{t+1}^c + \widehat{\varepsilon}_t^b - m^h \mathbf{E}_t \widehat{\gamma}_{t+1}^z - \widehat{\varepsilon}_t^C + m^h E_t \widehat{\varepsilon}_{t+1}^C \right) && \text{(A.60)} \\
\widehat{c}_t^{rot} &= (w_{ss} L_{ss} / (p_{ss}^C C_{ss}))(\widehat{w}_t + \widehat{l}_t - \widehat{p}_t^c) && \text{(A.61)} \\
\widehat{mon}_t &= 1/(1-\psi_C)(\widehat{c}_t^{opt} - \psi_C \widehat{c}_{t-1}^{opt}) - 1/(\varepsilon_C R_{ss})(\widehat{r}_t + \widehat{\varepsilon}_t^b) + \widehat{p}_t^c / \varepsilon_C && \text{(A.62)} \\
\widehat{q}_t &= m^h \mathbf{E}_t \widehat{q}_{t+1} + \widehat{r}_t - \widehat{r}_t^f - m^h \mathbf{E}_t \widehat{\pi}_{t+1}^z + m^h \mathbf{E}_t \widehat{\pi}_{t+1}^{cf} - \widehat{\varepsilon}_t^{bf} && \text{(A.63)} \\
\widehat{c}_t &= \omega^o \widehat{c}_t^{opt} + (1-\omega^o) \widehat{c}_t^{rot} && \text{(A.64)} \\
\widehat{z}_t &= \alpha_{cz}(C_{ss}/Z_{ss})\widehat{c}_t^z + (I_{ss}/Z_{ss})\widehat{i}_t + (G_{ss}/Z_{ss})\widehat{g}_t + (X_{ss}/Z_{ss})\widehat{x}_t + (I_{ss}^O/Z_{ss})\widehat{i}_t^O && \text{(A.65)} \\
\widehat{\pi}_t^c &= \widehat{\pi}_t^z + \widehat{p}_t^c - \widehat{p}_{t-1}^c && \text{(A.66)}
\end{aligned}$$

Investment and Capital

$$\begin{aligned}
\widehat{i}_t^O &= \widehat{i}_{t-1}^O - \widehat{\gamma}_t^z + (\rho_{io}-1)\widehat{i}_{t-1}^O + \widehat{\varepsilon}_t^{I^O} && \text{(A.67)} \\
\widehat{i}_t &= \frac{1}{1+\beta\Gamma_{ss}^H}(\widehat{i}_{t-1} - \widehat{\gamma}_t^z) + \frac{\beta\Gamma_{ss}^H}{1+\beta\Gamma_{ss}^H} m^h \mathbf{E}_t(\widehat{i}_{t+1} + \widehat{\gamma}_{t+1}^z) + \frac{1}{(1+\beta\Gamma_{ss}^H)(\Gamma_{ss}^H\Gamma_{ss}^z\Gamma_{ss}^I)^2}\left(\frac{1}{\psi^I}\widehat{tq}_t + \widehat{\varepsilon}_t^I\right) && \text{(A.68)} \\
\widehat{tq}_t &= (1-\delta_k)/(rk_{ss}+(1-\delta_k)) m^h \widehat{tq}_{t+1} - (\widehat{r}_t - m^h \mathbf{E}_t \widehat{\pi}_{t+1}^z + \widehat{\varepsilon}_t^b) + rk_{ss}/(rk_{ss}+(1-\delta_k)) m^h \mathbf{E}_t \widehat{rk}_{t+1} && \text{(A.69)} \\
\widehat{k}_t &= (1-\delta_k)/(\Gamma_{ss}^z\Gamma_{ss}^I\Gamma_{ss}^H)(\widehat{k}_{t-1} - \widehat{\gamma}_t^z) + (I_{ss}/(\Gamma_{ss}^H K_{ss}))(\widehat{i}_t + \widehat{\varepsilon}_t^I) && \text{(A.70)}
\end{aligned}$$

Labour Unions

$$\begin{aligned}
\widehat{mrs}_t &= \widehat{\varepsilon}_t^l + \varepsilon_L \widehat{l}_t + \varepsilon_C/(1-\psi_C)(\omega^o(\widehat{c}_t^{opt} - \psi_C \widehat{c}_{t-1}^{opt}) + (1-\omega^o)(\widehat{c}_t^{rot} - \psi_C \widehat{c}_{t-1}^{rot})) + \widehat{p}_t^c && \text{(A.71)} \\
\widehat{w}_t &= \widehat{\pi}_t^w - \widehat{\pi}_t^z - (\widehat{\gamma}_t^z - \widehat{\gamma}_t^L) + \widehat{w}_{t-1} && \text{(A.72)} \\
\widehat{\pi}_t^W &= \widehat{\mu}_t^W + \frac{(1-\phi_W)\left(1-\omega_W-(1-\omega_W)\bar{\Gamma}^H\beta\phi_W\right)}{\phi_W\left(1+\bar{\Gamma}^H\beta\xi_W\right)+(1-\phi_W)\omega_W}(\widehat{mrs}_t - \widehat{w}_t) \\
&\quad + \frac{\xi_W(\phi_W+(1-\phi_W)\omega_W)}{\phi_W\left(1+\bar{\Gamma}^H\beta\xi_W\right)+(1-\phi_W)\omega_W}\widehat{\pi}_{t-1}^W + \frac{\beta\bar{\Gamma}^H\phi_W}{\phi_W\left(1+\bar{\Gamma}^H\beta\xi_W\right)+(1-\phi_W)\omega_W} m^h \mathbf{E}_t \widehat{\pi}_{t+1}^W && \text{(A.73)}
\end{aligned}$$

Final-Output Firms

$$\begin{aligned}
\widehat{z}_t &= \alpha_{zz}\widehat{z}_t^z + (1-\alpha_{zz})(\widehat{e}_t^z) && \text{(A.74)} \\
\widehat{z}_t^z &= \alpha_v \widehat{v}_t + (1-\alpha_v)\widehat{m}_t^z && \text{(A.75)} \\
\widehat{m}_t^z &= \frac{\psi_M(\widehat{m}_{t-1}^z - \widehat{\gamma}_t^z)}{\frac{1}{\varepsilon_v(\Gamma_{ss}^H\Gamma_{ss}^z)^2} + \psi_M(1+\beta\Gamma_{ss}^H)} + \frac{\psi_M\beta\Gamma_{ss}^H m^f \mathbf{E}_t(\widehat{m}_{t+1}^z + \widehat{\gamma}_{t+1}^z)}{\frac{1}{\varepsilon_v(\Gamma_{ss}^H\Gamma_{ss}^z)^2} + \psi_M(1+\beta\Gamma_{ss}^H)} + \frac{\widehat{mc}_t^{zz} - \widehat{p}_t^m + \widehat{z}_t^z/\varepsilon_v}{\frac{1}{\varepsilon_v} + \psi_M(1+\beta\Gamma_{ss}^H)(\Gamma_{ss}^H\Gamma_{ss}^z)^2} - \widehat{\varepsilon}_t^M && \text{(A.76)} \\
\widehat{v}_t &= \frac{\psi_V(\widehat{v}_{t-1} - \widehat{\gamma}_t^z)}{\frac{1}{\varepsilon_v(\Gamma_{ss}^H\Gamma_{ss}^z)^2} + \psi_V(1+\beta\Gamma_{ss}^H)} + \frac{\psi_V\beta\Gamma_{ss}^H m^f \mathbf{E}_t(\widehat{v}_{t+1} + \widehat{\gamma}_{t+1}^z)}{\frac{1}{\varepsilon_v(\Gamma_{ss}^H\Gamma_{ss}^z)^2} + \psi_V(1+\beta\Gamma_{ss}^H)} + \frac{\widehat{mc}_t^{zz} - \widehat{p}_t^v + \widehat{z}_t^z/\varepsilon_v}{\frac{1}{\varepsilon_v} + \psi_V(1+\beta\Gamma_{ss}^H)(\Gamma_{ss}^H\Gamma_{ss}^z)^2} && \text{(A.77)} \\
\widehat{e}_t^z &= \frac{\psi_E(\widehat{e}_{t-1}^z - \widehat{\gamma}_t^z)}{\frac{1}{\varepsilon_e(\Gamma_{ss}^H\Gamma_{ss}^z)^2} + \psi_E(1+\beta\Gamma_{ss}^H)} + \frac{\psi_E\beta\Gamma_{ss}^H m^f \mathbf{E}_t(\widehat{e}_{t+1}^z + \widehat{\gamma}_{t+1}^z)}{\frac{1}{\varepsilon_e(\Gamma_{ss}^H\Gamma_{ss}^z)^2} + \psi_E(1+\beta\Gamma_{ss}^H)} + \frac{\widehat{mc}_t^z - \widehat{p}_t^e + \widehat{z}_t/\varepsilon_e}{\frac{1}{\varepsilon_e} + \psi_E(1+\beta\Gamma_{ss}^H)(\Gamma_{ss}^H\Gamma_{ss}^z)^2} && \text{(A.78)} \\
\widehat{mc}_t^z &= \alpha_{zz}\widehat{mc}_t^{zz} + (1-\alpha_{zz})\widehat{p}_t^e && \text{(A.79)} \\
\widehat{\pi}_t^z &= \frac{(1-\phi_Z)(1-\omega_Z-\phi_Z\beta\Gamma_{ss}^H(1-\omega_Z))}{\phi_Z(1+\xi_Z\beta\Gamma_{ss}^H)+(1-\phi_Z)\omega_Z}\widehat{mc}_t^z + \frac{\xi_Z(\phi_Z+(1-\phi_Z)\omega_Z)}{\phi_Z(1+\xi_Z\beta\Gamma_{ss}^H)+(1-\phi_Z)\omega_Z}\widehat{\pi}_{t-1}^z \\
&\quad + \frac{\beta\Gamma_{ss}^H\phi_Z}{\phi_Z(1+\xi_Z\beta\Gamma_{ss}^H)+(1-\phi_Z)\omega_Z} m^f \mathbf{E}_t \widehat{\pi}_{t+1}^z + \widehat{\mu}_t^z + \widehat{\mu}_t^{z,temp} && \text{(A.80)}
\end{aligned}$$

Value-added Firms

$$\widehat{v}_t = (1-\alpha_L)(\widehat{k}_{t-1}-\widehat{\gamma}_t^z)+\alpha_L\widehat{l}_t+\widehat{\varepsilon}_t^{tfp} \tag{A.81}$$

$$\widehat{l}_t = \psi^L\frac{\widehat{l}_{t-1}-\widehat{\gamma}_t^L}{1+\psi^L(1+\beta\Gamma^H)}+\psi^L\beta\Gamma^H m^f\frac{E_t\widehat{l}_{t+1}+E_t\widehat{\gamma}_{t+1}^L}{1+\psi^L(1+\beta\Gamma^H)}+\frac{\widehat{v}_t+\widehat{p}_t^v+\widehat{mc}_t^v-\widehat{w}_t}{1+\psi^L(1+\beta\Gamma^H)} \tag{A.82}$$

$$\widehat{k}_{t-1} = \widehat{v}_t+\widehat{p}_t^v+\widehat{mc}_t^v-\widehat{rk}_t+\widehat{\gamma}_t^z \tag{A.83}$$

$$\widehat{p}_t^v = \widehat{\pi}_t^v-\widehat{\pi}_t^z+\widehat{p}_{t-1}^v \tag{A.84}$$

$$\begin{aligned}\widehat{\pi}_t^v &= \frac{(1-\phi_V)(1-\omega_V-\phi_V\beta\Gamma_{ss}^H(1-\omega_V))}{\phi_V(1+\xi_V\beta\Gamma_{ss}^H)+(1-\phi_V)\omega_V}\widehat{mc}_t^v+\frac{\xi_V(\phi_V+(1-\phi_V)\omega_V)}{\phi_V(1+\xi_V\beta\Gamma_{ss}^H)+(1-\phi_V)\omega_V}\widehat{\pi}_{t-1}^v\\ &\quad+\frac{\beta\Gamma_{ss}^H\phi_V}{\phi_V(1+\xi_V\beta\Gamma_{ss}^H)+(1-\phi_V)\omega_V}m^f\mathbf{E}_t\widehat{\pi}_{t+1}^v+\widehat{\mu}_t^v\end{aligned} \tag{A.85}$$

Non-Energy Import Firms

$$\widehat{mc}_t^m = \widehat{p}_t^{xf}-\widehat{q}_t-\widehat{p}_t^m \tag{A.86}$$

$$\widehat{p}_t^{xf} = \widehat{\pi}_t^{xf}+\widehat{p}_{t-1}^{xf}-\widehat{\pi}_t^{cf} \tag{A.87}$$

$$\widehat{p}_t^m = \widehat{\pi}_t^m-\widehat{\pi}_t^z+\widehat{p}_{t-1}^m \tag{A.88}$$

$$\begin{aligned}\widehat{\pi}_t^m &= \frac{(1-\phi_M)(1-\omega_M-\phi_M\beta\Gamma_{ss}^H(1-\omega_M))}{\phi_M(1+\xi_M\beta\Gamma_{ss}^H)+(1-\phi_M)\omega_M}\widehat{mc}_t^m\\ &\quad+\frac{\xi_M(\phi_M+(1-\phi_M)\omega_M)}{\phi_M(1+\xi_M\beta\Gamma_{ss}^H)+(1-\phi_M)\omega_M}\widehat{\pi}_{t-1}^m+\frac{\beta\Gamma_{ss}^H\phi_M}{\phi_M(1+\xi_M\beta\Gamma_{ss}^H)+(1-\phi_M)\omega_M}m^f\mathbf{E}_t\widehat{\pi}_{t+1}^m+\widehat{\mu}_t^m\end{aligned} \tag{A.89}$$

Energy Import Firms

$$\widehat{mc}_t^e = \widehat{p}_t^{ef}-\widehat{q}_t-\widehat{p}_t^e \tag{A.90}$$

$$\widehat{p}_t^{ef} = \rho_{pe}\widehat{p}_{t-1}^{ef}+(1-\rho_{pe}^2)^{1/2}\sigma_{pe}\eta_t^{pe} \tag{A.91}$$

$$\begin{aligned}\widehat{\pi}_t^E &= \frac{(1-\phi_E)(1-\omega_E-\phi_E\beta\Gamma_{ss}^H(1-\omega_E))}{\phi_E(1+\xi_E\beta\Gamma_{ss}^H)+(1-\phi_E)\omega_E}\widehat{mc}_t^e\\ &\quad+\frac{\xi_E(\phi_E+(1-\phi_E)\omega_E)}{\phi_E(1+\xi_E\beta\Gamma_{ss}^H)+(1-\phi_E)\omega_E}\widehat{\pi}_{t-1}^E+\frac{\beta\Gamma_{ss}^H\phi_E}{\phi_E(1+\xi_E\beta\Gamma_{ss}^H)+(1-\phi_E)\omega_E}m^f\mathbf{E}_t\widehat{\pi}_{t+1}^E\end{aligned} \tag{A.92}$$

Export Firms

$$\widehat{p}_t^{exp} = \widehat{\pi}_t^{exp}-\widehat{\pi}_t^{cf}+\widehat{p}_{t-1}^{exp} \tag{A.93}$$

$$\begin{aligned}\widehat{\pi}_t^{exp} &= \frac{(1-\phi_X)(1-\omega_X-\phi_X\beta\Gamma_{ss}^H(1-\omega_X))}{\phi_X(1+\xi_X\beta\Gamma_{ss}^H)+(1-\phi_X)\omega_X}\widehat{mc}_t^x+\frac{\xi_X(\phi_X+(1-\phi_X)\omega_X)}{\phi_X(1+\xi_X\beta\Gamma_{ss}^H)+(1-\phi_X)\omega_X}\widehat{\pi}_{t-1}^{exp} \quad (A.94)\\ &\quad+\frac{\beta\Gamma_{ss}^H\phi_X}{\phi_X(1+\xi_X\beta\Gamma_{ss}^H)+(1-\phi_X)\omega_X}m^f\mathbf{E}_t\widehat{\pi}_{t+1}^{exp}+\widehat{\mu}_t^x\end{aligned}$$

$$\widehat{mc}_t^x = \widehat{q}_t-\widehat{p}_t^{exp} \tag{A.95}$$

Retailers

$$\widehat{c}_t^z = \widehat{c}_t+\varepsilon_{ce}\widehat{p}_t^c \tag{A.96}$$

$$\widehat{c}_t^e = \widehat{c}_t+\varepsilon_{ce}(\widehat{p}_t^c-\widehat{p}_t^e) \tag{A.97}$$

$$\widehat{p}_t^c = (1-\alpha_{cz})\widehat{p}_t^e \tag{A.98}$$

Fiscal and Monetary Policy

$$\widehat{g}_t = \widehat{g}_{t-1} - \widehat{\gamma}_t^z + (\rho_g - 1)\widehat{g}_{t-1} + \widehat{\varepsilon}_t^g \tag{A.99}$$

$$\widehat{r}_t = \widehat{rule}_t + \widehat{\varepsilon}_t^r \tag{A.100}$$

$$\widehat{rule}_t = (1-\theta_r)\left(\frac{\theta_\pi}{4}(\widehat{\pi}_t^{CPI,annual} - (1-\theta_E)\times\widehat{\text{econt}}_t - \widehat{\mu}_t^{z,temp,ann}) + \theta_y \widehat{y}_t^{gap}\right) + \theta_r \widehat{r}_{t-1} \tag{A.101}$$

$$\widehat{y}_t^{gap} = \widehat{v}_t - \widehat{v}_t^{flex} \tag{A.102}$$

$$\widehat{\pi}_t^{c,annual} = \widehat{\pi}_t^c + \widehat{\pi}_t^{c,l1} + \widehat{\pi}_t^{c,l2} + \widehat{\pi}_{t-1}^{c,l2} \tag{A.103}$$

$$\widehat{\pi}_t^{c,l1} = \widehat{\pi}_{t-1}^c \tag{A.104}$$

$$\widehat{\pi}_t^{c,l2} = \widehat{\pi}_{t-1}^{c,l1} \tag{A.105}$$

World

$$\widehat{v}_t^f = \frac{1}{1+\psi_C^f} m^h \mathbf{E}_t \widehat{v}_{t+1}^f + \frac{\psi_{C,f}}{1+\psi_C^f}\widehat{v}_{t-1}^f - \left(\frac{1-\psi_C^f}{\varepsilon_C^f(1+\psi_C^f)}\right)\left(\widehat{r}_t^f - m^h \mathbf{E}_t \widehat{\pi}_{t+1}^{cf} - m^h \mathbf{E}_t \widehat{\gamma}_{t+1}^{v,f}\right) + (1-\rho_{cf})\widehat{\varepsilon}_t^{cf} \tag{A.106}$$

$$\widehat{mc}_t^{v,f} = \left(\frac{\varepsilon_{lf} + 1 - \alpha_{lf}}{\alpha_{lf}} + \frac{\varepsilon_C^f}{1-\psi_C^f}\right)\widehat{v}_t^f - \left(\varepsilon_C^f \frac{\psi_C^f}{1-\psi_C^f}\right)\widehat{v}_{t-1}^f \tag{A.107}$$

$$\widehat{\pi}_t^{cf} = \frac{(1-\phi_{vf})(1-\phi_{vf}\beta\Gamma_{ss}^H)}{\phi_{vf}(1+\xi^{vf}\beta\Gamma_{ss}^H)}\widehat{mc}_t^{v,f} + \frac{\xi^{vf}}{1+\xi^{vf}\beta\Gamma_{ss}^H}\widehat{\pi}_{t-1}^{cf} + \frac{\beta\Gamma_{ss}^H}{1+\xi^{vf}\beta\Gamma_{ss}^H} m^h \mathbf{E}_t \widehat{\pi}_{t+1}^{cf} + \widehat{\mu}_t^{v,f} \tag{A.108}$$

$$\widehat{z}_t^f = \widehat{v}_t^f + \phi_{Zf}\varepsilon_t^{zf} \tag{A.109}$$

$$\widehat{x}_t = \widehat{v}_t^f + \phi_{Zf}\varepsilon_t^{zf} + \varepsilon_t^{\kappa,f} - \varepsilon_f\left(\widehat{p}_t^{exp} - \widehat{p}_t^{xf}\right) + \widehat{\omega}_t^f \tag{A.110}$$

$$\widehat{r}_t^f = \theta_{rf}\widehat{r}_{t-1}^f + (1-\theta_{rf})\left(\frac{\theta_{\pi,f}}{4}\widehat{\pi}_t^{cf,annual} + \theta_{vf}\left(\widehat{v}_t^f - \widehat{v}_t^{f,flex}\right)\right) + \varepsilon_t^{r,f} \tag{A.111}$$

$$\widehat{\pi}_t^{cf,annual} = \widehat{\pi}_t^{cf} + \widehat{\pi}_t^{cf,l1} + \widehat{\pi}_t^{cf,l2} + \widehat{\pi}_{t-1}^{cf,l2} \tag{A.112}$$

$$\widehat{\pi}_t^{cf,l1} = \widehat{\pi}_{t-1}^{cf} \tag{A.113}$$

$$\widehat{\pi}_t^{cf,l2} = \widehat{\pi}_{t-1}^{cf,l1} \tag{A.114}$$

Growth Rates

$$\widehat{\gamma}_t^{v,f} = \widehat{\varepsilon}_t^{lap} + \widehat{\gamma}_t^l \tag{A.115}$$

$$\widehat{\gamma}_t^z = \widehat{\varepsilon}_t^{lap} + \widehat{\gamma}_t^l - \left(\widehat{\omega}_t^f - \widehat{\omega}_{t-1}^f\right) \tag{A.116}$$

$$\widehat{\gamma}_t^l = \sigma^{nonL}\eta_t^{nonL} \tag{A.117}$$

Forcing Processes

$$\widehat{\varepsilon}_t^g = (1-\rho_g^2)^{1/2}\sigma_g\eta_t^g \qquad \text{(Government spending forcing process)}$$

$$\widehat{\varepsilon}_t^c = \rho_c\widehat{\varepsilon}_{t-1}^c + (1-\rho_c^2)^{1/2}\sigma_c\eta_t^c \qquad \text{(Consumption preference forcing process)}$$

$$\widehat{\varepsilon}_t^m = \rho_m\widehat{\varepsilon}_{t-1}^m + (1-\rho_m^2)^{1/2}\sigma_m\eta_t^m \qquad \text{(Import demand forcing process)}$$

$$\widehat{\varepsilon}_t^r = \sigma_r\eta_t^r \qquad \text{(Interest rate forcing process)}$$

$$\widehat{\varepsilon}_t^i = \rho_i\widehat{\varepsilon}_{t-1}^i + (1-\rho_i^2)^{1/2}\sigma_i\eta_t^i \qquad \text{(Investment forcing process)}$$

$$\widehat{\varepsilon}_t^{lap} = \rho_{lap}\widehat{\varepsilon}_{t-1}^{lap} + (1-\rho_{lap}^2)^{1/2}\sigma_{lap}\eta_t^{lap} \qquad \text{(Labour augmenting productivity growth forcing process)}$$

$$\widehat{\varepsilon}_t^l = \rho_l\widehat{\varepsilon}_{t-1}^l + (1-\rho_l^2)^{1/2}\sigma_l\eta_t^l \qquad \text{(Labour supply forcing process)}$$

$$\widehat{\mu}_t^m = \rho_{mum}\widehat{\mu}_{t-1}^m + (1-\rho_{mum}^2)^{1/2}\sigma_{mum}\eta_t^{mum} \qquad \text{(Import markup forcing process)}$$

$$\widehat{\mu}_t^x = \rho_{mux}\widehat{\mu}_{t-1}^x + (1-\rho_{mux}^2)^{1/2}\sigma_{mux}\eta_t^{mux} \qquad \text{(Export markup forcing process)}$$

$$\widehat{\mu}_t^z = \rho_{muz}\widehat{\mu}_{t-1}^z + (1-\rho_{muz}^2)^{1/2}\sigma_{muz}\eta_t^{muz} \qquad \text{(Final output markup forcing process)}$$

$$\widehat{\mu}_t^v = \rho_{muv}\widehat{\mu}_{t-1}^v + (1-\rho_{muv}^2)^{1/2}\sigma_{muv}\eta_t^{muv} \qquad \text{(Value added markup forcing process)}$$

$$\widehat{\mu}_t^w = \rho_{muw}\widehat{\mu}_{t-1}^w + (1-\rho_{muw}^2)^{1/2}\sigma_{muw}\eta_t^{muw} \qquad \text{(Wage markup forcing process)}$$

$$\widehat{\mu}_t^{vf} = \rho_{muvf}\widehat{\mu}_{t-1}^{vf} + (1-\rho_{muvf}^2)^{1/2}\sigma_{muvf}\eta_t^{muvf} \qquad \text{(World markup forcing process)}$$

$$\widehat{\varepsilon}_t^{io} = ((1-\rho_{io}^2)/(1-\rho_{etaio}^2))^{1/2}\sigma_{io}(etaiot - rhoetaiosetaiot - 1) \qquad \text{(Other investment forcing process)}$$

$$\omega_t^f = \rho_{omegaf}\omega_{t-1}^f + (1-\rho_{omegaf}^2)^{1/2}\sigma_{omegaf}\eta_t^{omegaf} \qquad \text{(Relative productivity)}$$

$$\widehat{\varepsilon}_t^b = \rho_b\widehat{\varepsilon}_{t-1}^b + (1-\rho_b^2)^{1/2}\sigma_b\eta_t^b \qquad \text{(Risk premium forcing process)}$$

$$setaio_t = etaio_t \qquad \text{(Shadow residual component of total final expenditure shock)}$$

$$\widehat{\varepsilon}_t^{tfp} = \rho_{tfp}\widehat{\varepsilon}_{t-1}^{tfp} + (1-\rho_{tfp}^2)^{1/2}\sigma_{tfp}\eta_t^{tfp} \qquad \text{(TFP forcing process)}$$

$$\widehat{\varepsilon}_t^{bf} = \rho_{bf}\widehat{\varepsilon}_{t-1}^{bf} + (1-\rho_{bf}^2)^{1/2}\sigma_{bf}\eta_t^{bf} \qquad \text{(UIP risk premium forcing process)}$$

$$\widehat{\varepsilon}_t^{cf} = \rho_{cf}\widehat{\varepsilon}_{t-1}^{cf} + (1-\rho_{cf}^2)^{1/2}\sigma_{cf}\eta_t^{cf} \qquad \text{(World consumption preference forcing process)}$$

$$\widehat{\varepsilon}_t^{zf} = \rho_{zf}\widehat{\varepsilon}_{t-1}^{zf} + (1-\rho_{zf}^2)^{1/2}\sigma_{zf}\eta_t^{zf} \qquad \text{(World demand forcing process)}$$

$$\widehat{p}_t^{xf} = \rho_{pxf}\widehat{p}_{t-1}^{xf} + (1-\rho_{pxf}^2)^{1/2}\sigma_{pxf}\eta_t^{pxf} \qquad \text{(World export price (relative to world final output price) process)}$$

$$\widehat{\varepsilon}_t^{rf} = \rho_{rf}\widehat{\varepsilon}_{t-1}^{rf} + (1-\rho_{rf}^2)^{1/2}\sigma_{rf}\eta_t^{rf} \qquad \text{(World monetary policy forcing process)}$$

$$\widehat{\varepsilon}_t^{kappaf} = \rho_{kappaf}\widehat{\varepsilon}_{t-1}^{kappaf} + (1-\rho_{kappaf}^2)^{1/2}\sigma_{kappaf}\eta_t^{kappaf} \qquad \text{(World preference forcing process)}$$

Auxiliary Variables and Definitions

$$\widehat{\mu}_t^{z,temp} \equiv \sigma^{\mu,z,temp}\eta_t^{muz,temp} \quad \text{(Temporary final output price markup forcing process)}$$

$$\widehat{\mu}_t^{z,temp,l1} \equiv \widehat{\mu}_{t-1}^{z,temp} \quad \text{(Definition of first lag of temp final output markup)}$$

$$\widehat{\mu}_t^{z,temp,l2} \equiv \widehat{\mu}_{t-1}^{z,temp,l1} \quad \text{(Definition of second lag of temp final output markup)}$$

$$\widehat{\mu}_t^{z,temp,ann} \equiv \widehat{\mu}_t^{z,temp} + \widehat{\mu}_t^{z,temp,l1} + \widehat{\mu}_t^{z,temp,l2} + \widehat{\mu}_{t-1}^{z,temp,l2} \quad \text{(Definition of annual temporary final output markup)}$$

$$\widehat{\pi}_t^{e} \equiv \widehat{\pi}_t^{z} + \widehat{p}_t^{e} - \widehat{p}_{t-1}^{e} \quad \text{(Energy price inflation)}$$

$$\widehat{\pi}_t^{e,annual} \equiv \widehat{\pi}_t^{e} + \widehat{\pi}_t^{e,l1} + \widehat{\pi}_t^{e,l2} + \widehat{\pi}_{t-1}^{e,l2} \quad \text{(Annual energy price inflation)}$$

$$\widehat{\pi}_t^{e,l1} \equiv \widehat{\pi}_{t-1}^{e} \quad \text{(1st lag of energy price inflation)}$$

$$\widehat{\pi}_t^{e,l2} \equiv \widehat{\pi}_{t-1}^{e,l1} \quad \text{(2nd lag of energy price inflation)}$$

$$enprcontr_t \equiv (1-\alpha_{cz})(\widehat{\pi}_t^{e}) \quad \text{(Direct contribution of energy prices to quarterly inflation)}$$

$$enprcontrann_t \equiv (1-\alpha_{cz})(\widehat{\pi}_t^{e} + \widehat{\pi}_t^{el} + \widehat{\pi}_t^{el2} + \widehat{\pi}_{t-1}^{el2}) \quad \text{(Direct contribution of energy prices to annual inflation)}$$

$$etapeimpact_t \equiv (1-\alpha_{cz})\widehat{p}_t^{e} \quad \text{(Shock based measure of energy shock impact)}$$

$$etapecont_t \equiv etapeimpact_t - etapeimpact_{t-1} \quad \text{(Inflation based shock measure)}$$

$$econt_t \equiv etapecont_t + etapecont_t^{l1} + etapecont_t^{l2} + etapecont_{t-1}^{l2} \quad \text{(Annual shock based contribution)}$$

$$etapecont_t^{l1} \equiv etapecont_{t-1} \quad \text{(Lag of inflation based shock measure)}$$

$$etapecont_t^{l2} \equiv etapecont_{t-1}^{l1} \quad \text{(Second lag of inflation based shock measure)}$$

$$dlngdp_t \equiv \widehat{v}_t - \widehat{v}_{t-1} + \widehat{\gamma}_t^{z} \quad \text{(Demeaned real GDP growth)}$$

$$dlnl_t \equiv \widehat{l}_t - \widehat{l}_{t-1} + \widehat{\gamma}_t^{L} \quad \text{(Demeaned hours worked growth)}$$

$$dlnlabprod_t \equiv dlngdp_t - dlnl_t \quad \text{(Demeaned labour productivity growth)}$$

$$dlne_t \equiv \widehat{q}_t - \widehat{q}_{t-1} - \widehat{\pi}_t^{z} + \widehat{\pi}_t^{cf} \quad \text{(Demeaned nominal exchange rate change)}$$

$$dlnwnom_t \equiv \widehat{\pi}_t^{w} \quad \text{(Demeaned nominal wage growth)}$$

$$dlnk_t \equiv \widehat{k}_t - \widehat{k}_{t-1} + \widehat{\gamma}_t^{z} \quad \text{(Demeaned real capital stock)}$$

$$dlnc_t \equiv \widehat{c}_t - \widehat{c}_{t-1} + \widehat{\gamma}_t^{z} \quad \text{(Demeaned real consumption growth)}$$

$$dlnx_t \equiv \widehat{x}_t - \widehat{x}_{t-1} + \widehat{\gamma}_t^{z} \quad \text{(Demeaned real export growth)}$$

$$dlnpx_t \equiv \widehat{p}_t^{exp} - \widehat{p}_{t-1}^{exp} - \widehat{q}_t + \widehat{q}_{t-1} + \widehat{\pi}_t^{z} \quad \text{(Demeaned export price inflation)}$$

$$dlng_t \equiv \widehat{g}_t - \widehat{g}_{t-1} + \widehat{\gamma}_t^{z} \quad \text{(Demeaned real government spending growth)}$$

$$dlnm_t \equiv \widehat{m}_t^{z} - \widehat{m}_{t-1}^{z} + \widehat{\gamma}_t^{z} \quad \text{(Demeaned real import growth)}$$

$$dlni_t \equiv \widehat{i}_t - \widehat{i}_{t-1} + \widehat{\gamma}_t^{z} \quad \text{(Demeaned real investment growth)}$$

$$dlnio_t \equiv \widehat{i}_t^{O} - \widehat{i}_{t-1}^{O} + \widehat{\gamma}_t^{z} \quad \text{(Demeaned residual component of real TFE growth)}$$

$$dlnw_t \equiv \widehat{w}_t - \widehat{w}_{t-1} + (\widehat{\gamma}_t^{z} - \widehat{\gamma}_t^{L}) \quad \text{(Demeaned wage growth)}$$

$$dlnyf_t \equiv \widehat{z}_t^{f} - \widehat{z}_{t-1}^{f} + \widehat{\gamma}_t^{yf} \quad \text{(Demeaned World trade growth)}$$

$$dlnvf_t \equiv \widehat{v}_t^{f} - \widehat{v}_{t-1}^{f} + \widehat{\gamma}_t^{yf} \quad \text{(Demeaned World GDP growth)}$$

$$dlngdpstar_t \equiv \widehat{v}_t^{flex} - \widehat{v}_{t-1}^{flex} + \widehat{\gamma}_t^{z} \quad \text{(Demeaned real potential GDP growth)}$$

Measurement Equations

$$dlnckp_t = dlnc_t + 100*(log(\Gamma^z_{ss}) + log(\Gamma^h_{ss}) + log(\Gamma^c_{ss})) \quad \text{(Consumption growth measurement equation)}$$
$$dlncpisa_t = \widehat{\pi}^c_t + 100*(log(pistar)) \quad \text{(CPI inflation measurement equation)}$$
$$dlnpxdef_t = dlnpx_t + 100*(log(pistar) - log(\Gamma^x_{ss})) \quad \text{(Export price inflation measurement equation)}$$
$$dlnxkp_t = dlnx_t + 100*(log(\Gamma^z_{ss}) + log(\Gamma^h_{ss}) + log(\Gamma^x_{ss})) \quad \text{(Exports growth measurement equation)}$$
$$dlngdpkp_t = dlngdp_t + 100*(log(\Gamma^z_{ss}) + log(\Gamma^h_{ss})) \quad \text{(GDP growth measurement equation)}$$
$$dlnpmdef_t = \widehat{\pi}^m_t + 100*(log(pistar) - log(\Gamma^x_{ss})) \quad \text{(Import price inflation measurement equation)}$$
$$dlnmkp_t = dlnm_t + 100*(log(\Gamma^z_{ss}) + log(\Gamma^h_{ss}) + log(\Gamma^x_{ss})) \quad \text{(Imports growth measurement equation)}$$
$$dlnikkp_t = dlni_t + 100*(log(\Gamma^z_{ss}) + log(\Gamma^h_{ss}) + log(\Gamma^i_{ss})) \quad \text{(Investment growth measurement equation)}$$
$$dlneer_t = dlne_t + 100*(log(pistar) - log(pistar)) \quad \text{(Nominal exchange rate measurement equation)}$$
$$robs_t = \widehat{r}_t + 100*(log(r_{ss})) \quad \text{(Nominal interest rate measurement equation)}$$
$$dlngonskp_t = dlng_t + 100*(log(\Gamma^z_{ss}) + log(\Gamma^g_{ss}) + log(\Gamma^h_{ss})) \quad \text{(Real government spending growth measurement equation)}$$
$$dlnhrs_t = dlnl_t + 100*log(\Gamma^h_{ss}) \quad \text{(Total hours worked growth measurement equation)}$$
$$dlnaweagg_t = dlnwnom_t + 100*(log(\Gamma^z_{ss}) + log(pistar)) \quad \text{(Wage inflation measurement equation)}$$
$$dlncpif_t = \widehat{\pi}^{cf}_t + 100*log(pistar) \quad \text{(World CPI inflation measurement equation)}$$
$$dlnyfkp_t = dlnyf_t + 100*(log(\Gamma^z_{ss}) + log(\Gamma^h_{ss})) \quad \text{(World demand growth measurement equation)}$$
$$dlnpxfdef_t = \widehat{\pi}^{xf}_t + 100*(log(pistar) - log(\Gamma^x_{ss})) \quad \text{(World export price inflation measurement equation)}$$
$$dlngdpfkp_t = dlnvf_t + 100*(log(\Gamma^z_{ss}) + log(\Gamma^h_{ss})) \quad \text{(World GDP growth measurement equation)}$$
$$robsf_t = \widehat{r}^f_t + 100*log(r_{ss}) \quad \text{(World interest rate measurement equation)}$$

A.14 Data Sources

Table A.1: DATA INPUTS

Code	Description	Data source	Conditioning path
gdpkp	Real GDP	ONS: ABMI	
ckp	Real consumption	ONS: ABJR+HAYO	
gonskp[1]	Real government expenditure	ONS: NMRY+DLWF	OBR and Bank calculations
ikkp[1]	Real business investment	ONS: GAN8	
mkp[2]	Real imports	ONS: IKBL - OFNN × (BQKO/BOKH)	
xkp[2]	Real exports	ONS: IKBK - OFNN × (BQKO/BOKH)	
hrs	Whole economy total hours worked	ONS: YBUS	
cpisa[3]	Consumer Price Index level	Bank calculations and ONS: D7BT	
aweagg[4]	Whole economy Average Weekly Earnings	ONS: KAB9	
encontqoq[5]	Direct energy contribution to CPI inflation	Bank calculation and ONS: D7CH and D7EC	Oil, gas and electricity wholesale futures prices
pmdef[2]	Imports deflator	ONS: (IKBI - OFNN) / (IKBL - (OFNN × BKQO / BOKH))	
pxdef[2]	Exports deflator	ONS: (IKBH - OFNN) / (IKBL - (OFNN × BKQO / BOKH))	
rgawqe[6]	Bank Rate (adjusted for QE)	Bank of England: IUQABEDR and Bank calculations	UK instantaneous nominal forward curve (OIS) and Bank calculations
eer	Sterling effective exchange rate	Bank of England: XUQABK67	50/50 UIP / random walk
gdpfkp	UK-weighted world GDP	Bank calculations: ONS Pink Book and national sources	IMF WEO and Bank calculations
yf	UK-weighted world imports	Bank calculations: ONS Pink Book and national sources	IMF WEO and Bank calculations
cpif	UK-weighted world consumer price	Bank calculations: ONS Pink Book and national sources	IMF WEO and Bank calculations
pxfdef	UK-weighted world export price	Bank calculations: ONS Pink Book and national sources	Bank calculations
rgaf	UK-weighted world policy rate	Bank calculations: ONS Pink Book and national sources	OIS curves for UK trade partners

[a] Series includes adjustment for exceptional transfer of ownership.
[b] Series includes adjustment for Missing Trader Intra-Community (MTIC) fraud.
[c] Staff seasonally adjust ONS CPI data at monthly frequency.
[d] Staff project series backwards using Average Earnings Index prior to inception of AWE.
[e] Projection incorporates OFGEM's default tariff price cap mechanisms.
[f] QE adjustment reflects staff estimate of shadow policy rate.

B Parameter Estimation

Table B.1: ESTIMATED STRUCTURAL PARAMETERS

Parameters	Prior mean	Posterior mean	90% HPD interval		Distribution	Posterior standard deviation
θ_π	1.500	1.4990	1.4203	1.5783	norm	0.0500
θ_y	0.200	0.2556	0.1955	0.3106	invg	0.0250
θ_E	0.250	0.2464	0.1619	0.3337	beta	0.0500
$\theta_{\pi f}$	1.500	1.4911	1.4158	1.5770	norm	0.0500
θ_{vf}	0.200	0.2763	0.1693	0.3759	invg	0.0500
θ_{rf}	0.950	0.8948	0.8721	0.9170	beta	0.0250
ε_C	1.000	1.3336	1.0893	1.5895	invg	0.1000
ϕ_Z	0.660	0.8554	0.8297	0.8810	beta	0.0500
ϕ_V	0.660	0.6077	0.5270	0.6954	beta	0.0500
ϕ_M	0.660	0.4569	0.4079	0.5105	beta	0.0500
ϕ_E	0.660	0.6244	0.5725	0.6777	beta	0.0500
ϕ_W	0.800	0.8637	0.8298	0.9005	beta	0.0500
ϕ_{vf}	0.800	0.9002	0.8741	0.9276	beta	0.0500
ϕ_X	0.850	0.3293	0.2368	0.4207	beta	0.0750
ξ_Z	0.150	0.1252	0.0569	0.1938	beta	0.0500
ξ_V	0.150	0.1502	0.0661	0.2280	beta	0.0500
ξ_W	0.150	0.0827	0.0402	0.1236	beta	0.0500
ξ_M	0.150	0.0794	0.0366	0.1241	beta	0.0500
ξ_X	0.150	0.0955	0.0417	0.1458	beta	0.0500
ξ_E	0.150	0.0901	0.0396	0.1442	beta	0.0500
ξ^{vf}	0.150	0.1484	0.0713	0.2215	beta	0.0500
ω_Z	0.200	0.2457	0.1534	0.3359	beta	0.0500
ω_V	0.200	0.1837	0.1080	0.2587	beta	0.0500
ω_W	0.400	0.4081	0.3322	0.4834	beta	0.0500
ω_M	0.200	0.1133	0.0664	0.1603	beta	0.0500
ω_X	0.200	0.1200	0.0705	0.1709	beta	0.0500
ω_E	0.200	0.1996	0.1242	0.2798	beta	0.0500
ψ_C	0.200	0.3696	0.2690	0.4689	beta	0.0500
ψ_M	6.000	0.8474	0.4635	1.2434	norm	1.0000
ψ_I	6.000	6.5172	4.4460	8.5219	norm	1.5000
ψ_V	6.000	10.5388	8.8265	12.4488	norm	1.5000
ψ_L	6.000	7.6636	5.5608	10.0857	norm	1.5000
ψ_E	100.000	99.9486	96.4881	103.3741	norm	2.0000
ε_C^f	1.000	1.0755	0.8874	1.2584	invg	0.1000
ε_f	0.350	0.2619	0.1983	0.3348	invg	0.1000
ψ_C^f	0.500	0.8826	0.8394	0.9217	beta	0.1000

Notes: This table presents the prior and posterior moments for the estimated parameters.

Table B.2: ESTIMATED SHOCK VOLATILITY PARAMETERS

Parameters	Prior mean	Posterior mean	90% HPD interval		Distribution	Posterior Standard Deviation
σ_b	0.500	0.6979	0.5509	0.8279	invg	0.2500
σ_{muz}	0.150	0.0786	0.0549	0.1016	invg	0.5000
σ_r	0.100	0.1162	0.1044	0.1284	invg	0.5000
σ_{pe}	30.000	39.1436	28.6024	49.2742	invg	5.0000
σ_g	2.000	3.1392	2.7213	3.5443	invg	0.2500
σ_{bf}	1.250	0.9131	0.7786	1.0313	invg	0.2500
σ_{cf}	0.300	0.3182	0.2308	0.3979	invg	0.2500
σ_{rf}	0.150	0.0755	0.0664	0.0849	invg	0.5000
σ_{muvf}	0.250	0.1703	0.1323	0.2035	invg	0.5000
σ_{zf}	3.500	3.2130	2.7558	3.6640	invg	0.5000
σ_{pxf}	3.000	3.9082	3.2991	4.5408	invg	0.5000
σ_{mum}	1.000	3.0216	2.2263	3.8262	invg	0.2500
σ_{mux}	2.000	4.7106	2.8961	6.2491	invg	0.5000
σ_{muw}	1.000	0.8434	0.7697	0.9143	invg	0.2500
σ_{tfp}	0.500	0.4144	0.2337	0.6012	invg	0.2500
σ_i	2.000	3.1196	2.8129	3.4308	invg	0.2500
σ_m	2.000	2.8217	2.2600	3.3607	invg	0.2500
σ_{kappaf}	2.000	4.1144	3.5451	4.7037	invg	0.2500
σ_{io}	10.000	37.0428	31.3660	43.0562	invg	5.0000
σ_c	0.100	0.2437	0.0456	0.4181	invg	1.0000
σ_{lap}	0.250	0.1468	0.0992	0.1901	invg	0.1500
σ_{nonL}	0.250	0.1604	0.1104	0.2073	invg	0.1500
σ_l	0.750	0.7604	0.4286	1.1046	invg	0.2500
σ_{omegaf}	2.000	1.7792	1.5118	2.0586	invg	0.2500
σ_{muv}	0.100	0.0718	0.0249	0.1232	invg	0.2500
$\sigma_{muztemp}$	0.200	0.2679	0.2291	0.3067	invg	0.2500

Notes: This table presents the prior and posterior moments for the estimated parameters that govern the volatility of the shock processes.

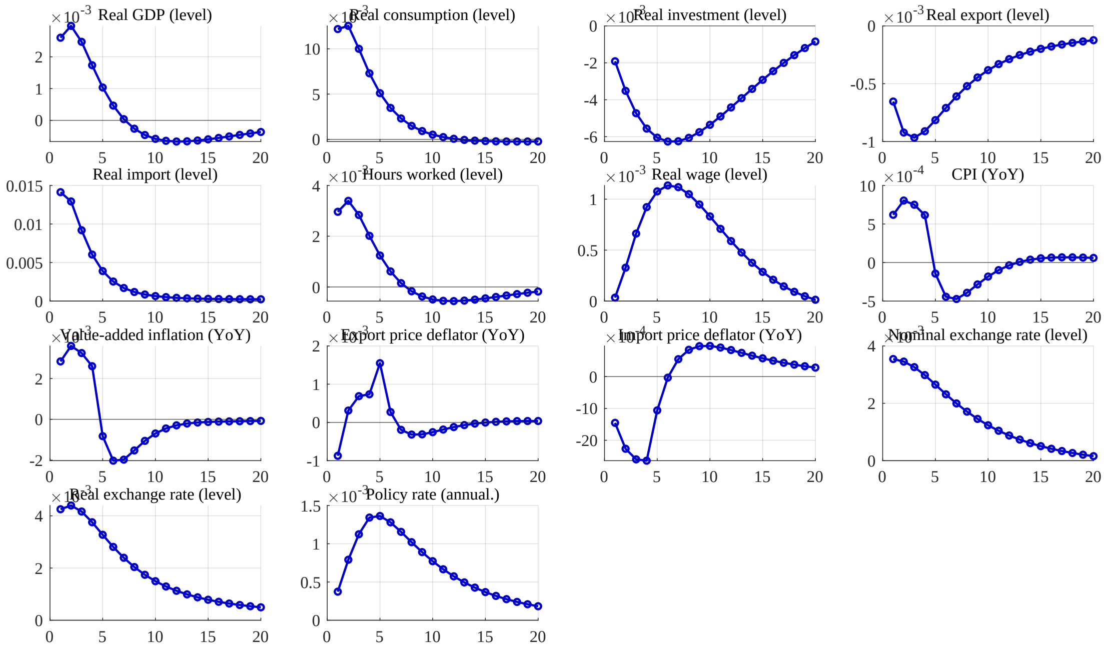
Table B.3: ESTIMATED SHOCK PERSISTENCE PARAMETERS

Parameters	Prior mean	Posterior mean	90% HPD interval		Distribution	Posterior standard deviation
ρ_b	0.700	0.9074	0.8839	0.9324	beta	0.1000
ρ_{pe}	0.400	0.5328	0.4315	0.6411	beta	0.1000
ρ_g	0.700	0.8921	0.8643	0.9192	beta	0.1000
ρ_{bf}	0.700	0.8602	0.8215	0.8967	beta	0.1000
ρ_{cf}	0.700	0.7695	0.7120	0.8369	beta	0.1000
ρ_{rf}	0.250	0.2654	0.1286	0.4022	beta	0.1500
ρ_{muvf}	0.250	0.5387	0.3812	0.7137	beta	0.1500
ρ_{zf}	0.600	0.8780	0.8421	0.9149	beta	0.1000
ρ_{pxf}	0.500	0.9420	0.9232	0.9619	beta	0.2000
ρ_{mum}	0.500	0.9057	0.8717	0.9447	beta	0.2000
ρ_{mux}	0.500	0.7606	0.6673	0.8811	beta	0.2500
ρ_{muw}	0.250	0.0404	0.0021	0.0790	beta	0.1500
ρ_{tfp}	0.500	0.6483	0.3827	0.8995	beta	0.2000
ρ_i	0.700	0.2836	0.1814	0.3756	beta	0.1000
ρ_m	0.700	0.7638	0.7015	0.8250	beta	0.1000
ρ_{kappaf}	0.700	0.8400	0.7945	0.8821	beta	0.1000
ρ_c	0.700	0.6878	0.5209	0.8510	beta	0.1000
ρ_{io}	0.700	0.8095	0.7570	0.8657	beta	0.1000
ρ_l	0.700	0.7157	0.5579	0.8677	beta	0.1000
ρ_{omegaf}	0.700	0.9443	0.9209	0.9711	beta	0.1000

Notes: This table presents the prior and posterior moments for the estimated parameters that govern the persistence of the shock processes.

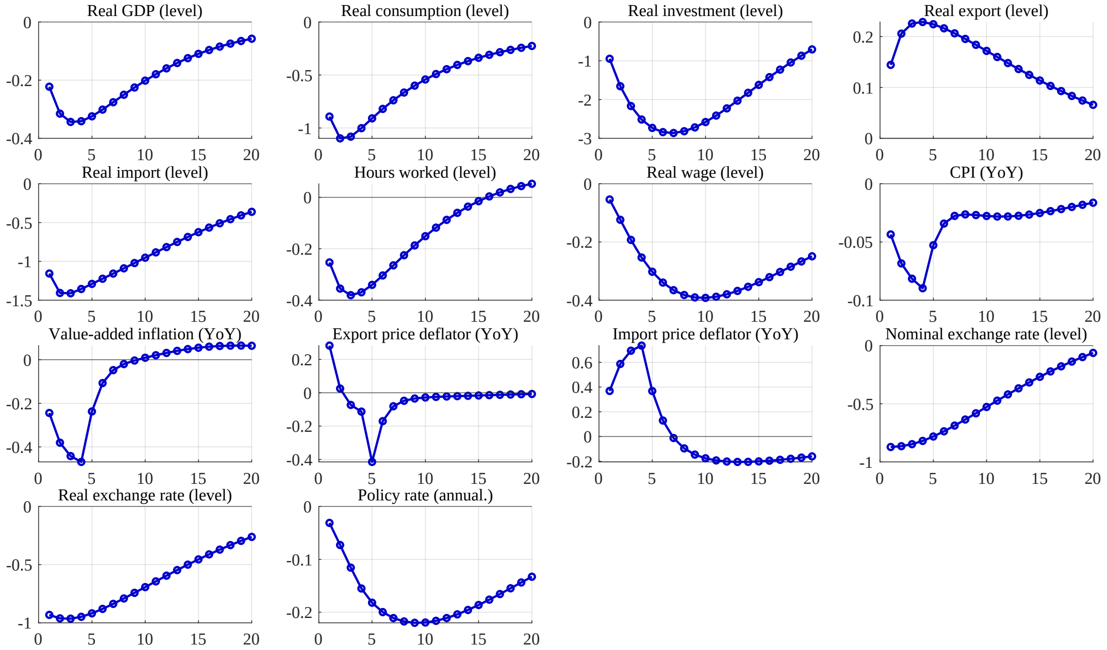
C Model Impulse Response Functions

Figure C.1: CONSUMPTION PREFERENCE SHOCK



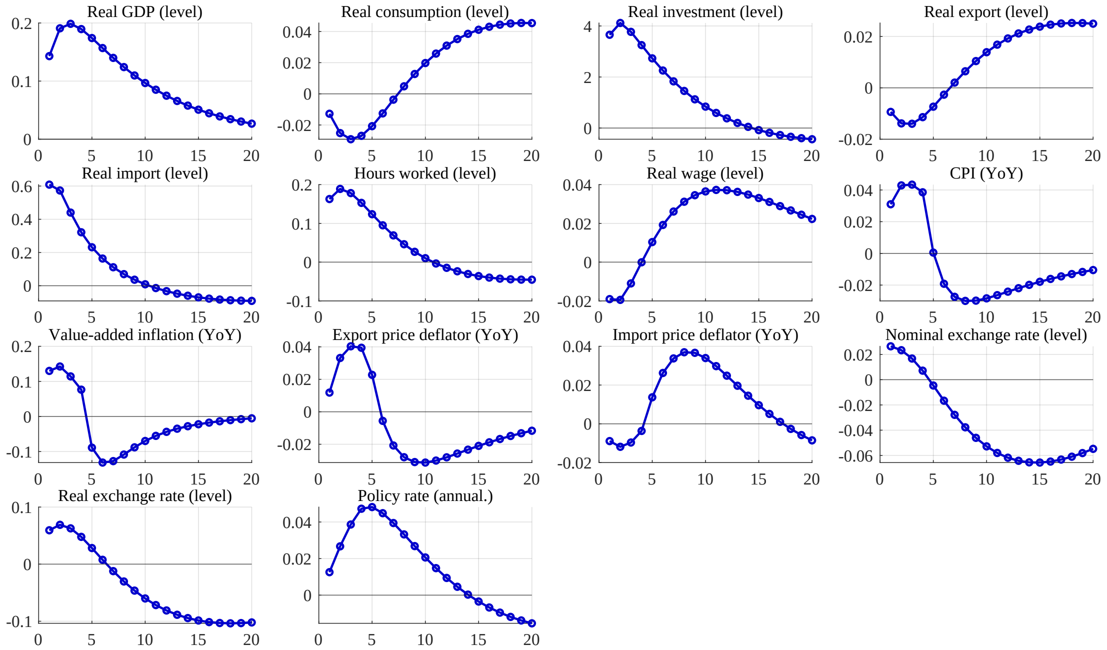
Notes: The charts show the mean responses to a one standard deviation shock. Responses are measured as percentage changes from the steady state for variables expressed in levels, or as percentage points for variables expressed in year-on-year terms and for the policy rate.

Figure C.2: RISK PREMIUM SHOCK



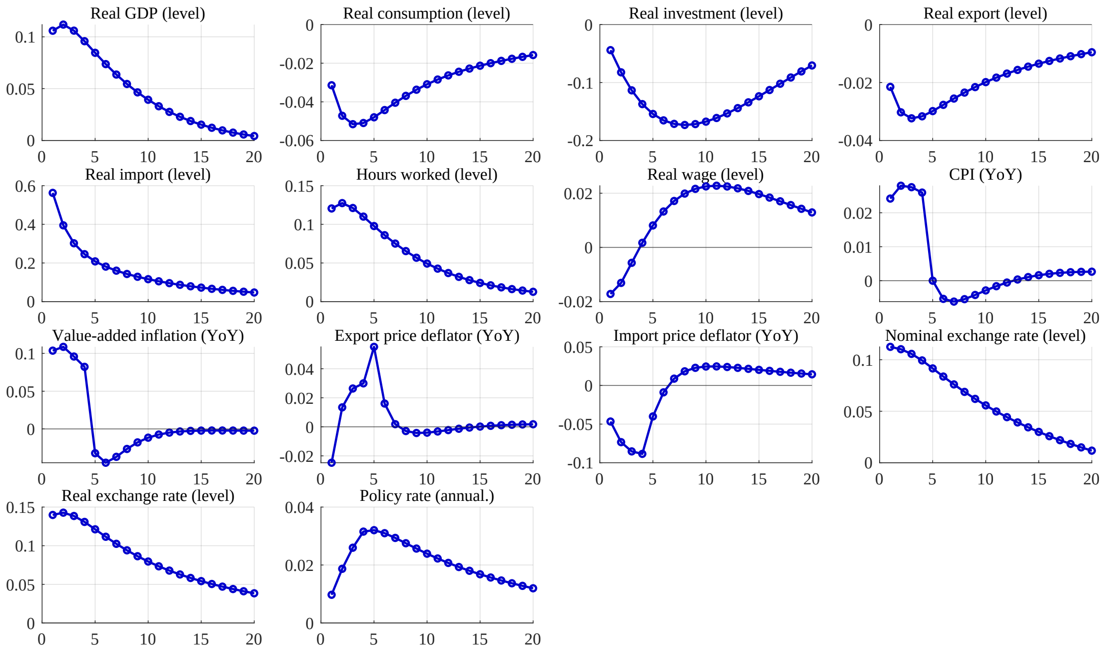
Notes: The charts show the mean responses to a one standard deviation shock. Responses are measured as percentage changes from the steady state for variables expressed in levels, or as percentage points for variables expressed in year-on-year terms and for the policy rate.

Figure C.3: INVESTMENT ADJUSTMENT COST SHOCK



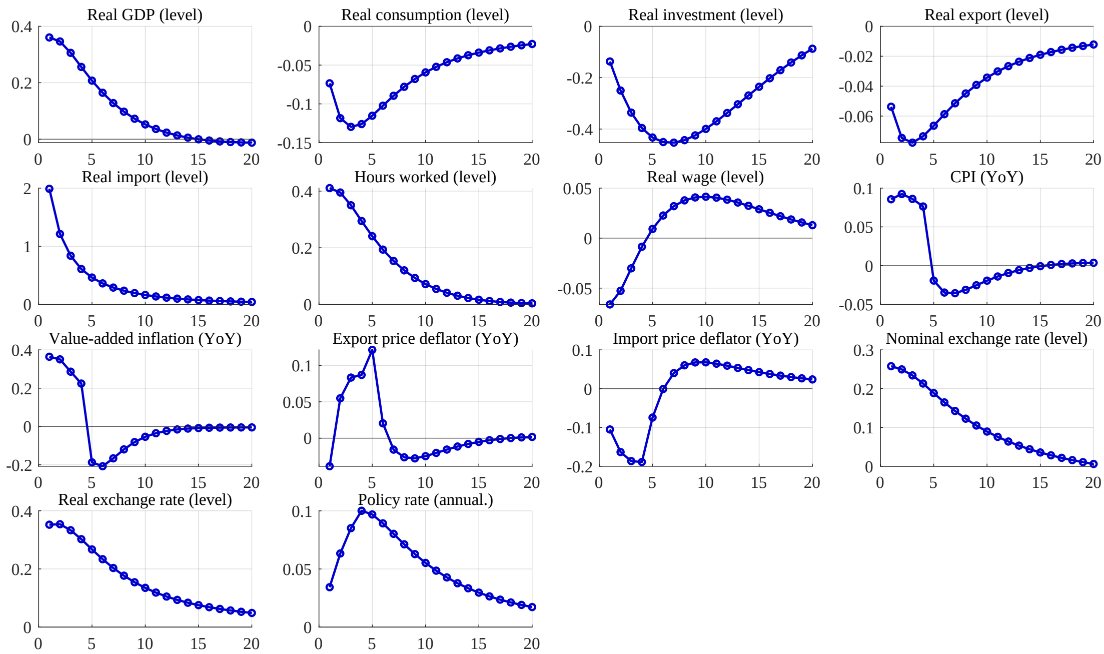
Notes: The charts show the mean responses to a one standard deviation shock. Responses are measured as percentage changes from the steady state for variables expressed in levels, or as percentage points for variables expressed in year-on-year terms and for the policy rate.

Figure C.4: GOVERNMENT SPENDING SHOCK



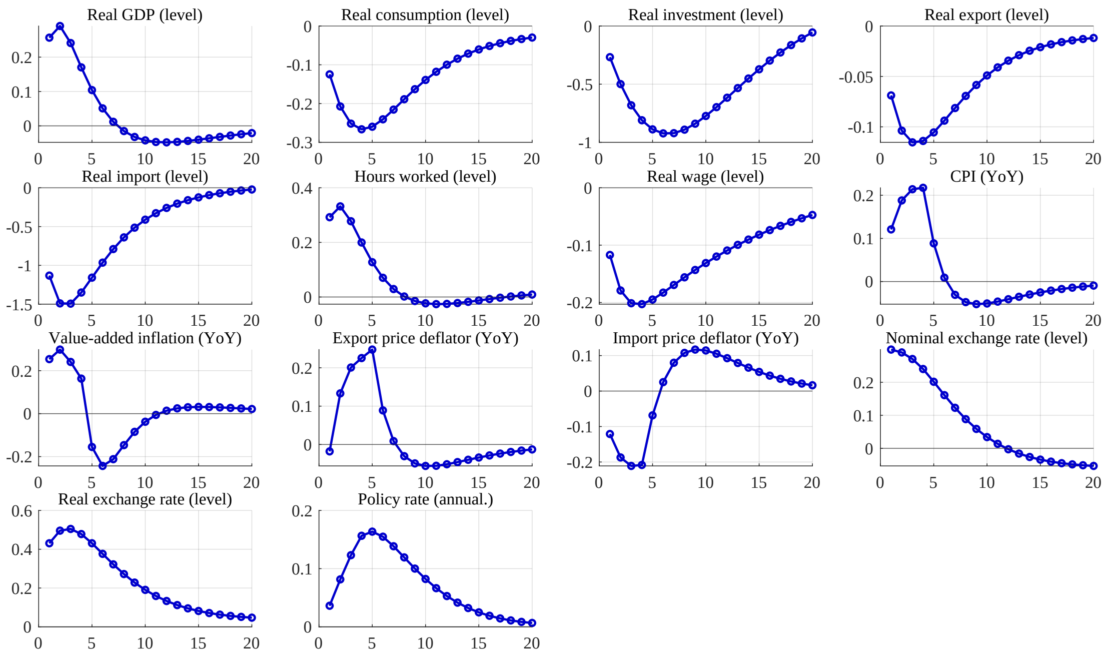
Notes: The charts show the mean responses to a one standard deviation shock. Responses are measured as percentage changes from the steady state for variables expressed in levels, or as percentage points for variables expressed in year-on-year terms and for the policy rate.

Figure C.5: RESIDUAL EXPENDITURE COMPONENT SHOCK



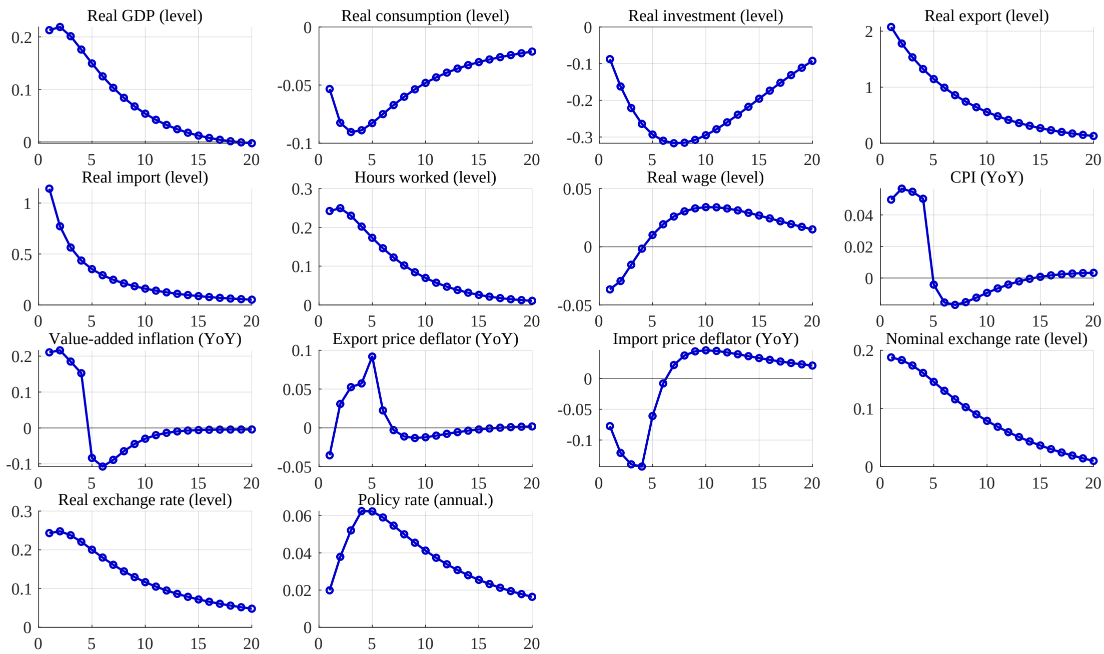
Notes: The charts show the mean responses to a one standard deviation shock. Responses are measured as percentage changes from the steady state for variables expressed in levels, or as percentage points for variables expressed in year-on-year terms and for the policy rate.

Figure C.6: IMPORT PREFERENCE SHOCK



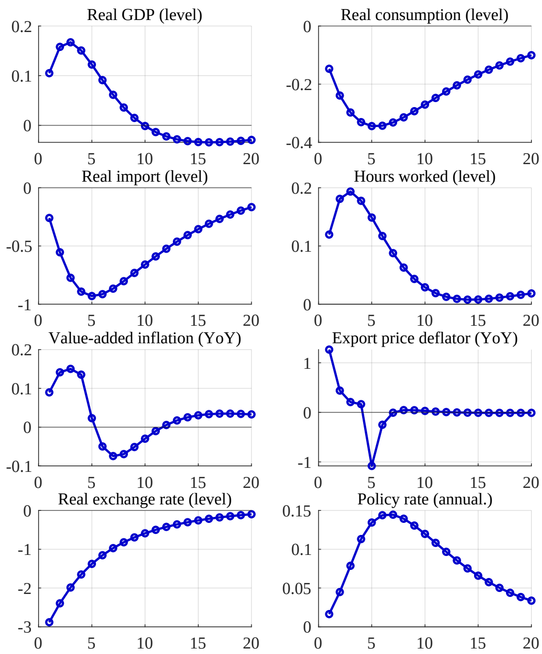
Notes: The charts show the mean responses to a one standard deviation shock. Responses are measured as percentage changes from the steady state for variables expressed in levels, or as percentage points for variables expressed in year-on-year terms and for the policy rate.

Figure C.7: EXPORT PREFERENCE SHOCK



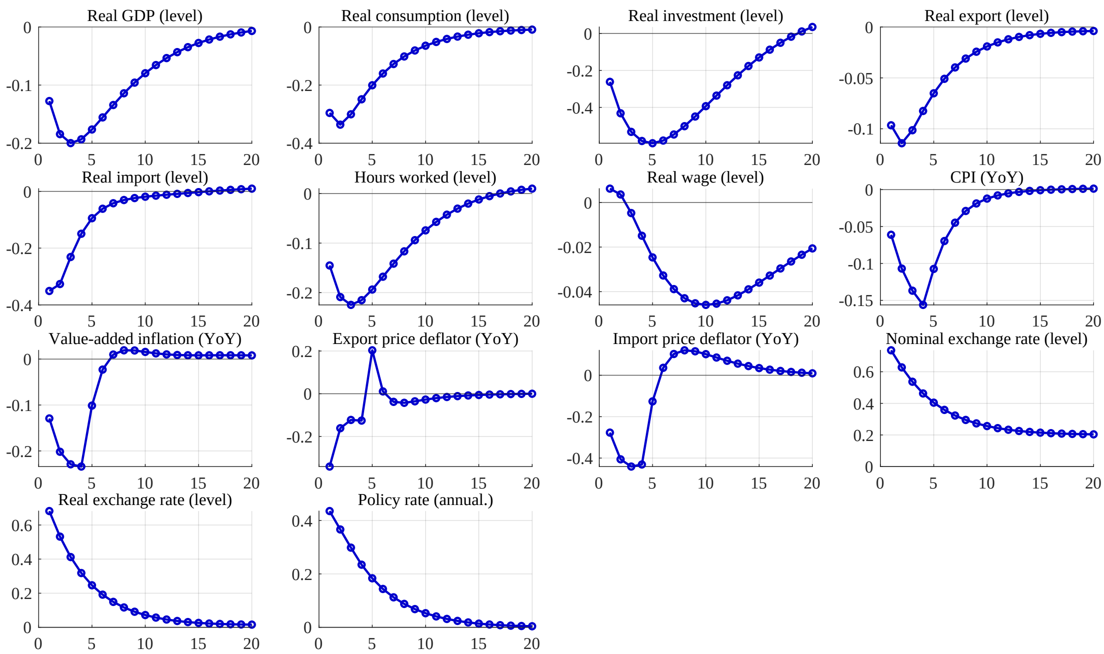
Notes: The charts show the mean responses to a one standard deviation shock. Responses are measured as percentage changes from the steady state for variables expressed in levels, or as percentage points for variables expressed in year-on-year terms and for the policy rate.

Figure C.8: EXCHANGE RATE RISK PREMIUM SHOCK



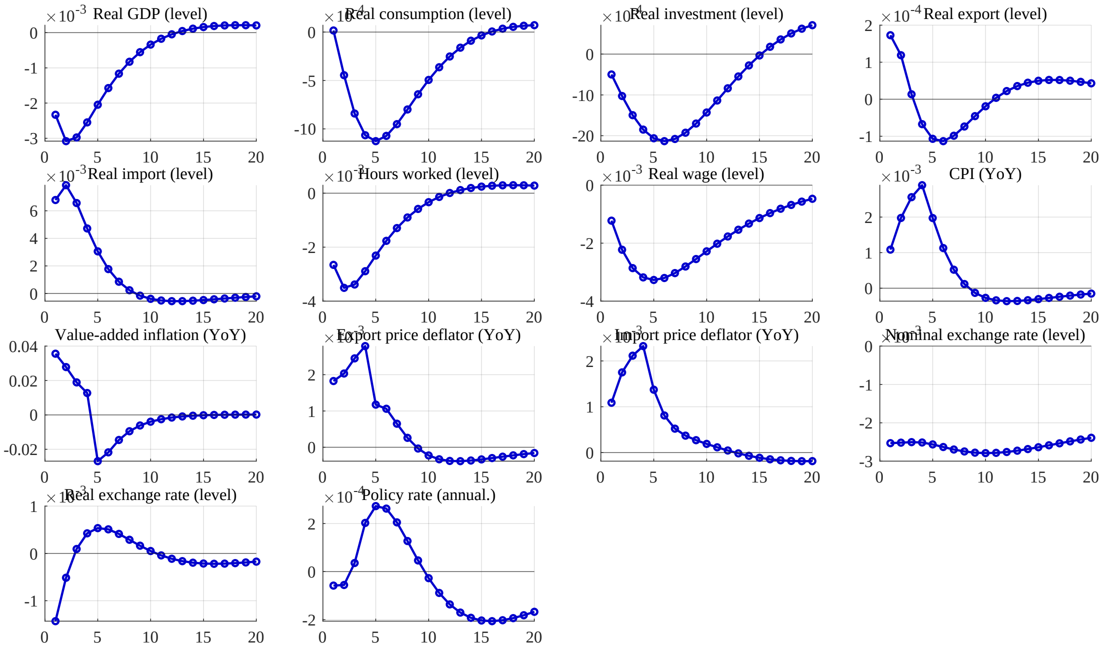
Notes: The charts show the mean responses to a one standard deviation shock. Responses are measured as percentage changes from the steady state for variables expressed in levels, or as percentage points for variables expressed in year-on-year terms and for the policy rate.

Figure C.9: MONETARY POLICY SHOCK



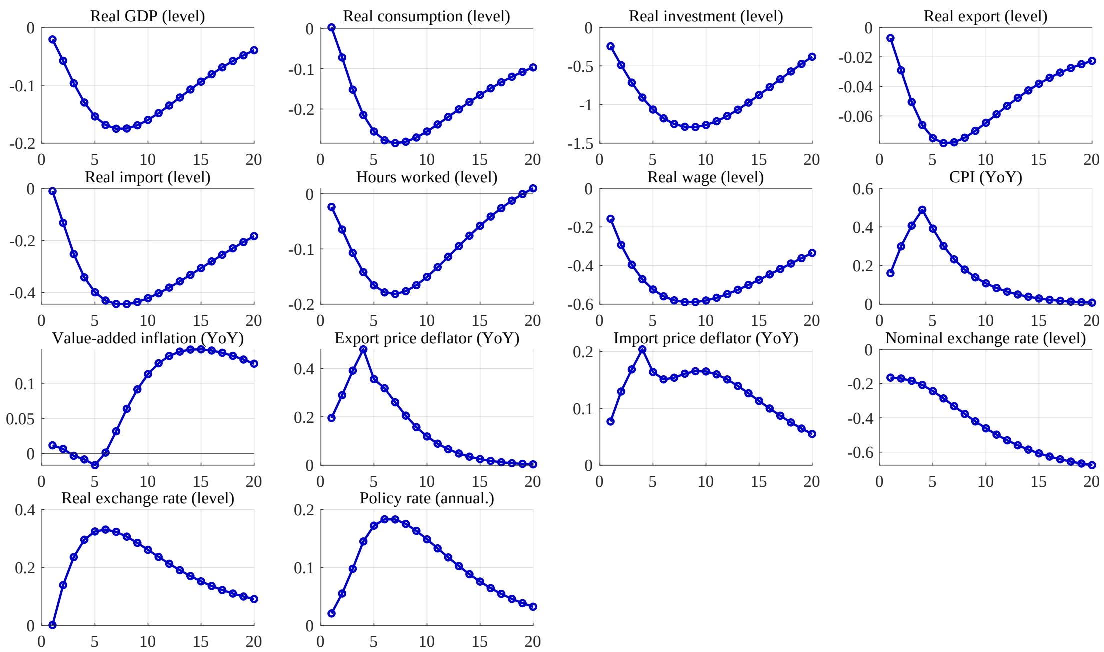
Notes: The charts show the mean responses to a one standard deviation shock. Responses are measured as percentage changes from the steady state for variables expressed in levels, or as percentage points for variables expressed in year-on-year terms and for the policy rate.

Figure C.10: VALUE-ADDED PRICE MARKUP SHOCK



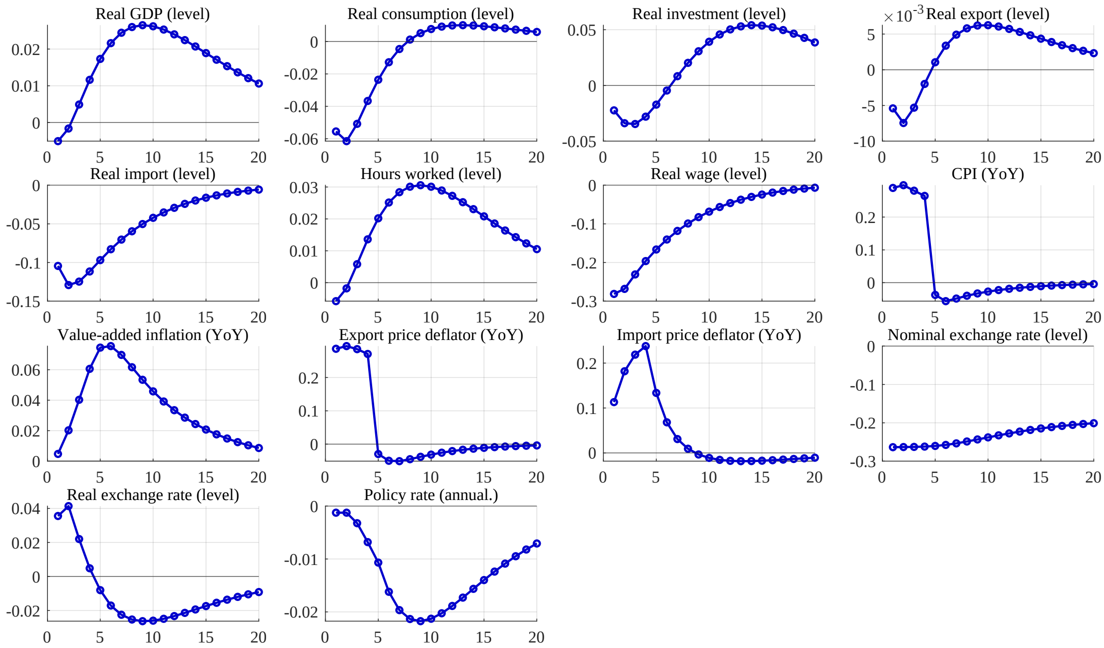
Notes: The charts show the mean responses to a one standard deviation shock. Responses are measured as percentage changes from the steady state for variables expressed in levels, or as percentage points for variables expressed in year-on-year terms and for the policy rate.

Figure C.11: FINAL OUTPUT PRICE MARKUP SHOCK



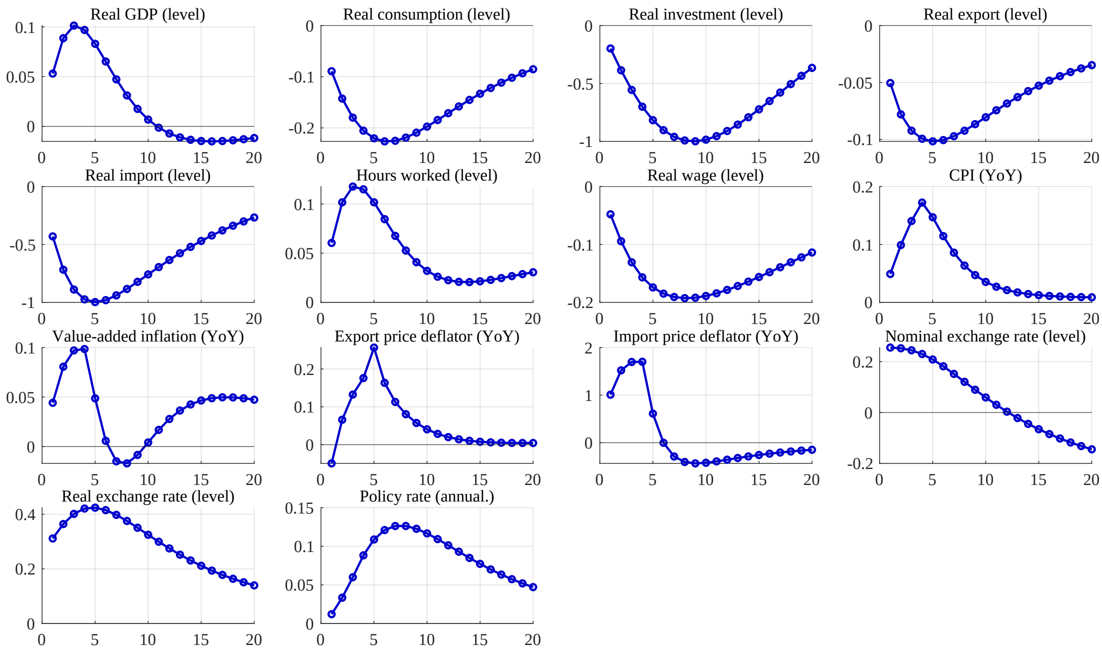
Notes: The charts show the mean responses to a one standard deviation shock. Responses are measured as percentage changes from the steady state for variables expressed in levels, or as percentage points for variables expressed in year-on-year terms and for the policy rate.

Figure C.12: TEMPORARY FINAL OUTPUT PRICE MARKUP SHOCK



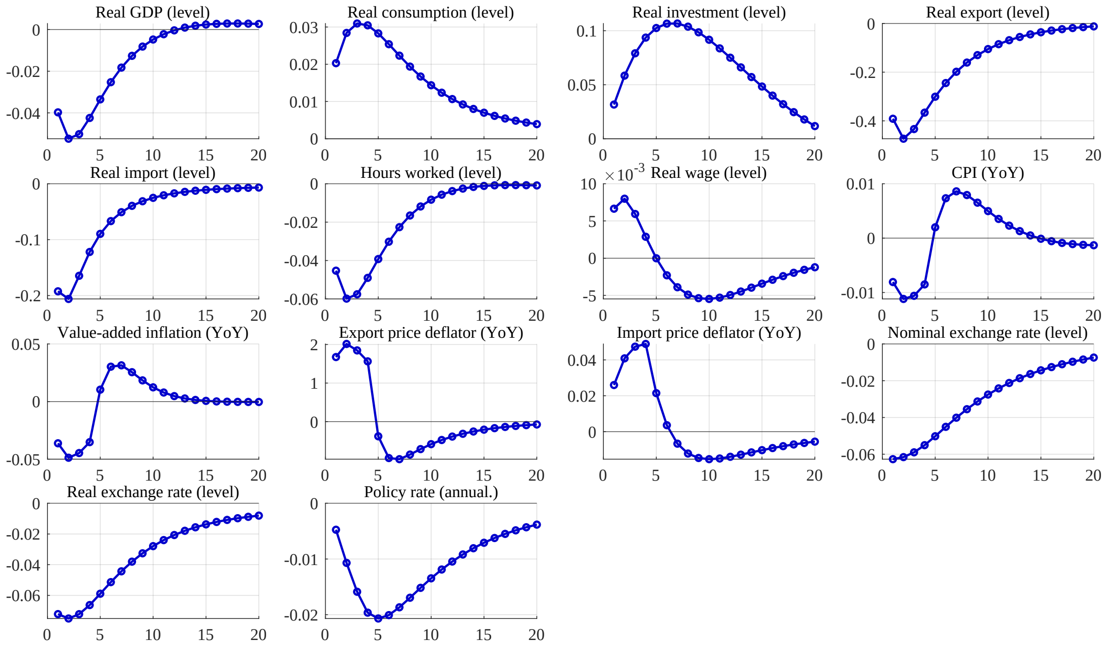
Notes: The charts show the mean responses to a one standard deviation shock. Responses are measured as percentage changes from the steady state for variables expressed in levels, or as percentage points for variables expressed in year-on-year terms and for the policy rate.

Figure C.13: IMPORT PRICE MARKUP SHOCK



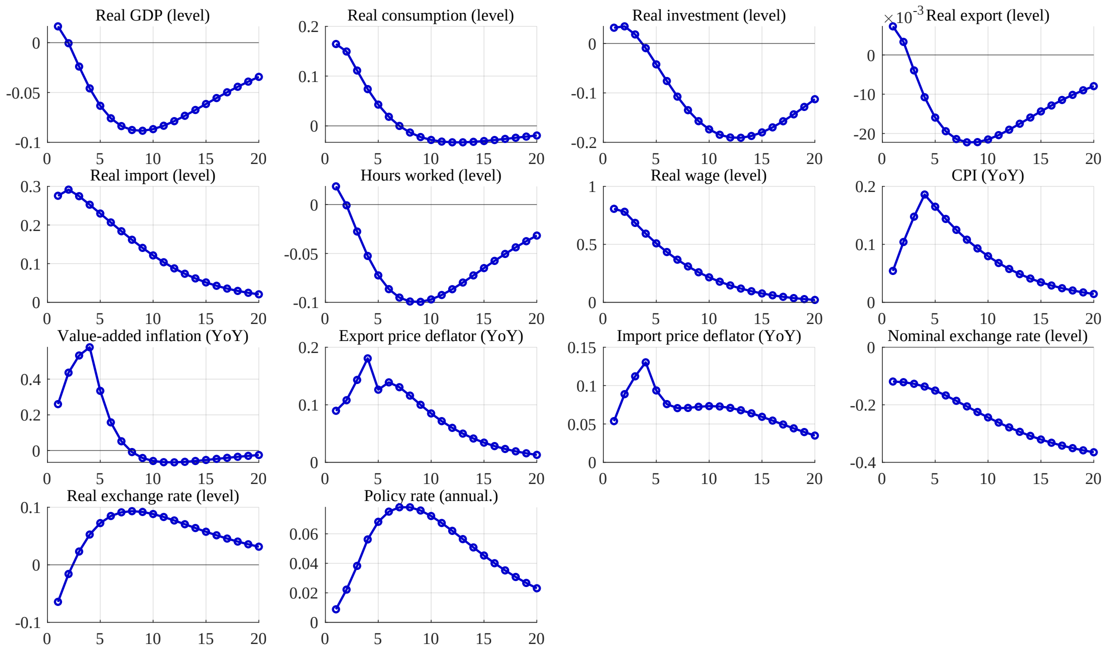
Notes: The charts show the mean responses to a one standard deviation shock. Responses are measured as percentage changes from the steady state for variables expressed in levels, or as percentage points for variables expressed in year-on-year terms and for the policy rate.

Figure C.14: EXPORT PRICE MARKUP SHOCK



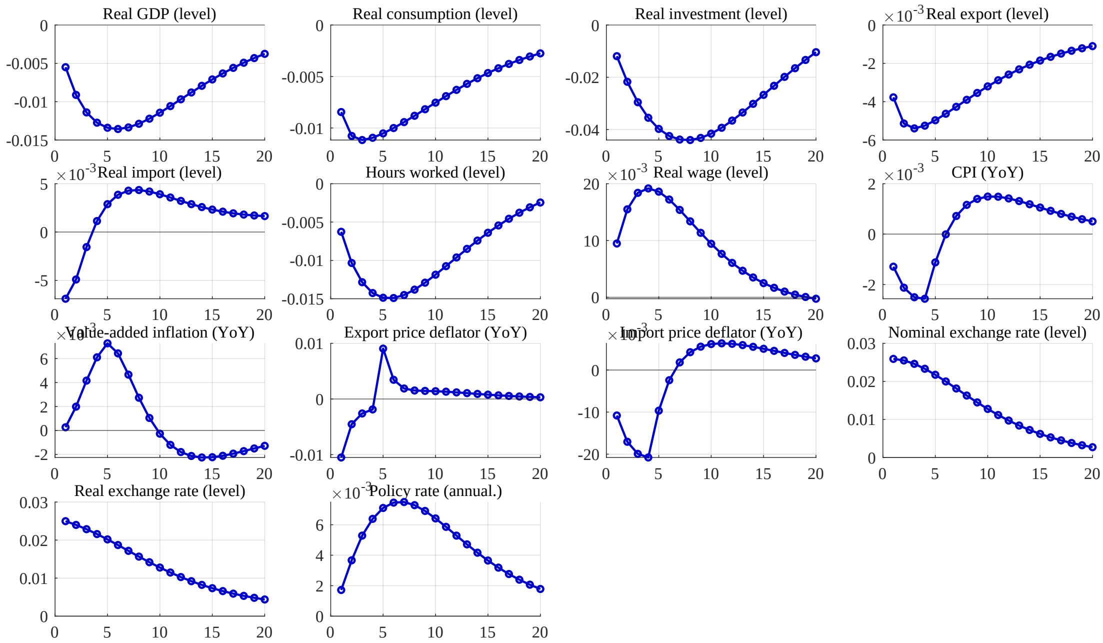
Notes: The charts show the mean responses to a one standard deviation shock. Responses are measured as percentage changes from the steady state for variables expressed in levels, or as percentage points for variables expressed in year-on-year terms and for the policy rate.

Figure C.15: WAGE MARKUP SHOCK



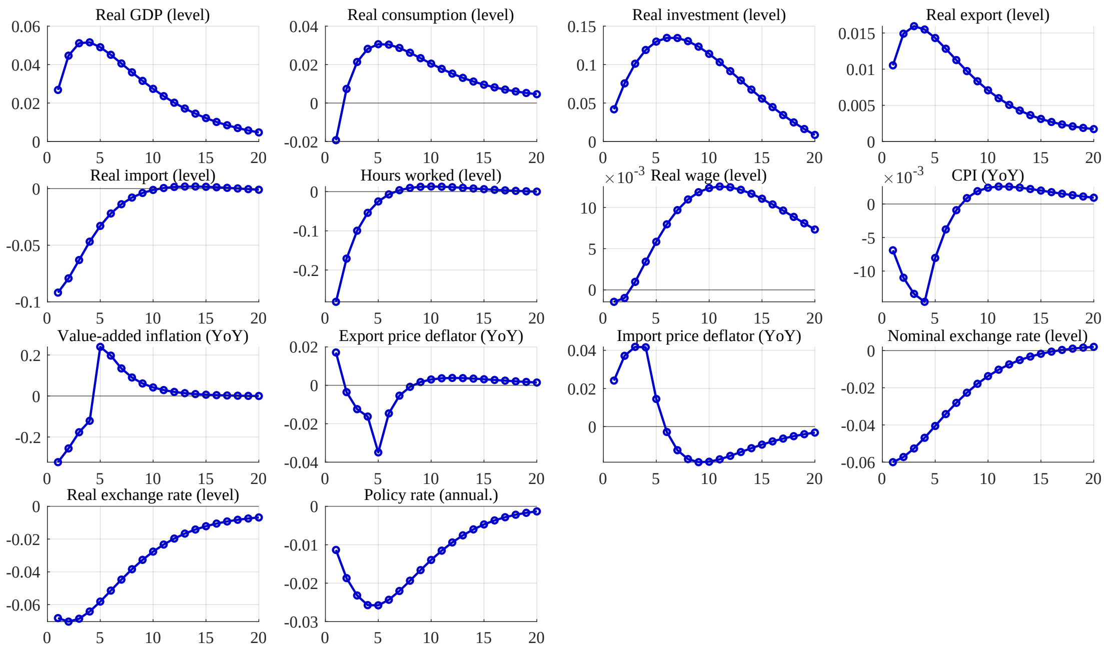
Notes: The charts show the mean responses to a one standard deviation shock. Responses are measured as percentage changes from the steady state for variables expressed in levels, or as percentage points for variables expressed in year-on-year terms and for the policy rate.

Figure C.16: Labour supply shock



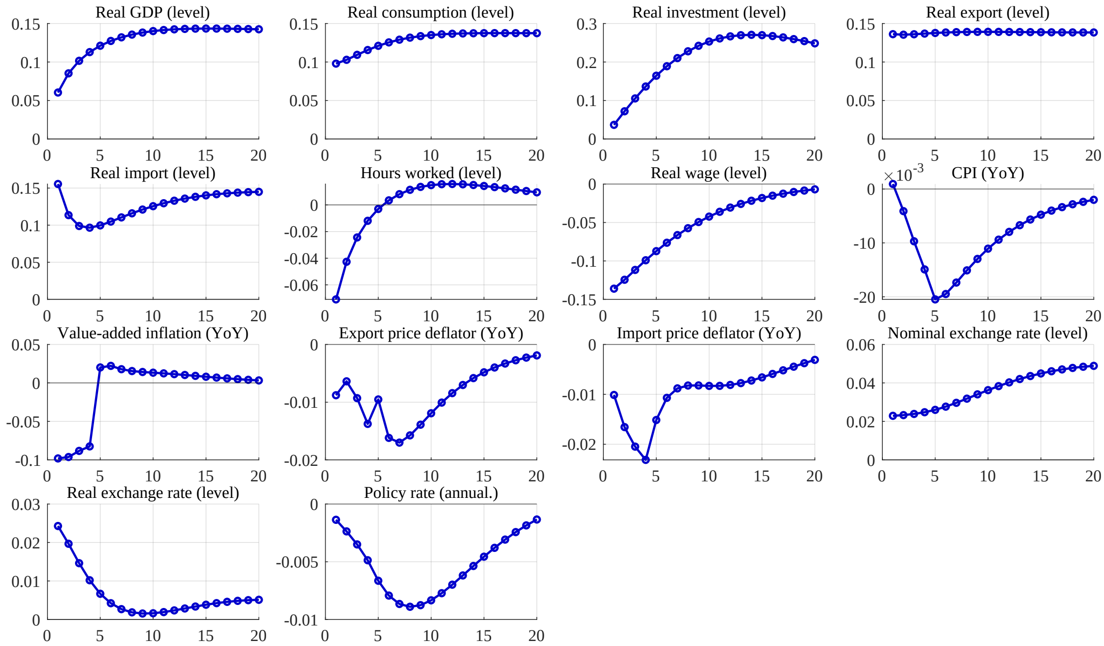
Notes: The charts show the mean responses to a one standard deviation shock. Responses are measured as percentage changes from the steady state for variables expressed in levels, or as percentage points for variables expressed in year-on-year terms and for the policy rate.

Figure C.17: Total factor productivity shock



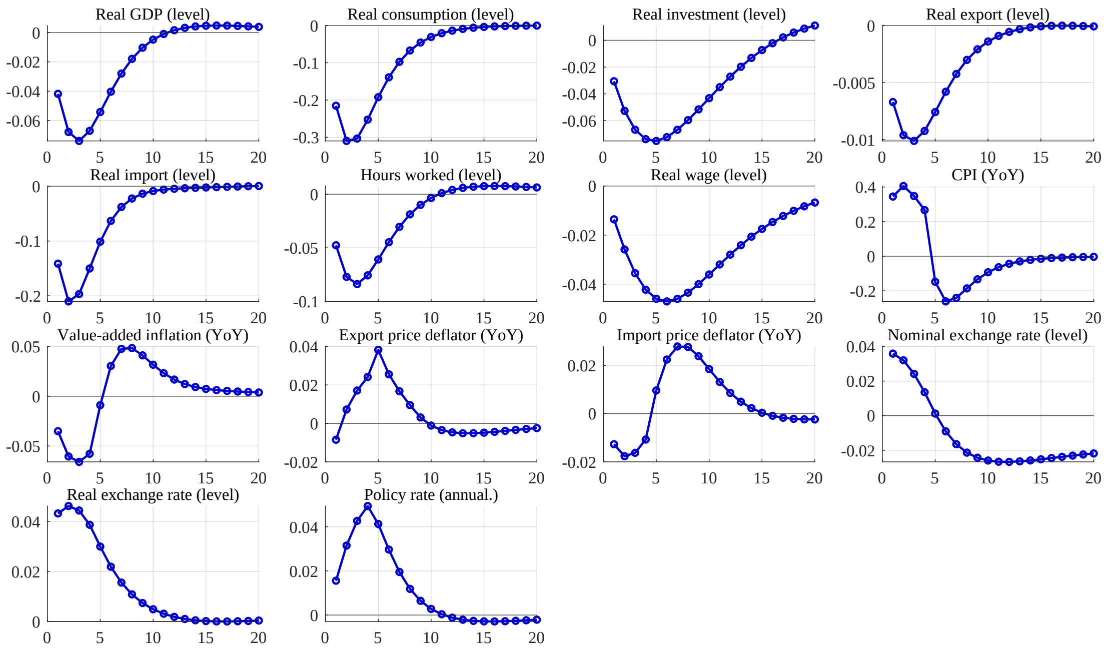
Notes: The charts show the mean responses to a one standard deviation shock. Responses are measured as percentage changes from the steady state for variables expressed in levels, or as percentage points for variables expressed in year-on-year terms and for the policy rate.

Figure C.18: LABOUR-AUGMENTING PRODUCTIVITY SHOCK



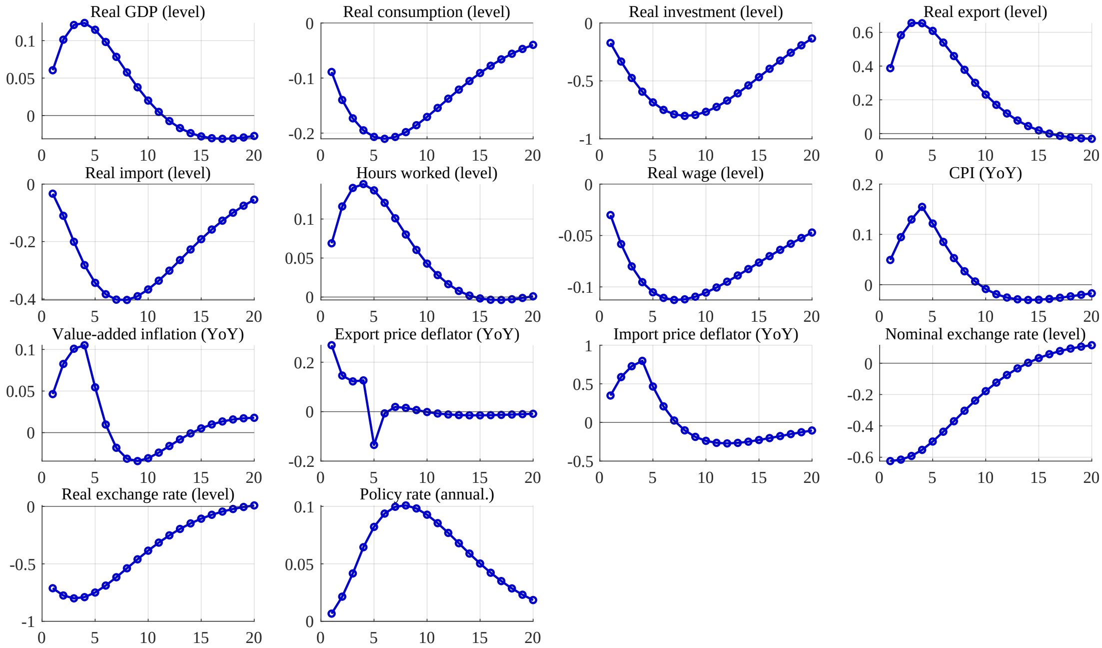
Notes: The charts show the mean responses to a one standard deviation shock. Responses are measured as percentage changes from the steady state for variables expressed in levels, or as percentage points for variables expressed in year-on-year terms and for the policy rate.

Figure C.19: ENERGY PRICE SHOCK



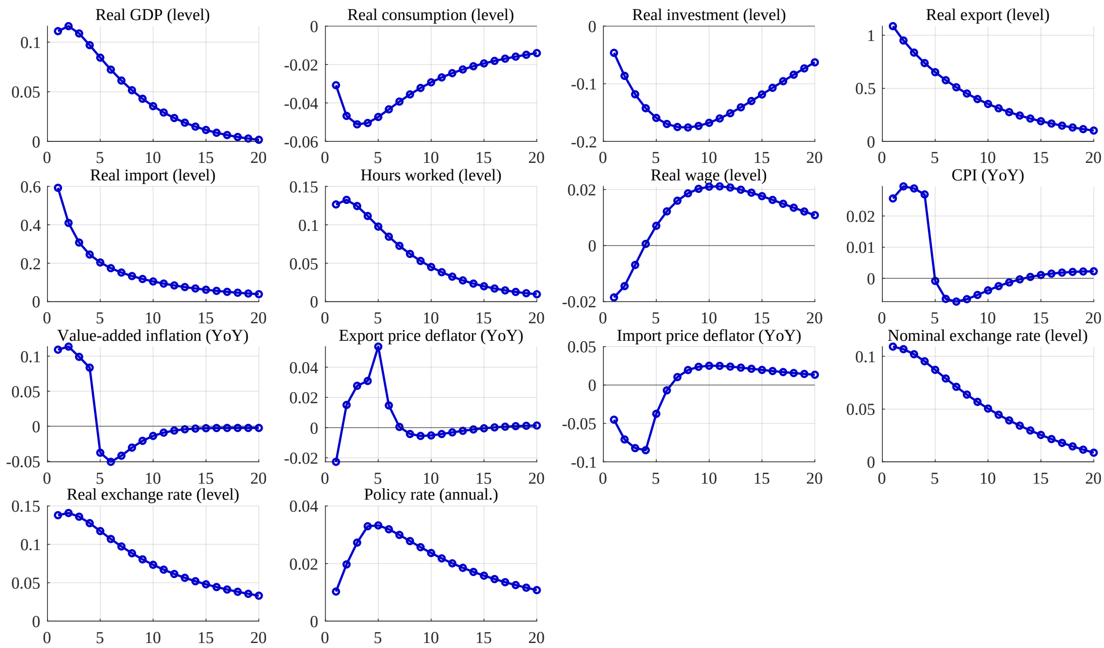
Notes: The charts show the mean responses to a one standard deviation shock. Responses are measured as percentage changes from the steady state for variables expressed in levels, or as percentage points for variables expressed in year-on-year terms and for the policy rate.

Figure C.20: WORLD DEMAND SHOCK



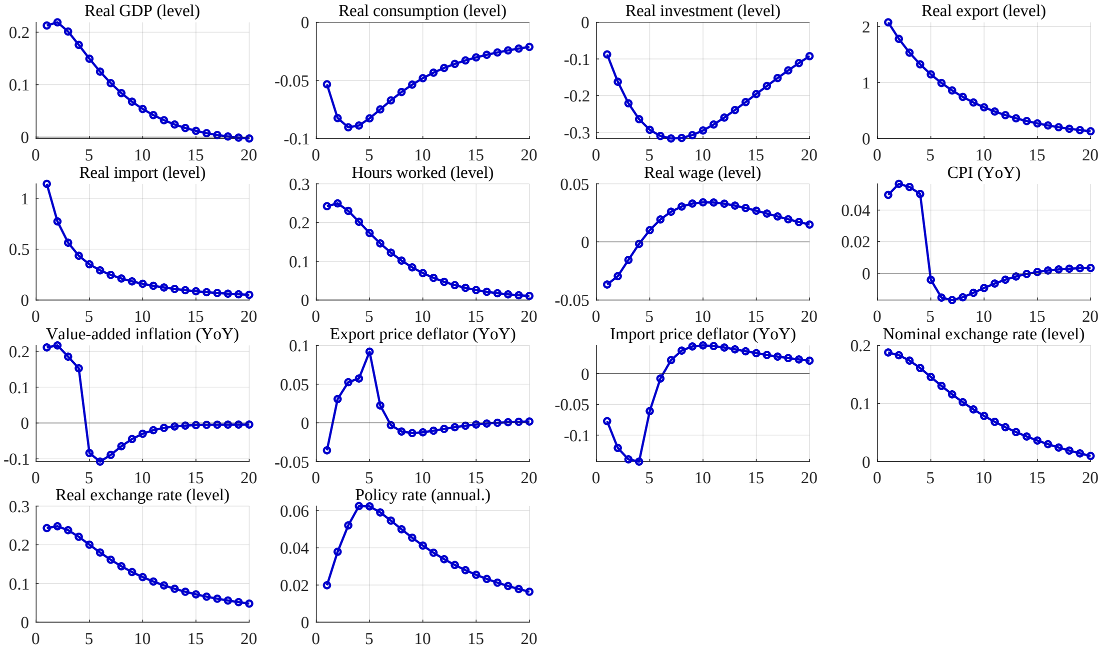
Notes: The charts show the mean responses to a one standard deviation shock. Responses are measured as percentage changes from the steady state for variables expressed in levels, or as percentage points for variables expressed in year-on-year terms and for the policy rate.

Figure C.21: WORLD TRADE SHOCK



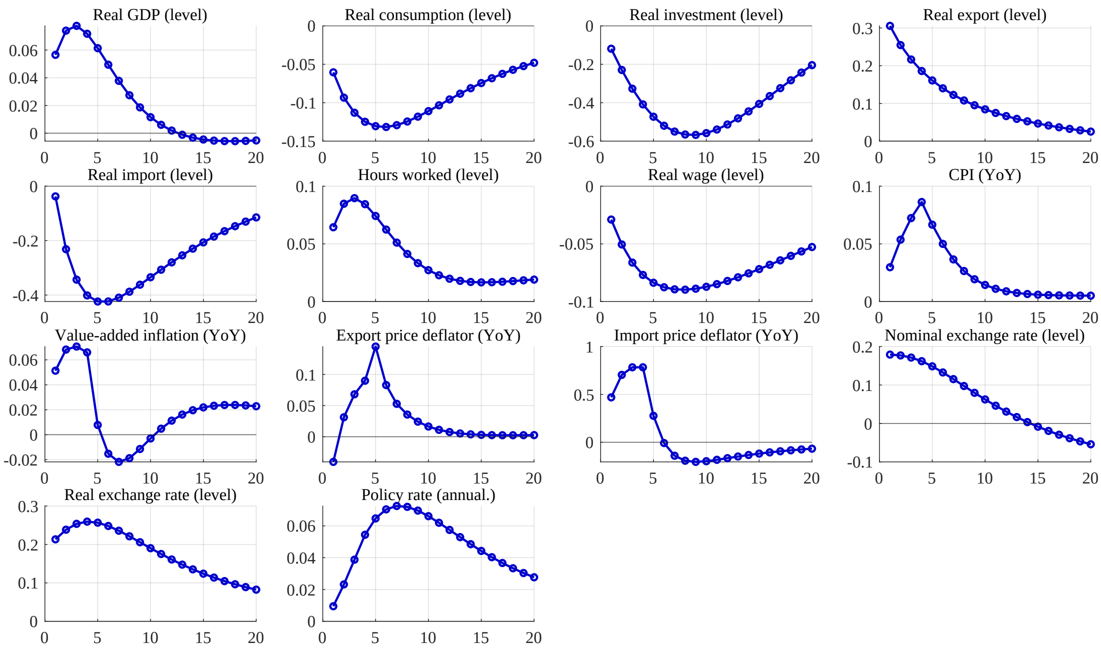
Notes: The charts show the mean responses to a one standard deviation shock. Responses are measured as percentage changes from the steady state for variables expressed in levels, or as percentage points for variables expressed in year-on-year terms and for the policy rate.

Figure C.22: WORLD PREFERENCE FOR UK EXPORTS SHOCK



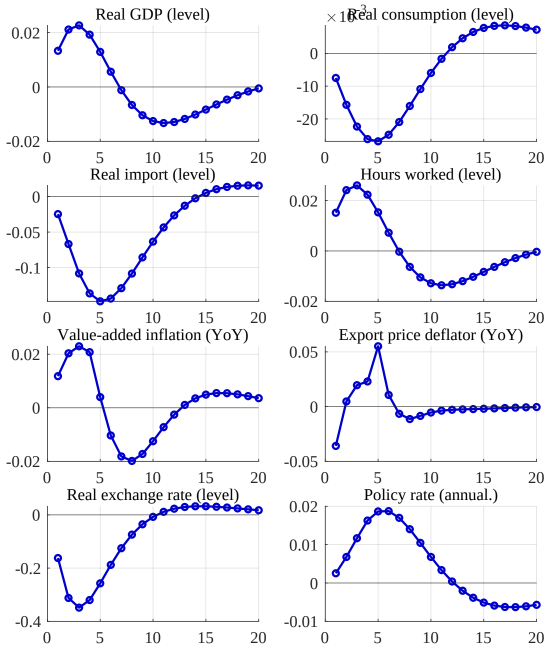
Notes: The charts show the mean responses to a one standard deviation shock. Responses are measured as percentage changes from the steady state for variables expressed in levels, or as percentage points for variables expressed in year-on-year terms and for the policy rate.

Figure C.23: WORLD EXPORT PRICE SHOCK



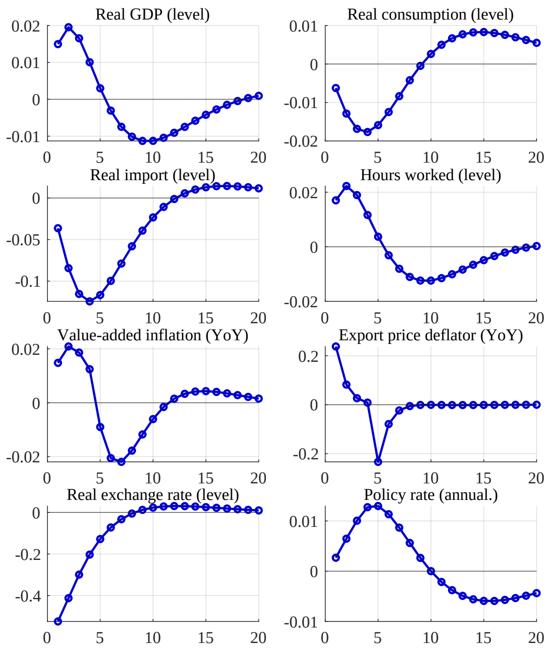
Notes: The charts show the mean responses to a one standard deviation shock. Responses are measured as percentage changes from the steady state for variables expressed in levels, or as percentage points for variables expressed in year-on-year terms and for the policy rate.

Figure C.24: WORLD PRICE MARKUP SHOCK



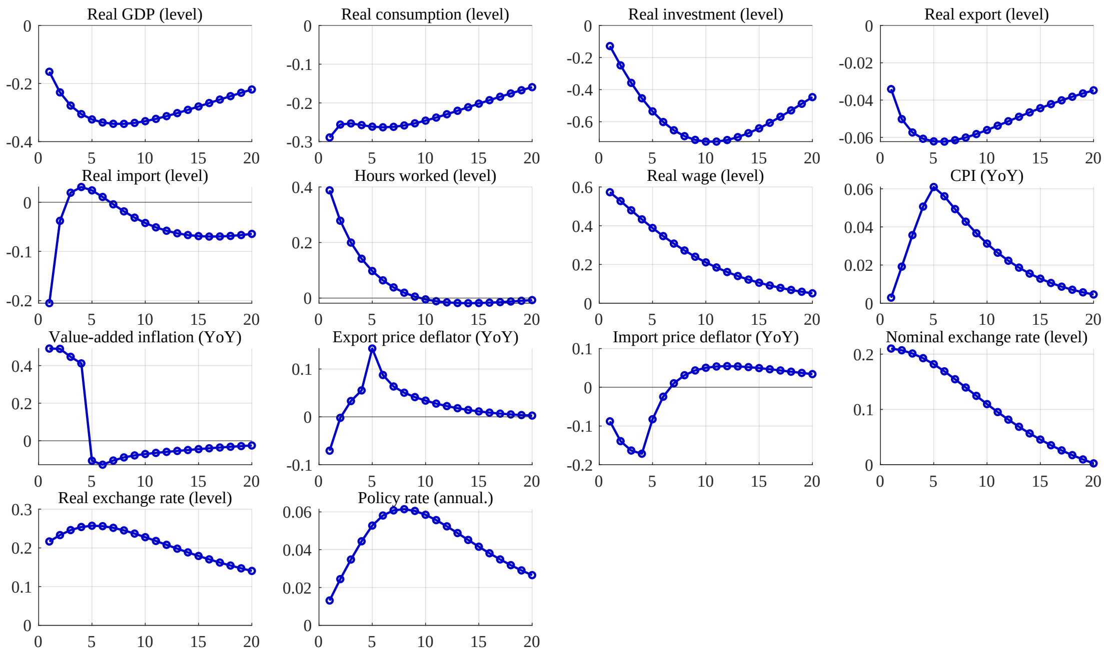
Notes: The charts show the mean responses to a one standard deviation shock. Responses are measured as percentage changes from the steady state for variables expressed in levels, or as percentage points for variables expressed in year-on-year terms and for the policy rate.

Figure C.25: WORLD MONETARY POLICY SHOCK



Notes: The charts show the mean responses to a one standard deviation shock. Responses are measured as percentage changes from the steady state for variables expressed in levels, or as percentage points for variables expressed in year-on-year terms and for the policy rate.

Figure C.26: RELATIVE PRODUCTIVITY SHOCK



Notes: The charts show the mean responses to a one standard deviation shock. Responses are measured as percentage changes from the steady state for variables expressed in levels, or as percentage points for variables expressed in year-on-year terms and for the policy rate.